HEAT AND MASS TRANSFER

Subject Code : 10ME63                IA Marks : 25
Hours/Week : 04                    Exam Hours : 03
Total Hours : 52                  Exam Marks : 100

PART – A

UNIT - 1
Introductory Concepts and Definitions:
Modes of heat transfer: Basic laws governing conduction, convection, and radiation heat transfer;
Thermal conductivity; convective heat transfer coefficient; radiation heat transfer; combined heat
transfer mechanism. Boundary conditions of 1st, 2nd and 3rd kind
Conduction: Derivation of general three dimensional conduction equation in Cartesian coordinate,
special cases, discussion on 3-D conduction in cylindrical and spherical coordinate systems (No
derivation). One dimensional conduction equations in rectangular, cylindrical and spherical coordinates
for plane and composite walls. Overall heat transfer coefficient. Thermal contact resistance.
07 Hours

UNIT - 2
Variable Thermal Conductivity:
Derivation for heat flow and temperature distribution in plane wall. Critical thickness of insulation
without heat generation, Thermal resistance concept & its importance. Heat transfer in extended
surfaces of uniform cross-section without heat generation. Long fin, short fin with insulated tip and
without insulated tip and fin connected between two heat sources. Fin efficiency and effectiveness.
Numerical problems.
06 Hours

UNIT - 3
One-Dimensional Transient Conduction:
Conduction in solids with negligible internal temperature gradient (Lumped system analysis), Use of
Transient temperature charts (Heisler’s charts) for transient conduction in slab, long cylinder and
sphere; use of transient temperature charts for transient conduction in semi-infinite solids. Numerical
Problems.
06 Hours

UNIT - 4
Concepts And Basic Relations In Boundary Layers:
Flow over a body velocity boundary layer; critical Reynolds number; general expressions for drag
coefficient and drag force; thermal boundary layer; general expression for local heat transfer
coefficient; Average heat transfer coefficient; Nusselt number. Flow inside a duct- velocity boundary
layer, hydrodynamic entrance length and hydro dynamically developed flow; flow through tubes
(internal flow discussion only). Numerical based on empirical relation given in data
Handbook.
Free Or Natural Convection: Application of dimensional analysis for free convection- physical
significance of Grashoff number; use of correlations of free convection in vertical, horizontal and
inclined flat plates, vertical and horizontal cylinders and spheres, Numerical problems.
07 Hours

PART – B

UNIT - 5
Forced Convections:
Applications of dimensional analysis for forced convection. Physical significance of Reynolds, Prandtl,
Nusselt and Stanton numbers. Use of various correlations for hydro dynamically and thermally
developed flows inside a duct, use of correlations for flow over a flat plate, over a cylinder and sphere.
Numerical problems.
06 Hours
UNIT - 6
Heat Exchangers:
Classification of heat exchangers; overall heat transfer coefficient, fouling and fouling factor; LMTD, Effectiveness-NTU methods of analysis of heat exchangers. Numerical problems. 06 Hours

UNIT - 7
Condensation and Boiling:
Types of condensation (discussion only) Nusselt’s theory for laminar condensation on a vertical flat surface; use of correlations for condensation on vertical flat surfaces, horizontal tube and horizontal tube banks; Reynolds number for condensate flow; regimes of pool boiling, pool boiling correlations. Numerical problems. Mass transfer definition and terms used in mass transfer analysis, Ficks First law of diffusion (no numerical). 07 Hours

UNIT - 8
Radiation Heat Transfer:
Thermal radiation; definitions of various terms used in radiation heat transfer; Stefan-Boltzman law, Kirchoff’s law, Planck’s law and Wein’s displacement law. Radiation heat exchange between two parallel infinite black surfaces, between two parallel infinite gray surfaces; effect of radiation shield; intensity of radiation and solid angle; Lambert’s law; radiation heat exchange between two finite surfaces configuration factor or view factor. Numerical problems. 07 Hours

TEXT BOOKS:

REFERENCE BOOKS:
1. Heat transfer, a practical approach, Yunus A- Cengel Tata Mc-Graw Hill
2. Principles of heat transfer, Kreith Thomas Learning 2001
DEPARTMENT OF MECHANICAL ENGINEERING

HEAT AND MASS TRANSFER NOTES
(10ME63)

UNIT-1
INTRODUCTORY CONCEPTS AND DEFINITIONS
1.1 Introduction

“Heat is a form of energy that can be transferred from one system to another as a result of temperature difference” is known as heat transfer. Larger the temperature gradient, higher the rate of heat transfer also heat transfer takes place from higher temperature region to lower temperature region. Heat cannot be measured or observed directly, but the effect it produces is amenable to observation and measurement.

1.2 Difference between heat and temperature

<table>
<thead>
<tr>
<th>Heat</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition</strong></td>
<td><strong>Definition</strong></td>
</tr>
<tr>
<td>Heat is energy that is transferred from one body to another as the result of a difference in temperature.</td>
<td>Temperature is a measure of hotness or coldness expressed in terms of any of several arbitrary scales like Celsius and Fahrenheit.</td>
</tr>
<tr>
<td><strong>Symbol</strong></td>
<td><strong>Symbol</strong></td>
</tr>
<tr>
<td>Q</td>
<td>T</td>
</tr>
<tr>
<td><strong>Unit</strong></td>
<td><strong>Unit</strong></td>
</tr>
<tr>
<td>Joules</td>
<td>Kelvin, Celsius or Fahrenheit</td>
</tr>
<tr>
<td><strong>SI unit</strong></td>
<td><strong>SI unit</strong></td>
</tr>
<tr>
<td>Joule</td>
<td>Kelvin</td>
</tr>
<tr>
<td><strong>Particles</strong></td>
<td><strong>Particles</strong></td>
</tr>
<tr>
<td>Heat is a measure of how many atoms there are in a substance multiplied by how much energy each atom possesses.</td>
<td>Temperature is related to how fast the atoms within a substance are moving. The ‘temperature’ of an object is like the water level – it determines the direction in which ‘heat’ will flow.</td>
</tr>
<tr>
<td><strong>Ability to do work</strong></td>
<td><strong>Ability to do work</strong></td>
</tr>
<tr>
<td>Heat has the ability to do work.</td>
<td>Temperature can only be used to measure the degree of heat.</td>
</tr>
</tbody>
</table>

