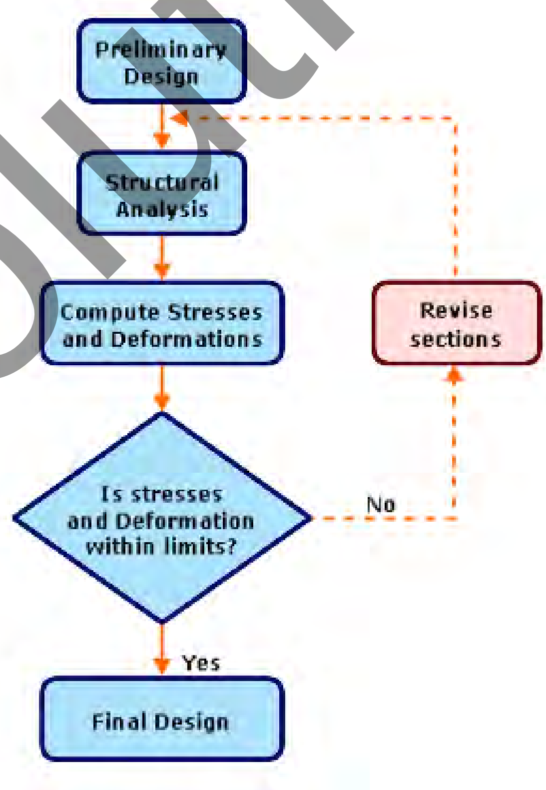
Introduction

A structure can be defined as a body which can resist the applied loads without appreciable deformations. Civil engineering structures are created to serve some specific functions like human habitation, transportation, bridges, storage etc. in a safe and economical way. A structure is an assemblage of individual elements like pin ended elements (truss elements), beam element, column, shear wall slab cable or arch. Structural engineering is concerned with the planning, designing and the construction of structures. Structural analysis involves the determination of the forces and displacements of the structures or components of a structure. Design process involves the selection and or detailing of the components that make up the structural system. The cyclic process of analysis and design is illustrated in the flow chart given below.

Flow Chart



Objectives

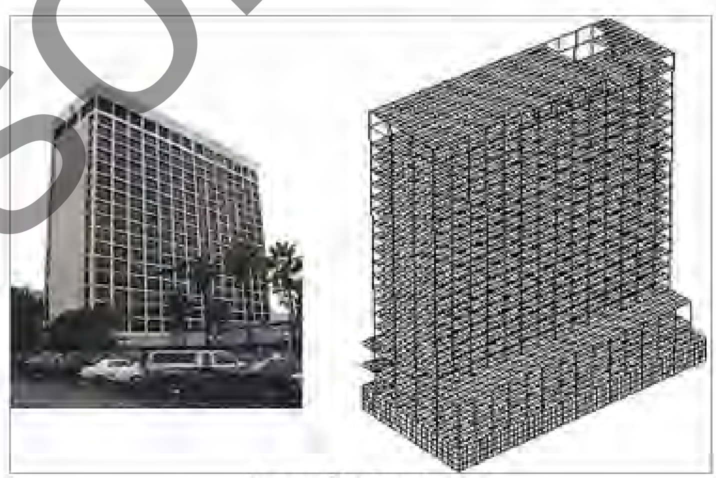
After completing this chapter you will be able to:

- Identify the different forms of structures
- Compute Static and Kinematic Indeterminacy of Structures
- Learn about linear and non-linear structures

Forms of Structures

Engineering structure is an assemblage of individual members. Assemblage of members forming a frame to support the forces acting is called framed structure. Assemblage of continuous members like flat plates, curved members etc., are called continuous system. Buildings, bridges, transmission towers, space crafts, aircrafts etc., are idealized as framed structures. Shells, domes, plates, retaining walls, dams, cooling towers etc., are idealized as continuous systems.

A frame work is the skeleton of the complete structure. This frame work supports all intended loads safely and economically. Continuous system structures transfers loads through the in-plane or membrane action to the boundaries. The images given below illustrates framed structure and continuous system.



Framed Structure

Actual structure is generally converted to simple single line structures and this process is called idealization of structures. The idealization consists of identifying the members of structure as well known individual structural elements. This process requires considerable experience and judgment. Structural analyst may be required to idealize the structure as one or more of following.

Continuous System

I. Real structure
II. A physical model
III. A mathematical model

I. Real structure

In a real structure the response of the structure is studied under the actual forces like gravity loads and lateral loads. The load test is performed using elaborate loading equipment. Strains and deformations of structural elements under loads are measured. This is very expensive and time consuming procedure, hence performed in only exceptional cases. Load testing carried out on a slab system is illustrated in the figure given below.