1.3 Difference between thermodynamics and heat transfer

<table>
<thead>
<tr>
<th><em>Thermodynamics tells us:</em></th>
<th><em>Heat transfer tells us:</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>• How much heat is transferred ((\delta Q))</td>
<td>• How (with what <strong>modes</strong>) (\delta Q) is transferred</td>
</tr>
<tr>
<td>• How much work is done ((\delta W))</td>
<td>• At what <strong>rate</strong> (\delta Q) is transferred</td>
</tr>
<tr>
<td>• Final state of the system</td>
<td>• Temperature distribution inside the body</td>
</tr>
</tbody>
</table>
1.4 Modes of heat transfer

**Conduction:** Heat conduction is a mechanism of heat transfer from a region of high temperature to a region of low temperature within a medium or between different medium in direct physical contact. Examples: Heating a Rod.

**Convection:** It is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures. It is possible only in the presence of fluid medium. Example: Cooling of Hot Plate by air.

**Radiation:** The heat transfer from one body to another without any transmitting medium. It is an electromagnetic wave phenomenon. Example: Radiation sun to earth.

1.5 Basic laws of heat transfer governing conduction

**Basic law of governing conduction:** This law is also known as Fourier’s law of conduction.

The rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temperature gradient in that direction.

\[ q = k \frac{A}{L} \left( T_i - T_e \right) \]
Where, \( A \) – Area in \( \text{m}^2 \)

\( \nabla \) – Temperature gradient, \( \text{K} / \text{m} \)

\( \lambda \) – Thermal conductivity, \( \text{W} / \text{m} \text{ K} \)

**Basic law of governing convection:** This law is also known as **Newton’s law of convection.**

An energy transfer across a system boundary due to a temperature difference by the combined mechanisms of intermolecular interactions and bulk transport. Convection needs fluid matter.

Newton’s Law of Cooling:

\[
q = h A (T_s - T_i)
\]

Where:

- \( q \) = heat flow from surface, a scalar, (W)
- \( h \) = heat transfer coefficient (which is not a thermodynamic property of the material, but may depend on geometry of surface, flow characteristics, thermodynamic properties of the fluid, etc. (W/m\(^2\) K)
- \( A \) = Surface area from which convection is occurring. (m\(^2\))
- \( T_s \) = Temperature Difference between surface and coolant. (K)

**Basic law of governing convection:** This law is also known as **Stefan Boltzman law.**

According to the Stefan Boltzman law the radiation energy emitted by a body is proportional to the fourth power of its absolute temperature and its surface area.
Where:
\( \varepsilon = \) Surface Emissivity
\( \sigma = \) Steffan Boltzman constant
\( A = \) Surface Area
\( T_s = \) Absolute temperature of surface. (K)
\( T_{sur} = \) Absolute temperature of surroundings. (K)

**Thermal conductivity:** Thermal conductivity is a thermodynamic property of a material “the amount of energy conducted through a body of unit area and unit thickness in unit time when the difference in temperature between faces causing heat flow is unit temperature difference”.

### 1.6 Derivation of general three dimensional conduction equation in Cartesian coordinate

Consider a small rectangular element of sides \( dx, dy \) and \( dz \) as shown in figure. The energy balance of this rectangular element is obtained from first law of thermodynamics

Consider the differential control element shown below. Heat is assumed to flow through the element in the positive directions as shown by the 6 heat vectors.
In the equation above we substitute the 6 heat inflows/outflows using the appropriate sign:

\[
\rho \cdot c_p \cdot \left( \Delta x \cdot \Delta y \cdot \Delta z \right) \cdot \frac{dT}{dt} \bigg|_{\text{q,gen}} = q_z - q_{z+\Delta z} + q_y - q_{y+\Delta y} + q_x - q_{x+\Delta x} + Q_{\text{gen}}
\]

Substitute for each of the conduction terms using the Fourier Law:

\[
\rho \cdot c_p \cdot \left( \Delta x \cdot \Delta y \cdot \Delta z \right) \cdot \frac{dT}{dt} \bigg|_{\text{q,gen}} = \left\{ -k \cdot (\Delta y \cdot \Delta z) \cdot \frac{\partial T}{\partial x} \left[ -k \cdot (\Delta y \cdot \Delta z) \cdot \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left( -k \cdot (\Delta y \cdot \Delta z) \cdot \frac{\partial T}{\partial x} \right) \cdot \Delta x \right] \right.
\]

\[
+ \left\{ -k \cdot (\Delta x \cdot \Delta z) \cdot \frac{\partial T}{\partial y} \left[ -k \cdot (\Delta x \cdot \Delta z) \cdot \frac{\partial T}{\partial y} + \frac{\partial}{\partial y} \left( -k \cdot (\Delta x \cdot \Delta z) \cdot \frac{\partial T}{\partial y} \right) \cdot \Delta y \right] \right.
\]

\[
+ \left\{ -k \cdot (\Delta x \cdot \Delta y) \cdot \frac{\partial T}{\partial z} \left[ -k \cdot (\Delta x \cdot \Delta y) \cdot \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left( -k \cdot (\Delta x \cdot \Delta y) \cdot \frac{\partial T}{\partial z} \right) \cdot \Delta z \right] \right\}
\]

where \( \bar{q} \) is defined as the internal heat generation per unit volume.

The above equation reduces to:

\[
\rho \cdot c_p \cdot \left( \Delta x \cdot \Delta y \cdot \Delta z \right) \cdot \frac{dT}{dt} \bigg|_{\text{q,gen}} = \left\{ - \left[ \frac{\partial}{\partial x} \left( -k \cdot (\Delta y \cdot \Delta z) \cdot \frac{\partial T}{\partial x} \right) \right] \cdot \Delta x \right\}
\]

### 1.7 Discussion on 3-D conduction in cylindrical and spherical coordinate systems

**Cylindrical coordinate system:**
The 3-Dimensional conduction equation in cylindrical co-ordinates is given by,

\[
\frac{1}{\alpha} \frac{\partial T}{\partial \tau} = \frac{1}{r \partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \partial \theta^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2 \sin \theta \partial \phi^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial}{\partial z} \frac{T}{k} + q
\]

Spherical coordinate systems:

The 3-Dimensional conduction equation in cylindrical co-ordinates is given by,

\[
\frac{1}{\alpha} \frac{\partial T}{\partial \tau} = \frac{1}{r \partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta \partial \theta^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2 \sin \theta \partial \phi^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial}{\partial \tau} \frac{T}{k} + q
\]

In each equation the dependent variable, \( T \), is a function of 4 independent variables, \((x,y,z,\tau); (r,\theta ,z,\tau); (r,\phi,\theta,\tau)\) and is a 2nd order, partial differential equation. The solution of such equations will normally require a numerical solution. For the present, we shall simply look at the simplifications that can be made to the equations to describe specific problems.

- **Steady State**: Steady state solutions imply that the system conditions are not changing with time. Thus \( \frac{\partial T}{\partial \tau} = 0 \).
- **One dimensional**: If heat is flowing in only one coordinate direction, then it follows that there is no temperature gradient in the other two directions. Thus the two partials associated with these directions are equal to zero.
- **Two dimensional**: If heat is flowing in only two coordinate directions, then it follows that there is no temperature gradient in the third direction. Thus the partial derivative associated with this third direction is equal to zero.
- **No Sources**: If there are no heat sources within the system then the term, \( q=0 \).
Note that the equation is 2\text{nd} order in each coordinate direction so that integration will result in 2 constants of integration. To evaluate these constants two additional equations must be written for each coordinate direction based on the physical conditions of the problem. Such equations are termed “boundary conditions”.

### 1.8 Boundary and Initial Conditions:

- The objective of deriving the heat diffusion equation is to determine the temperature distribution within the conducting body.
- We have set up a differential equation, with \( T \) as the dependent variable. The solution will give us \( T(x,y,z) \). Solution depends on boundary conditions (BC) and initial conditions (IC).
- How many BC’s and IC’s?
  - Heat equation is second order in spatial coordinate. Hence, 2 BC’s needed for each coordinate.
    - 1D problem: 2 BC in x-direction
    - 2D problem: 2 BC in x-direction, 2 in y-direction
    - 3D problem: 2 in x-dir., 2 in y-dir., and 2 in z-dir.
  - Heat equation is first order in time. Hence one IC needed.
HEAT AND MASS TRANSFER NOTES
(10ME63)

UNIT-2
VARIABLE THERMAL CONDUCTIVITY
2.1 Heat Diffusion Equation for a One Dimensional System:

Consider the system shown above. The top, bottom, front and back of the cube are insulated, so that heat can be conducted through the cube only in the x direction. The internal heat generation per unit volume is \( q \) (W/m\(^3\)).

Consider the heat flow through an arbitrary differential element of the cube.

From the 1st Law we write for the element:

\[
\left( \dot{E}_m - \dot{E}_{ext} \right) + \dot{E}_{gen} = \dot{E}_{int}
\]

\[
q_x - q_{x+\Delta x} + A_x (\Delta x) \dot{q} = \frac{\partial \dot{E}}{\partial t}
\]

\[
q_x = -kA_x \frac{\partial T}{\partial x}
\]

\[
q_{x+\Delta x} = q_x + \frac{\partial q_x}{\partial x} \Delta x
\]

\[
- kA \frac{\partial T}{\partial x} + kA \frac{\partial T}{\partial x} + \left( k \frac{\partial T}{\partial x} \right) \Delta x + A \Delta x \dot{q} = \rho c A \Delta x \frac{\partial T}{\partial t}
\]

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q} = \rho c \Delta x \frac{\partial T}{\partial t}
\]

Longitudinal conduction  Internal heat generation  Thermal inertia

If \( k \) is a constant, then

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{\rho c \partial T}{\partial t} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]
2.2 One Dimensional Steady State Heat Conduction:

The plane wall:

The differential equation governing heat diffusion is:

\[ \frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0 \]

With constant \( k \), the above equation may be integrated twice to obtain the general solution:

\[ T(x) = C_1 x + C_2 \]

where \( C_1 \) and \( C_2 \) are constants of integration. To obtain the constants of integration, we apply the boundary conditions at \( x = 0 \) and \( x = L \), in which case

\[ T(0) = T_{s,1} \quad \text{and} \quad T(L) = T_{s,2} \]

Once the constants of integration are substituted into the general equation, the temperature distribution is obtained:

\[ T(x) = \left( T_{s,2} - T_{s,1} \right) \frac{x}{L} + T_{s,1} \]

The heat flow rate across the wall is given by:

\[ q = -kA \frac{dT}{dx} = \frac{kA}{L} (T_{s,2} - T_{s,1}) = \frac{T_{s,2} - T_{s,1}}{L/kA} \]

Thermal resistance (electrical analogy):

Physical systems are said to be analogous if that obey the same mathematical equation. The above relations can be put into the form of Ohm’s law:

\[ V = IR_{\text{elec}} \]

Using this terminology it is common to speak of a thermal resistance:

\[ \Delta T = qR_{\text{therm}} \]
A thermal resistance may also be associated with heat transfer by convection at a surface. From Newton’s law of cooling,

The thermal resistance for convection is then

\[ R_{t,con} = \frac{T_1 - T_\infty}{q} = \frac{1}{hA} \]

Applying thermal resistance concept to the plane wall, the equivalent thermal circuit for the plane wall with convection boundary conditions is shown in the figure below.