Sand Bag Loading Ceiling View of Filler (Tile) Slab

Load Test on Slab

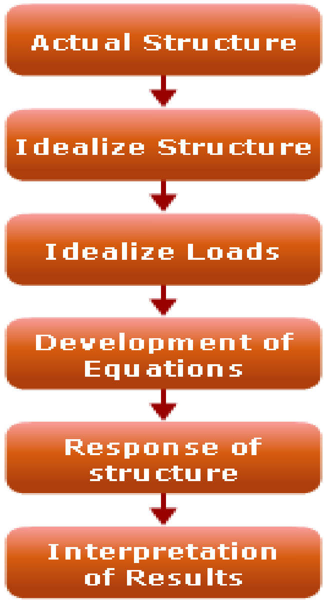
II. A physical model

Physical models which are scaled down and made up of plastic, metal or other suitable materials are used to study the response of structure under loading. These models are tested in laboratories. This study requires special techniques and is expensive. This study is carried out under compelling circumstances. Examples includes laboratory testing of small scale building frames, shake table test of bridges and building, photo elastic testing of a dam model, wind tunnel testing of small scale models of high rise buildings, towers or chimneys. Fig. 1.5 shows testing of a slab model under uniformly distributed load.

Testing of a Model

III. A mathematical model

A mathematical model is the development of mathematical equations. These equations describe the structure loads and connections. Equations are then solved using suitable algorithm. These solutions generally require electronic computers. The process of mathematical modeling is shown in block diagram given below.



A structure is generally idealized as either two dimensional structure (Plane frame) or as three dimensional structure (Space frame). The selection of idealization depends on the desire and experience of structural engineer. A two dimensional structure or a plane frame structure is that which has all members and forces are in one plane. Space frame or a three dimensional structure has members and forces in different planes. All structures in practice are three dimensional structures. However, analyst finds more convenient to analyze a plane structure rather than a space structure. This image shows two dimensional and three dimensional structures used in mathematical modeling.

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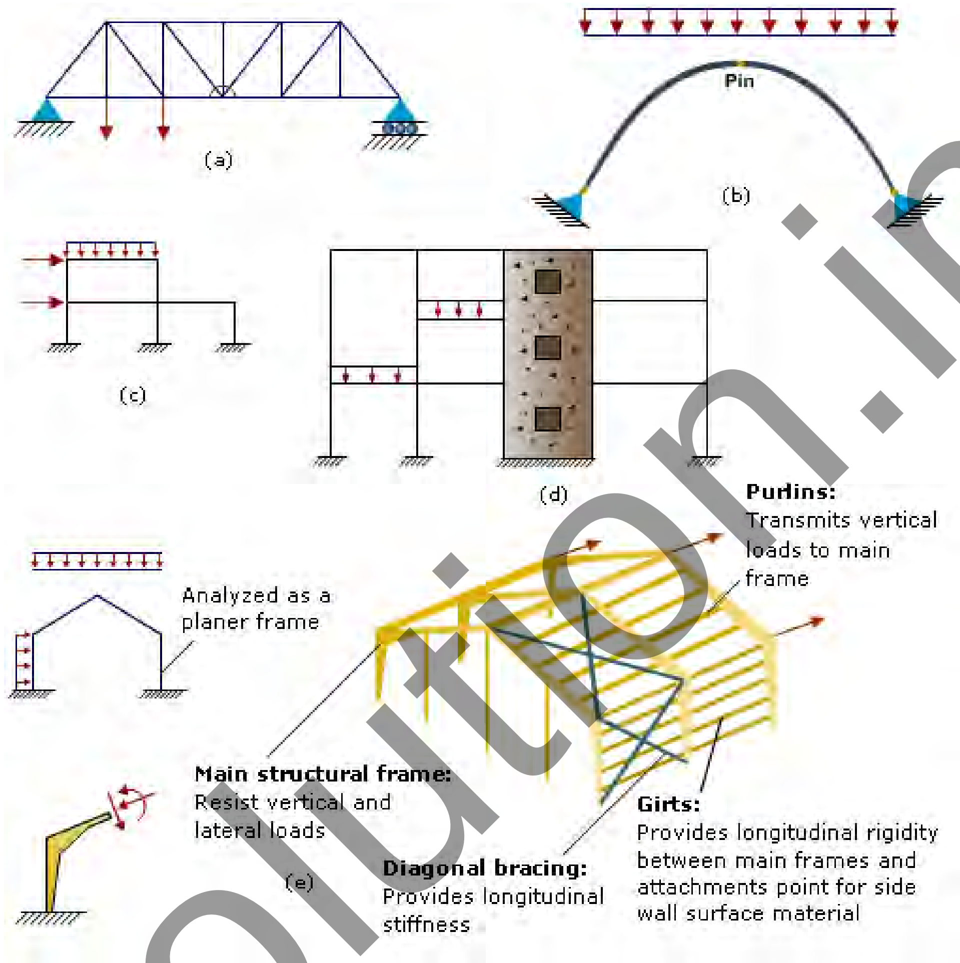
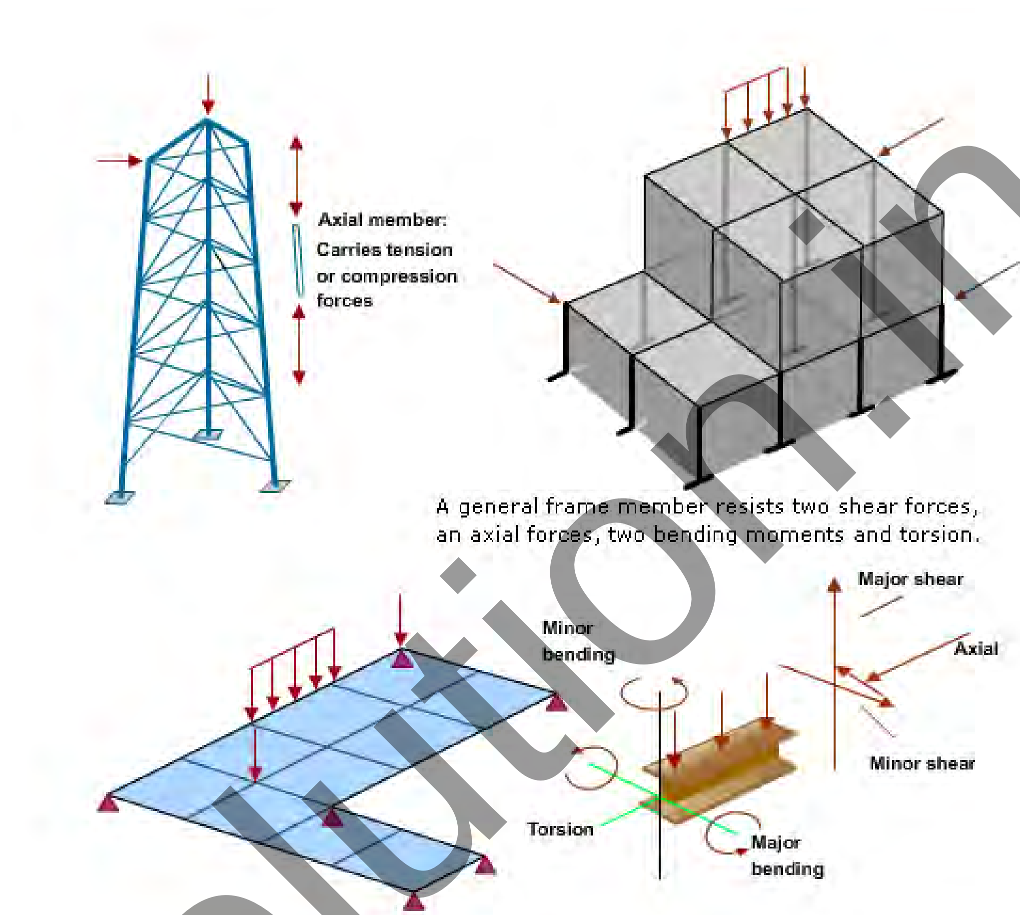
Two Dimensional