The heat transfer rate may be determined from separate consideration of each element in the network. Since \( q_x \) is constant throughout the network, it follows that

\[ q_x = \frac{T_{w,1} - T_{w,2}}{1/h_1A} = \frac{T_{z,2} - T_{z,1}}{L/kA} = \frac{T_{z,2} - T_{w,2}}{1/h_2A} \]

In terms of the overall temperature difference \( T_{w,1} - T_{w,2} \) and the total thermal resistance \( R_{tot} \), the heat transfer rate may also be expressed as

\[ q_x = \frac{T_{w,1} - T_{w,2}}{R_{tot}} \]

Since the resistance are in series, it follows that

\[ R_{tot} = \sum R_i = \frac{1}{h_1A} + \frac{T}{kA} + \frac{1}{h_2A} \]
Composite walls:

Thermal Resistances in Series:
Consider three blocks, A, B and C, as shown. They are insulated on top, bottom, front and back. Since the energy will flow first through block A and then through blocks B and C, we say that these blocks are thermally in a series arrangement.

\[
q_x = \frac{T_{\infty, 1} - T_{\infty, 2}}{\sum R_i} = \frac{T_{\infty, 1} - T_{\infty, 2}}{\frac{1}{h_1 A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A} + \frac{1}{h_2 A}} = U A \Delta T
\]

where \( U = \frac{1}{R_{\text{net}} A} \) is the overall heat transfer coefficient. In the above case, \( U \) is expressed as

\[
U = \frac{1}{\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_2}}
\]

Series-parallel arrangement:
The following assumptions are made with regard to the above thermal resistance model:

1) Face between B and C is insulated.
2) Uniform temperature at any face normal to X.

2.3 1-D radial conduction through a cylinder:

One frequently encountered problem is that of heat flow through the walls of a pipe or through the insulation placed around a pipe. Consider the cylinder shown. The pipe is either insulated on the ends or is of sufficient length, L, that heat losses through the ends are negligible. Assume no heat sources within the wall of the tube. If $T_1 > T_2$, heat will flow outward, radially, from the inside radius, $R_1$, to the outside radius, $R_2$. The process will be described by the Fourier Law.

The differential equation governing heat diffusion is:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr}\right) = 0$$

With constant $k$, the solution is:

The heat flow rate across the wall is given by:

$$q_r = -k A \frac{dT}{dx} = \frac{k A}{L} (T_{r,1} - T_{r,2}) = \frac{T_{r,1} - T_{r,2}}{L/kA}$$

Hence, the thermal resistance in this case can be expressed as:

$$\frac{\ln \frac{R_1}{R_2}}{2\pi k L}$$
2.4 Critical Insulation Thickness:

Objective: decrease $q$, increase $R_{\text{Total}}$

Vary $r_o$; as $r_o$ increases, first term increases, second term decreases.

This is a maximum – minimum problem. The point of extreme can be found by setting

$$\frac{dR_{\text{net}}}{dr_o} = 0$$

or,

$$\frac{1}{2\pi r_o L} - \frac{1}{2\pi h r_o L} = 0$$

or,

$$r_o = \frac{k}{h}$$

In order to determine if it is a maxima or a minima, we make the second derivative zero:

$$\frac{d^2 R_{\text{net}}}{dr_o^2} = 0 \quad \text{at} \quad r_o = \frac{k}{h}$$

$$\frac{d^2 R_{\text{net}}}{dr_o^2} = -\frac{1}{2\pi r_o^2 L} \left[ \frac{1}{\pi^2 r_o^2 L} \right]_{r_o = \frac{k}{h}} = -\frac{h^2}{2\pi L k^2} \neq 0$$

Minimum $q$ at $r_o = (k/h) = r_c$ (critical radius)
1-D radial conduction in a sphere:

\[ \frac{1}{r^2} \frac{d}{dr} \left( kr^2 \frac{dT}{dr} \right) = 0 \]

\[ T(r) = T_{r,1} - \left\{ T_{r,2} - T_{r,1} \right\} \left[ \frac{1}{r_2} - \frac{1}{r_1} \right] \]

\[ q_r = -kA \frac{dT}{dr} = \frac{4\pi k(T_{r,1} - T_{r,2})}{(1/r_1 - 1/r_2)} \]

\[ R_{r,cond} = \frac{1}{r_1 - 1/r_2} \frac{1}{4\pi k} \]

### 2.5 Summary of Electrical Analogy:

<table>
<thead>
<tr>
<th>System</th>
<th>Current</th>
<th>Resistance</th>
<th>Potential Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartesian Conduction</td>
<td>q</td>
<td>( L ) ( kA )</td>
<td>( \Delta T )</td>
</tr>
<tr>
<td>Cylindrical Conduction</td>
<td>q</td>
<td>( \ln \frac{r_2}{r_1} ) ( 2\pi kL )</td>
<td>( \Delta T )</td>
</tr>
<tr>
<td>Conduction through sphere</td>
<td>q</td>
<td>( \frac{1}{r_1 - 1/r_2} ) ( 4\pi k )</td>
<td>( \Delta T )</td>
</tr>
<tr>
<td>Convection</td>
<td>q</td>
<td>( \frac{1}{h \cdot A_s} )</td>
<td>( \Delta T )</td>
</tr>
</tbody>
</table>

### 2.6 Heat transfer in extended surfaces of uniform cross-section without heat generation:

Convection: Heat transfer between a solid surface and a moving fluid is governed by the Newton’s cooling law: \( q = hA(T_s - T_\infty) \), where \( T_s \) is the surface temperature and \( T_\infty \) is the fluid temperature. Therefore, to increase the convective heat transfer, one can

- Increase the temperature difference \( (T_s - T_\infty) \) between the surface and the fluid.
- Increase the convection coefficient \( h \). This can be accomplished by increasing the fluid flow over the surface since \( h \) is a function of the flow velocity and the higher the velocity, the higher the \( h \). Example: a cooling fan.
• Increase the contact surface area $A$. Example: a heat sink with fins.