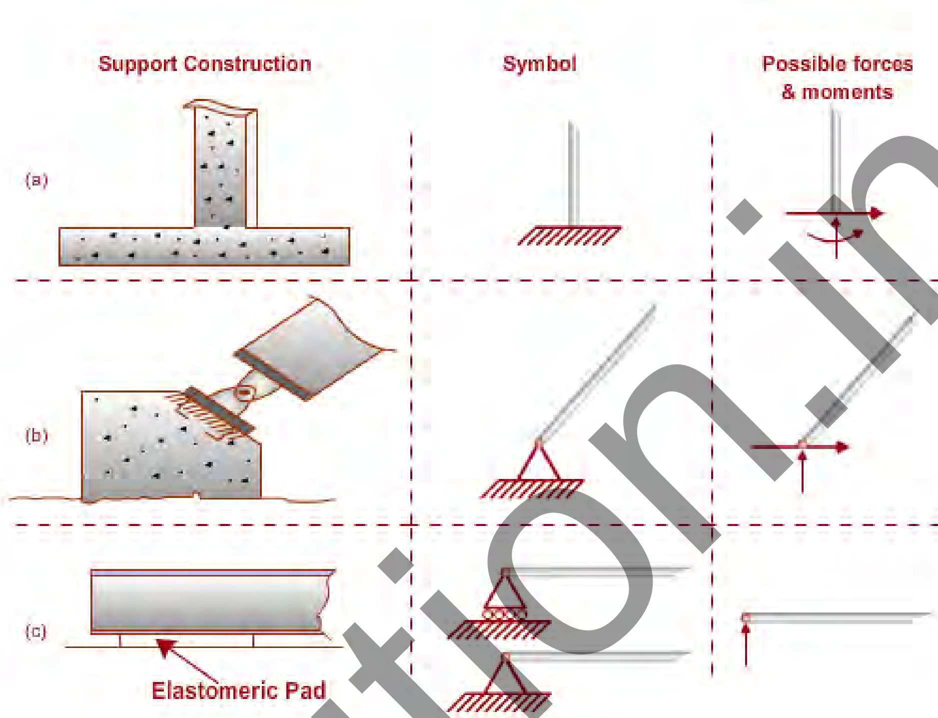


Three Dimensional



A mathematical modeling should also idealize the supports of the structure. Roller supports or simple supports, pinned supports or hinged supports and fixed supports are generally assumed type of supports in practice. In the figure given below shows the different types of supports. In a roller support the reaction is perpendicular to the surface of the roller. Two components of reaction are developed in hinged support and three reaction component, one moment and two forces parallel to horizontal and vertical axis are developed in fixed support.

Typical Support Conditions



Mathematical modeling requires to consider the loads acting on structure. Determination of the loads acting on the structure is often difficult task. Minimum loading guidance exists in codes and standards. Bureau of Indian standards, Indian road congress and Indian railways have published loading standards for building, for roads and for railway bridges respectively. Loads are generally modeled as concentrated point loads, line loads or surface loads. Loads are divided into two groups viz., dead loads and live loads. Dead loads are the weight of structural members, where as live loads are the forces that are not fixed. Snow loads, Wind loads, Occupancy loads, Moving vehicular loads, Earth quake loads, Hydrostatic pressure, earth pressure, temperature and fabrication errors are the live loads. All the live loads may not act on the structure simultaneously. Judgment of analyst on this matter is essential to avoid high loads.

Conditions of Equilibrium and Static Indeterminacy

A body is said to be under static equilibrium, when it continues to be under rest after application of loads. During motion, the equilibrium condition is called dynamic equilibrium. In two dimensional system, a body is in equilibrium when it satisfies following equation.

$\sum Fx=0$; $\sum Fy=0$; $\sum Mo=0$ ---1.1

To use the equation 1.1, the force components along x and y axes are considered. In three dimensional system equilibrium equations of equilibrium are

$\sum Fx=0$; $\sum Fy=0$; $\sum Fz=0$;

$\sum Mx=0$; $\sum My=0$; $\sum Mz=0$; ---1.2

To use the equations of equilibrium (1.1 or 1.2), a free body diagram of the structure as a whole or of any part of the structure is drawn. Known forces and unknown reactions with assumed direction is shown on the sketch while drawing free body diagram. Unknown forces are computed using either equation 1.1 or 1.2

Before analyzing a structure, the analyst must ascertain whether the reactions can be computed using equations of equilibrium alone. If all unknown reactions can be uniquely determined from the simultaneous solution of the equations of static equilibrium, the reactions of the structure are referred to as statically determinate. If they cannot be determined using equations of equilibrium alone then such structures are called statically indeterminate structures. If the number of unknown reactions are less than the number of equations of equilibrium then the structure is statically unstable.

The degree of indeterminacy is always defined as the difference between the number of unknown forces and the number of equilibrium equations available to solve for the unknowns. These extra forces are called redundants. Indeterminacy with respect external forces and reactions are called externally indeterminate and that with respect to internal forces are called internally indeterminate.

A general procedure for determining the degree of indeterminacy of two-dimensional structures are given below:

NUK = Number of unknown forces

NEQ = Number of equations available

IND = Degree of indeterminacy

IND = NUK – NEQ

Indeterminacy of Planar Frames

For entire structure to be in equilibrium, each member and each joint must be in equilibrium
(See the Figure given below)

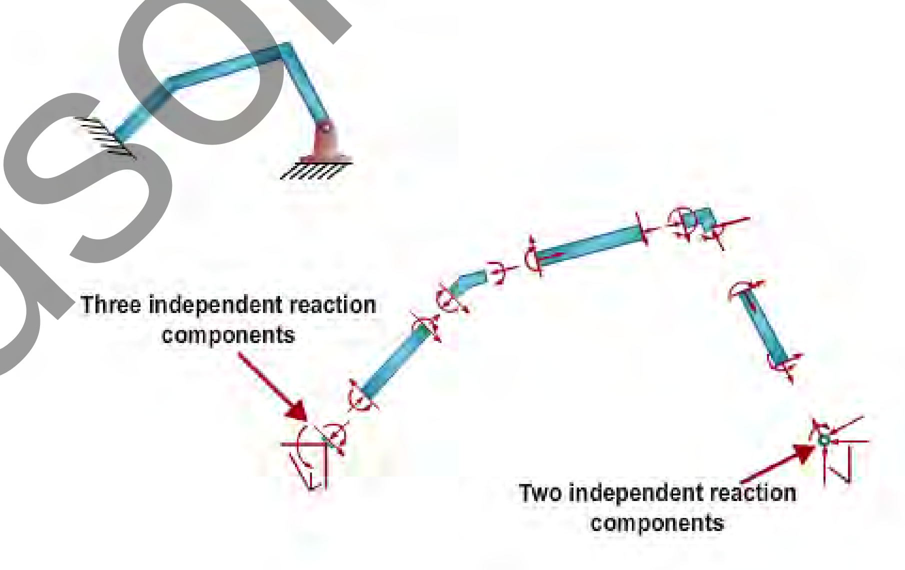
NEQ = 3NM+3NJ

NUK = 6NM+NR

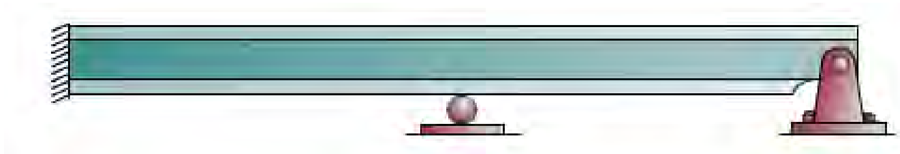
IND = NUK – NEQ = (6NM+NR)-(3NM+3NJ)

IND = 3NM+NR-3NJ ----- 1.3

Free body diagram of Members and Joints



Degree of Indeterminacy is reduced due to introduction of internal hinge



NC = Number of additional conditions

NEQ = 3NM+3NJ+NC

NUK = 6NM+NR

IND = NUK-NEQ = 3NM+NR-3NJ-NC -----1.3a

Indeterminacy of Planar Trusses

Members carry only axial forces

NEQ = 2NJ

NUK = NM+NR

IND = NUK – NEQ

IND = NM + NR - 2NJ -----1.4

Indeterminacy of 3D FRAMES

A member or a joint has to satisfy 6 equations of equilibrium

NEQ = 6NM + 6NJ-NC

NUK = 12NM + NR

IND = NUK – NEQ

IND = 6NM + NR - 6NJ - NC ----- 1.5

Indeterminacy of 3D Trusses

A joint has to satisfy 3 equations of equilibrium

NEQ = 3NJ

NUK = NM+NR

IND = NUK – NEQ

IND = NM + NR - 3NJ -----1.6

Stable Structure

Another condition that leads to a singular set of equations arises when the body or structure is improperly restrained against motion. In some instances, there may be an adequate number of support constraints, but their arrangement may be such that they cannot resist motion due to applied load. Such situation leads to instability of structure. A structure may be considered as externally stable and internally stable.

Externally Stable

Supports prevents large displacements

No. of reactions ≥ No. of equations

Internally Stable

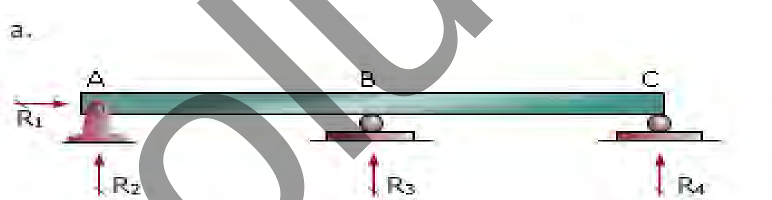
Geometry of the structure does not change appreciably