Many times, when the first option is not in our control and the second option (i.e. increasing $h$) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces. Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid. Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere.

![Diagram of a heat sink with fins]

The fin is situated on the surface of a hot surface at $T_s$ and surrounded by a coolant at temperature $T_\infty$, which cools with convective coefficient, $h$. The fin has a cross sectional area, $A_c$, (This is the area through with heat is conducted.) and an overall length, $L$.

Note that as energy is conducted down the length of the fin, some portion is lost, by convection, from the sides. Thus the heat flow varies along the length of the fin.

We further note that the arrows indicating the direction of heat flow point in both the $x$ and $y$ directions. This is an indication that this is truly a two- or three-dimensional heat flow, depending on the geometry of the fin. However, quite often, it is convenient to analyse a fin by examining an equivalent one-dimensional system. The equivalent system will involve the introduction of heat sinks (negative heat sources), which remove an amount of energy Equivalent to what would be lost through the sides by convection.
Across this segment the heat loss will be \( h \cdot (P \cdot x) \cdot (T - T_\infty) \), where \( P \) is the perimeter around the fin. The equivalent heat sink would be \( \frac{q}{(A \cdot x)} \).

Equating the heat source to the convective loss:

\[
\bar{q} = \frac{-h \cdot P \cdot (T - T_\infty)}{A_c}
\]

Substitute this value into the General Conduction Equation as simplified for One-Dimension, Steady State Conduction with Sources:

\[
\frac{d^2 T}{dx^2} = \frac{h \cdot P}{k \cdot A_c} (T - T_\infty) = 0
\]

which is the equation for a fin with a constant cross sectional area. This is the Second Order Differential Equation that we will solve for each fin analysis. Prior to solving, a couple of simplifications should be noted. First, we see that \( h, P, k \) and \( A_c \) are all independent of \( x \) in the defined system (They may not be constant if a more general analysis is desired.). We replace this ratio with a constant. Let

\[
m^2 = \frac{h \cdot P}{k \cdot A_c}
\]

\[
\frac{d^2 T}{dx^2} - m^2 \cdot (T - T_\infty) = 0
\]
Next we notice that the equation is non-homogeneous (due to the $T_\infty$ term). Recall that non-homogeneous differential equations require both a general and a particular solution. We can make this equation homogeneous by introducing the temperature relative to the surroundings:

$$\theta = T - T_\infty$$

Differentiating this equation we find:

$$\frac{d\theta}{dx} = \frac{dT}{dx} + 0$$

Differentiate a second time:

$$\frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$$

Substitute into the Fin Equation:

$$\frac{d^2\theta}{dx^2} - m^2 \cdot \theta = 0$$

This equation is a Second Order, Homogeneous Differential Equation.

2.7 Solution of the Fin Equation:

We apply a standard technique for solving a second order homogeneous linear differential equation.

Try $\theta = e^{\alpha x}$. Differentiate this expression twice:

$$\frac{d\theta}{dx} = \alpha \cdot e^{\alpha x}$$

$$\frac{d^2\theta}{dx^2} = \alpha^2 \cdot e^{\alpha x}$$

Substitute this trial solution into the differential equation:

$$\alpha^2 e^{\alpha x} - m^2 e^{\alpha x} = 0$$

Equation (13) provides the following relation:

$$\alpha = \pm m$$

We now have two solutions to the equation. The general solution to the above differential equation will be a linear combination of each of the independent solutions.
Then:
\[ \theta = A \cdot e^m \cdot x + B \cdot e^{-m} \cdot x. \]
where A and B are arbitrary constants which need to be determined from the boundary conditions. Note that it is a 2nd order differential equation, and hence we need two boundary conditions to determine the two constants of integration.

An alternative solution can be obtained as follows: Note that the hyperbolic sin, sinh, the hyperbolic cosine, cosh, are defined as:

\[ \sinh(m \cdot x) = \frac{e^{m \cdot x} - e^{-m \cdot x}}{2} \]
\[ \cosh(m \cdot x) = \frac{e^{m \cdot x} + e^{-m \cdot x}}{2} \]

We may write:

\[ C \cdot \cosh(m \cdot x) + D \cdot \sinh(m \cdot x) = \frac{C \cdot e^{m \cdot x} + C \cdot e^{-m \cdot x}}{2} + \frac{D \cdot e^{m \cdot x} - D \cdot e^{-m \cdot x}}{2} = \frac{C + D}{2} \cdot e^{m \cdot x} + \frac{C - D}{2} \cdot e^{-m \cdot x} \]

We see that if \( (C+D)/2 \) replaces A and \( (C-D)/2 \) replaces B then the two solutions are equivalent.

\[ \theta = C \cdot \cosh(m \cdot x) + D \cdot \sinh(m \cdot x) \]

Generally the exponential solution is used for very long fins, the hyperbolic solutions for other cases.

**Boundary Conditions:**
Since the solution results in 2 constants of integration we require 2 boundary conditions. The first one is obvious, as one end of the fin will be attached to a hot surface and will come into thermal equilibrium with that surface. Hence, at the fin base,

\[ \theta (0) = T_0 - T_\infty \equiv \theta_0 \]

The second boundary condition depends on the condition imposed at the other end of the fin.

There are various possibilities, as described below.

**Very long fins:**
For very long fins, the end located a long distance from the heat source will approach the temperature of the surroundings. Hence,

\[ \theta (\infty) = 0 \]

Substitute the second condition into the exponential solution of the fin equation:
The first exponential term is infinite and the second is equal to zero. The only way that this equation can be valid is if \( A = 0 \). Now apply the second boundary condition.

\[
\theta(0) = \theta_0 = B \cdot e^{-m \cdot 0} \Rightarrow B = \theta_0
\]

The general temperature profile for a very long fin is then:

\[
\theta(x) = \theta_0 \cdot e^{-m \cdot x}
\]

If we wish to find the heat flow through the fin, we may apply Fourier Law:

\[
q = -k \cdot A_c \cdot \frac{dT}{dx} = -k \cdot A_c \cdot \frac{d\theta}{dx}
\]

Differentiate the temperature profile:

\[
\frac{d\theta}{dx} = -\theta_0 \cdot m \cdot e^{-m \cdot x}
\]

So that:

\[
q = k \cdot A_c \cdot \theta_0 \left[ \frac{h \cdot P}{k \cdot A_c} \right] \cdot \frac{\theta_1}{e^{-m \cdot x}} = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot e^{-m \cdot x} \cdot \theta_0 = M \theta_0 e^{-m \cdot x}
\]

where \( M = \sqrt{h \cdot P \cdot k \cdot A_c} \).

Often we wish to know the total heat flow through the fin, i.e. the heat flow entering at the base \((x=0)\).

\[
q = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \theta_0 = M \theta_0
\]

**The insulated tip fin:**

Assume that the tip is insulated and hence there is no heat transfer:

\[
\frac{d\theta}{dx} \bigg|_{x=L} = 0
\]

The solution to the fin equation is known to be:

\[
\theta = C \cdot \cosh(m \cdot x) + D \cdot \sinh(m \cdot x)
\]

Differentiate this expression.

Apply the first boundary

\[
\frac{d\theta}{dx} = C \cdot m \cdot \sinh(m \cdot x) + D \cdot m \cdot \cosh(m \cdot x)
\]
condition at the base:

\[ \theta(0) = \theta_0 = C \sinh(m \cdot 0) + D \cosh(m \cdot 0) \]

So that \( D = 0 \). Now apply the second boundary condition at the tip to find the value of \( C \):

\[ \frac{d\theta}{dx}(L) = 0 = Cm \sinh(m \cdot L) + \theta_0 m \cosh(m \cdot L) \]

This requires that

\[ C = -\frac{\theta_0 \cosh(mL)}{\sinh(mL)} \]

We may find the heat flow at any value of \( x \) by differentiating the temperature profile and substituting it into the Fourier Law:

\[ q = -k \cdot A_c \cdot \frac{dT}{dx} = -k \cdot A_c \cdot \frac{d\theta}{dx} \]

So that the energy flowing through the base of the fin is:

\[ q = h P k A_c \theta_0 \tanh(mL) = M \theta_0 \tanh(mL) \]

If we compare this result with that for the very long fin, we see that the primary difference in form is in the hyperbolic tangent term. That term, which always results in a number equal to or less than one, represents the reduced heat loss due to the shortening of the fin.
2.8 Fin Effectiveness:

How effective a fin can enhance heat transfer is characterized by the fin effectiveness, \( \varepsilon_f \), which is as the ratio of fin heat transfer and the heat transfer without the fin. For an adiabatic fin:

\[
\varepsilon_f = \frac{q_f}{q} = \frac{q_f}{hA_c(T_b - T_m) - \frac{\sqrt{hPA_c}}{hA_c} \tanh(mL)} = \frac{kP}{\sqrt{hA_c}} \tanh(mL)
\]

If the fin is long enough, \( mL > 2 \), \( \tanh(mL) \rightarrow 1 \), and hence it can be considered as infinite fin (case D in table). Hence, for long fins,

\[
\varepsilon_f \rightarrow \frac{kP}{\sqrt{hA_c}} = \frac{k}{\sqrt{hA_c}} P
\]

2.9 Fin Efficiency:

The fin efficiency is defined as the ratio of the energy transferred through a real fin to that transferred through an ideal fin. An ideal fin is thought to be one made of a perfect or infinite conductor material. A perfect conductor has an infinite thermal conductivity so that the entire fin is at the base material temperature.

\[
\eta = \frac{q_{\text{real}}}{q_{\text{ideal}}} = \frac{\sqrt{h \cdot P \cdot k \cdot A_c \cdot \theta_L \cdot \tanh(m \cdot L)}}{h \cdot (P \cdot L) \cdot \theta_L}
\]

\[
\eta = \frac{\sqrt{k \cdot A_c \cdot \theta_L \cdot \tanh(m \cdot L)}}{h \cdot P \cdot L \cdot \theta_L} = \frac{\tanh(m \cdot L)}{m \cdot L}
\]
UNIT-4

CONCEPTS AND BASIC RELATIONS IN BOUNDARY LAYERS
4.1 Introduction:

Convection is the mode of heat transfer between a surface and a fluid moving over it. The energy transfer in convection is predominately due to the bulk motion of the fluid particles; through the molecular conduction within the fluid itself also contributes to some extent. If this motion is mainly due to the density variations associated with temperature gradients within the fluid, the mode of heat transfer is said to be due to free or natural convection. On the other hand if this fluid motion is principally produced by some superimposed velocity field like fan or blower, the energy transport is said to be due to forced convection.