For a 2D truss $NM \geq 2Nj - 3$ $(NR \geq 3)$

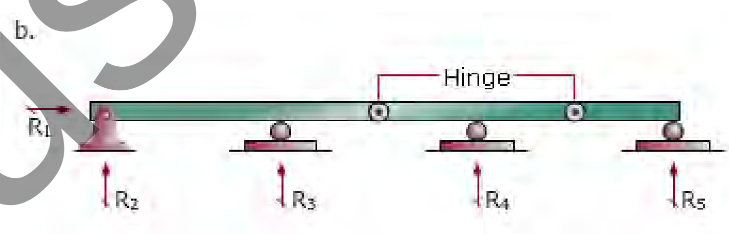
For a 3D truss $NM \geq 3Nj - 6$ $(NR \geq 3)$

Examples

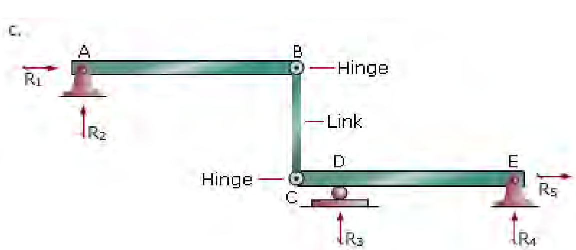
Determine Degrees of Statical indeterminacy and classify the structures



NM =2;NJ=3;NR=4;NC=0
IND=3NM+NR-3NJ-NC
IND=3x2+4-3x3-0=1
INDETERMINATE

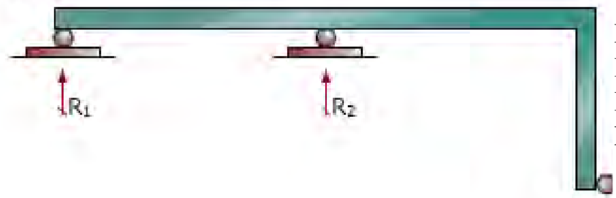


NM =3;NJ=4;NR=5; NC=2
IND=3NM+NR-3NJ-NC
IND=3x3+5-3x4-2=0
DETERMINATE



NM =3;NJ=4;NR=5;NC=2
IND=3NM+NR-3NJ-NC
IND=3x3+5-3x4-2=0
DETERMINATE

d.



NM =3; NJ=4; NR=3; NC=0
IND=3NM+NR-3NJ-NC
IND=3x3+3-3x4-0=0
DETERMINATE

e.

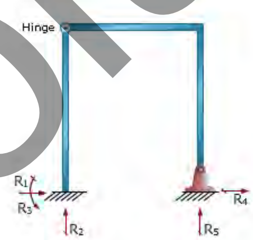
NM =1;NJ=2;NR=6;NC=2
IND=3NM+NR-3NJ-NC
IND=3x1+6-3x2-2=1
INDETERMINATE

f.

NM =1;NJ=2;NR=5;NC=1

IND=3NM+NR-3NJ-NC
IND=3x1+5-3x2-1=1
INDETERMINATE

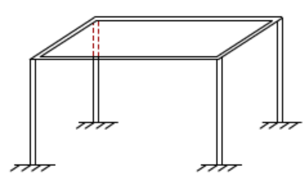
g.



NM =1;NJ=2;NR=5;NC=1

IND=3NM+NR-3NJ-NC
IND=3x1+5-3x2-1=1
INDETERMINATE

h. Each support has 6 reactions



NM =8;NJ=8;NR=24;NC=0

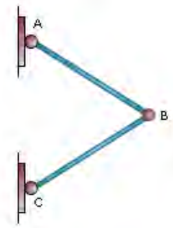
IND=6NM+NR-6NJ-NC
IND=6x8+24-6x8-0=24
INDETERMINATE

i. **Each support has 3 reactions**

NM =18;NJ=15;NR=18;NC=0

IND=6NM+NR-6NJ-NC
IND=6x18+18-6x15=32
INDETERMINATE

j. Each support has 3 reactions

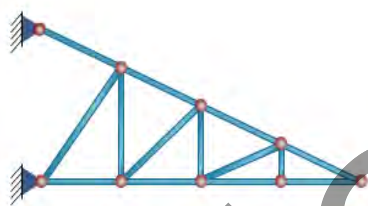


Truss

NM =2;NJ=3;NR=4;

IND=NM+NR-2NJ
IND=2+4-2x3=0
DETERMINATE

k.

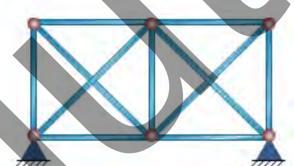


Truss

NM =14;NJ=9;NR=4;

IND=NM+NR-2NJ
IND=14+4-2x9=0
DETERMINATE

l.



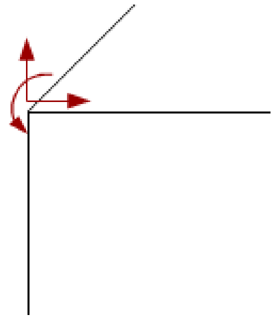
Truss

NM =11;NJ=6;NR=4;

IND=NM+NR-2NJ
IND=11+4-2x6=3
INDETERMINATE

Degree of freedom or Kinematic Indeterminacy

Members of structure deform due to external loads. The minimum number of parameters required to uniquely describe the deformed shape of structure is called "**Degree of Freedom**". Displacements and rotations at various points in structure are the parameters considered in describing the deformed shape of a structure. In framed structure the deformation at joints is first computed and then shape of deformed structure. Deformation at intermediate points on the structure is expressed in terms of end deformations. At supports the deformations corresponding to a reaction is zero. For example hinged support of a two dimensional system permits only rotation and translation along x and y directions are zero. Degree of freedom of a structure is expressed as a number equal to number of free displacements at all joints. For a two dimensional structure each rigid joint has three displacements as shown in Fig.



In case of three dimensional structure each rigid joint has six displacement.

Expression for degrees of freedom

2D Frames: NDOF = 3NJ – NR   NR ≥ 3

3D Frames: NDOF = 6NJ – NR   NR ≥ 6

2D Trusses: NDOF = 2NJ – NR   NR ≥ 3

3D Trusses: NDOF = 3NJ – NR   NR ≥ 6

Where, NDOF is the number of degrees of freedom

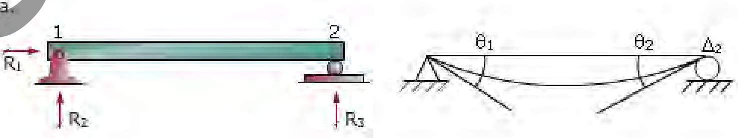
In 2D analysis of frames some times axial deformation is ignored.

Then NAC=No. of axial condition is deducted from NDOF.

Examples

1.2 Determine Degrees of Kinematic Indeterminacy of the structures given below

a.



Extensible

Nj=2; NR=3;

INDOF=3NJ-NR

INDOF=3x2−3=3(θ_1, θ_2, Δ_2)

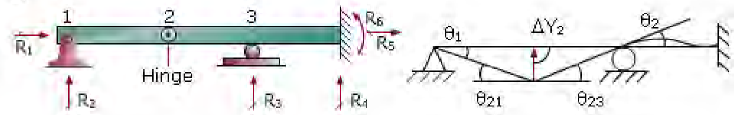
Inextensible

Nj=2; NR=3;NAC=1

INDOF=3NJ-NR-NAC

INDOF=3x2-3-1=2(θ_1, θ_2)

b.



Extensible

Nj=4; NR=5;

INDOF=3NJ-NR

INDOF=3x4-5=7 ($\theta_1,\theta_{21},\theta_{23}\theta_3,\Delta_{y2},e_1,e_2$)

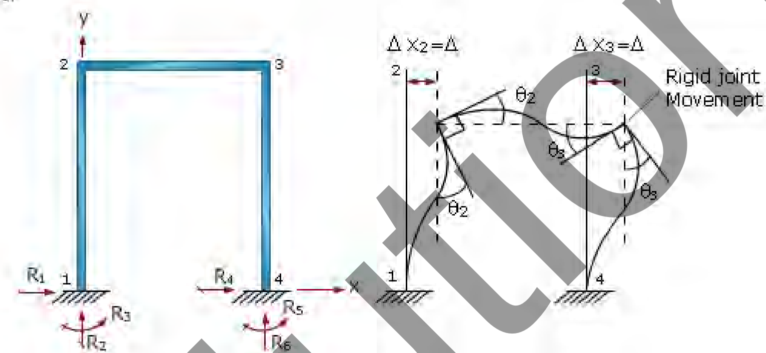
Inextensible

Nj=4; NR=5;NAC=2

INDOF=3NJ-NR-NAC

INDOF=3x4-5-2=5($\theta_1,\theta_{21},\theta_{23}\theta_3,\Delta_{y2}$)

c.



Portal Frame Deflected shape

Extensible

Nj=4; NR=6;

INDOF=3NJ-NR

INDOF=3x4-6=6($\theta_2,\theta_3,\Delta,e_1,e_2,e_3$)

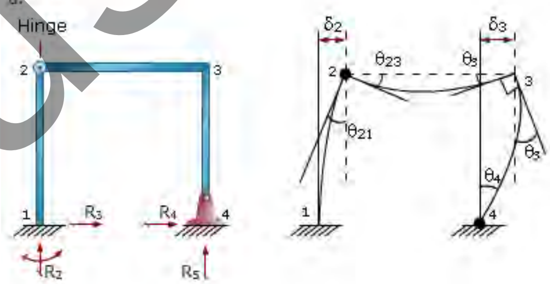
Inextensible

Nj=4; NR=6;NAC=3

INDOF=3NJ-NR-NAC

INDOF=3x4-6-3=3(θ_2,θ_3,Δ)

d.