4.2 Convection Boundary Layers:

**Velocity Boundary Layer:** Consider the flow of fluid over a flat plate as shown in the figure. The fluid approaches the plate in x direction with uniform velocity $u_\infty$. The fluid particles in the fluid layer adjacent to the surface get zero velocity. This motionless layer acts to retract the motion of particles in the adjoining fluid layer as a result of friction between the particles of these two adjoining fluid layers at two different velocities. This fluid layer then acts to restart the motion of particles of next fluid layer and so on, until a distance $y = \delta$ from the surface reaches, where these effects become negligible and the fluid velocity $u$ reaches the free stream velocity $u_\infty$. as a result of frictional effects between the fluid layers, the local fluid velocity $u$ will vary from $x = 0$, $y = 0$ to $y = \delta$.

The region of the flow over the surface bounded by $\delta$ in which the effects of viscous shearing forces caused by fluid viscosity are observed, is called velocity boundary layer or hydro dynamic boundary layer. The thickness of boundary layer $\delta$ is
generally defined as a distance from the surface at which local velocity \( u = 0.99 \) of free stream velocity \( u_\infty \). The retardation of fluid motion in the boundary layer is due to the shear stresses acting in opposite direction with increasing the distance \( y \) from the surface shear stress decreases, the local velocity \( u \) increases until approaches \( u_\infty \). With increasing the distance from the leading edge, the effect of viscosity penetrates further into the free stream and boundary layer thickness grows.

**Thermal boundary Layer:** If the fluid flowing on a surface has a different temperature than the surface, the thermal boundary layer developed is similar to the velocity boundary layer. Consider a fluid at a temperature \( T_\infty \) flows over a surface at a constant temperature \( T_s \). The fluid particles in adjacent layer to the plate get the same temperature that of surface. The particles exchange heat energy with particles in adjoining fluid layers and so on. As a result, the temperature gradients are developed in the fluid layers and a temperature profile is developed in the fluid flow, which ranges from \( T_s \) at the surface to fluid temperature \( T_\infty \) sufficiently far from the surface in \( y \) direction.

The flow region over the surface in which the temperature variation in the direction, normal to surface is observed is called thermal boundary layer. The thickness of thermal boundary layer \( \delta \) th at any location along the length of flow is defined as a distance \( y \) from the surface at which the temperature difference \( (T-T_s) \) equal 0.99 of \( (T_\infty - T_s) \). With increasing the distance from leading edge the effect of heat transfer penetrates further into the free stream and the thermal boundary layer grows as shown in the figure. The convection heat transfer rate anywhere along the surface is directly related to the temperature gradient at that location. Therefore, the shape of the temperature profile in the thermal boundary layer leads to the local convection heat transfer between surface and flowing fluid.
4.3 Development of velocity boundary layer on a flat plate:

It is most essential to distinguish between laminar and turbulent boundary layers. Initially, the boundary layer development is laminar as shown in figure for the flow over a flat plate. Depending upon the flow field and fluid properties, at some critical distance from the leading edge small disturbances in the flow begin to get amplified, a transition process takes place and the flow becomes turbulent. In laminar boundary layer, the fluid motion is highly ordered whereas the motion in the turbulent boundary layer is highly irregular with the fluid moving to and from in all directions. Due to fluid mixing resulting from these macroscopic motions, the turbulent boundary layer is thicker and the velocity profile in turbulent boundary layer is flatter than that in laminar flow.

The critical distance $x_c$, beyond which the flow cannot retain its laminar character is usually specified in term of critical Reynolds number $Re$. Depending upon surface and turbulence level of free stream the critical Reynolds number varies between $10^5$ and $3 \times 10^6$. In the turbulent boundary layer, as seen three distinct regimes exist. A laminar sub-layer, existing next to the wall, has a nearly linear velocity profile. The convective transport in this layer is mainly molecular. In the buffer layer adjacent to the sub-layer, the turbulent mixing and diffusion effects are comparable. Then there is the turbulent core with large scale turbulence.

4.4 Application of dimensional analysis for free convection:

Dimensional analysis is a mathematical method which makes use of the study of the dimensions for solving several engineering problems. This method can be applied to all types of fluid resistances, heat flow problems in fluid mechanics and thermodynamics.
Let us assume that heat transfer coefficient ‘h’ in fully developed forced convection in tube is function of following variables;

\[ h = f(D, V, k, \rho, \mu, cp,) \text{or} \quad f1(h, D,V, \rho, k, \mu, cp) \]

**Nusselt Number (Nu):**

It is defined as the ratio of the heat flow by convection process under a unit temperature gradient to the heat flow rate by conduction under a unit temperature gradient through a stationary thickness (L).

**Grashof number (Gr):**

It is defined as the ratio of product of inertia force and buoyancy force to the square of viscous force.

**Prandtl number (Pr):**

It is the ratio of the momentum diffusivity to the thermal diffusivity.
HEAT AND MASS TRANSFER NOTES
(10ME63)

UNIT-5
FORCED CONVECTIONS
5.1 Applications of dimensional analysis for forced convection:

Dimensional analysis is a mathematical method which makes use of the study of the dimensions for solving several engineering problems. This method can be applied to all types of fluid resistances, heat flow problems in fluid mechanics and thermodynamics.