Extensible

NJ=4; NR=5;

INDOF=3NJ-NR

INDOF=3x4-5+1=8($\theta_{21},\theta_{23},\theta_3,\delta_2,\delta_3,e_1,e_2,e_3$)

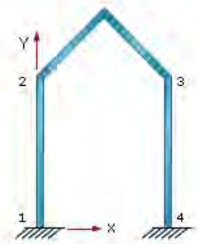
Inextensible

NJ=4; NR=5;NAC=3

INDOF=3NJ-NR-NAC

INDOF=3x4-5-3=4($\theta_{21},\theta_{23},\theta_3,\delta_2=\delta_3=\Delta$)

e.



Extensible

Nj=5; NR=6;

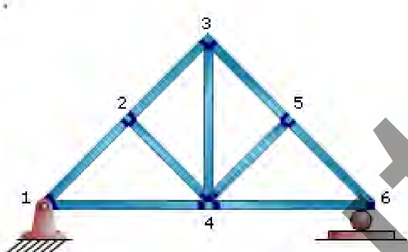
INDOF=3NJ-NR
INDOF=3x5−6=9($\theta_2,\theta_3,\theta_4,\delta_{x2},\delta_{x3},\delta_{y3},\delta_{x4}$e1,e4)

Inextensible

Nj=5; NR=6;NAC=3

INDOF=3NJ-NR-NAC
INDOF=3x5-6-3=6($\theta_2,\theta_3,\theta_4,\Delta_{x2}=\Delta_{x4}=\Delta_1,\Delta_{x3},\Delta_{y3}$)

f.

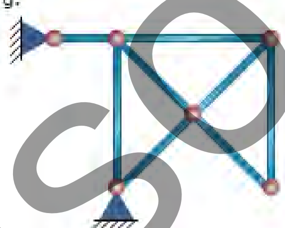


A Truss

Nj=6; NR=3;

INDOF=2NJ-NR
INDOF=2x6-3=9

g.



A Truss

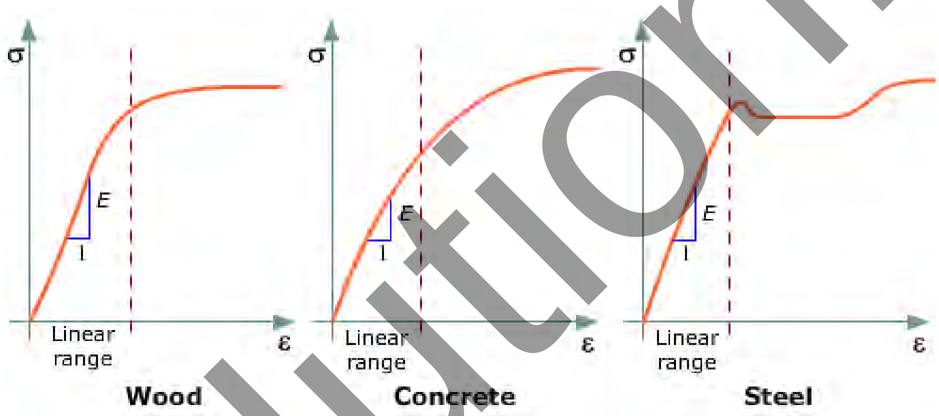
Nj=6; NR=4;

INDOF=2NJ-NR
INDOF=2x6-4=8

Linear and Non Linear Structures

Structural frameworks are commonly made of wood, concrete or steel. Each of them has different material properties that must be accounted for in the analysis and design. The modulus of elasticity E of each material must be known for any displacement computation. Typical stress-strain curve for these materials is shown in Fig.1.11. The structure in which the stresses developed is within the elastic limit, and then the structure is called **Linear Structure**. If the stress developed is in the plastic region, then the structure is said to **Non-Linear Structure**. In addition to material nonlinearities, some structures may behave in a nonlinear fashion due to change in the shape of the overall structure. This requires that the structure displace an amount significant enough to affect the equilibrium relations for the structure. When this occurs the structure is said to be **Geometrically nonlinear**. Cable structures are susceptible to this type of nonlinearity. A cantilever structure shown in Fig. 1.2 has geometrical nonlinearity

Stress-Strain Graph



Geometric Nonlinearity