Let us assume that heat transfer coefficient ‘h’ in fully developed forced convection in tube is function of following variables;

\[ h = f (D, V, k, \rho, \mu, c_p) \]

or

\[ f_1 (h, D, V, \rho, k, \mu, c_p) \]

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Variables</th>
<th>Symbols</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Heat transfer coefficient</td>
<td>( h )</td>
<td>( MT^{-3}T^{-1} )</td>
</tr>
<tr>
<td>02</td>
<td>Fluid density</td>
<td>( \rho )</td>
<td>( ML^{-3} )</td>
</tr>
<tr>
<td>03</td>
<td>Tube diameter</td>
<td>( D )</td>
<td>( L )</td>
</tr>
<tr>
<td>04</td>
<td>Fluid velocity</td>
<td>( V )</td>
<td>( LT^{-4} )</td>
</tr>
<tr>
<td>05</td>
<td>Fluid viscosity</td>
<td>( \mu )</td>
<td>( ML^{-1}T^{-1} )</td>
</tr>
<tr>
<td>06</td>
<td>Specific heat</td>
<td>( c_p )</td>
<td>( L^2T^{-2} )</td>
</tr>
<tr>
<td>07</td>
<td>Thermal conductivity</td>
<td>( k )</td>
<td>( MLT^{-3}T^{-1} )</td>
</tr>
</tbody>
</table>

Total no. of variables \( n = 7 \)

Fundamental dimensions in problem \( m = 4 \) (M, L, T, \( \theta \))

No. of dimensionless \( \pi \)-Term\( = n-m = 3 \)

Equation (2) can be written as;

\[ f_1(\pi_1, \pi_2, \pi_3) = 0 \]

Choosing \( h, D, V, \rho \) as group of repeating variables with unknown exponents.
Therefore,

\[ \pi_1 \text{-Term:} \]

Equating exponents of M, L, T, \( \theta \) respectively, we get;

\[ a_1 = 0, b_1 = -1, c_1 = -1, d_1 = -1 \]

Similarly for \( \pi_2 \) and \( \pi_3 \) Term

\[ \pi_2 \text{-Term:} \]

Since dimensions of \( h \) and \( k/D \) are same;

\[ D/K \]

\[ \pi_3 \text{-Term:} \]

According to \( \pi \) theorem:\( \pi \)

\[ D/K \]
where \( m' \) and \( n' \) are constants.

If \( m' > n' \), then

\[
\frac{D}{K} \quad \frac{D}{K} \quad \frac{D}{K}
\]

OR

\[
\left( \frac{D}{K} \right) \quad \frac{D}{K}
\]

OR

\[
\frac{D}{K}
\]

**Nusselt Number (Nu):**

It is defined as the ratio of the heat flow by convection process under a unit temperature gradient to the heat flow rate by conduction under a unit temperature gradient through a stationary thickness (L).

\[
\text{Nu} = \frac{D}{K}
\]

**Reynolds number (Re):**

It is defined as the ratio of inertia force to viscous force.

\[
\text{Re} = \frac{D}{K}
\]

**Prandtl number (Pr):**

It is the ratio of the momentum diffusivity to the thermal diffusivity.

\[
\text{Pr} = \frac{D}{K}
\]
UNIT-6
HEAT EXCHANGERS
6.1 Introduction:

The device used for exchange of heat between the two fluids that are at different temperatures, is called the heat exchanger. The heat exchangers are commonly used in wide range of applications, for example, in a car as radiator, where hot water from the engine is cooled by atmospheric air. In a refrigerator, the hot refrigerant from the compressor is cooled by natural convection into atmosphere by passing it through finned tubes. In a steam condenser, the latent heat of condensation is removed by circulating water through the tubes. The heat exchangers are also used in space heating and air-conditioning, waste heat recovery and chemical processing. Therefore, the different types of heat exchangers are needed for different applications.

The heat transfer in a heat exchanger usually involves convection on each side of fluids and conduction through the wall separating the two fluids. Thus for analysis of a heat exchanger, it is very convenient to work with an overall heat transfer coefficient $U$, that accounts for the contribution of all these effects on heat transfer. The rate of heat transfer between two fluids at any location in a heat exchanger depends on the magnitude of temperature difference at that location and this temperature difference varies along the length of heat exchanger. Therefore, it is also convenient to work with logarithmic mean temperature difference LMTD, which is an equivalent temperature difference between two fluids for entire length of heat exchanger.

6.2 Classification of heat exchangers:

Heat exchangers are designed in so many sizes, types, configurations and flow arrangements and used for so many purposes. These are classified according to heat transfer process, flow arrangement and type of construction.

According to Heat Transfer Process:

(i) Direct contact type: In this type of heat exchanger, the two immiscible fluids at different temperatures are come in direct contact. For the heat exchange between two fluids, one fluid is sprayed through the other. Cooling towers, jet condensers, desuperheaters, open feed water heaters and -scrubbers are the best examples of such heat exchangers. It cannot be used for transferring heat between two gases or between two miscible
liquids. A direct contact type heat exchanger (cooling tower) is shown in Figure 6.1.

![Diagram of direct contact type heat exchanger (cooling tower)](image)

**Figure 6.1: direct contact type heat exchanger (cooling tower)**

**(ii) Transfer type heat exchangers or recuperators:**

In this type of heat exchanger, the cold and hot fluids flow simultaneously through the device and the heat is transferred through the wall separating them. These types of heat exchangers are most commonly used in almost all fields of engineering.

**(iii) Regenerators or storage type heat exchangers:**

In these types of heat exchangers, the hot and cold fluids flow alternatively on the same surface. When hot fluid flows in an interval of time, it gives its heat to the surface, which stores it in the form of an increase in its internal energy. This stored energy is transferred to cold fluid as it flows over the surface in next interval of time. Thus the same surface is subjected to periodic heating and cooling. In many applications, a rotating disc type matrix is used, the continuous flow of both the hot and cold fluids are maintained. These are pre heaters for steam power plants, blast furnaces, oxygen producers etc. A stationary and rotating matrix shown in Figure 6.2 are examples of storage type of heat exchangers.

The storage type of heat exchangers is more compact than the transfer type of heat exchangers with more surface area per unit volume. However, some mixing of hot and cold fluids is always there.
According to Constructional Features:

(i) **Tubular heat exchanger:**

These are also called tube in tube or concentric tube or double pipe heat exchanger as shown in Figure 6.3. These are widely used in many sizes and different flow arrangements and type.

(ii) **Shell and tube type heat exchanger:**

These are also called surface condensers and are most commonly used for heating, cooling, condensation or evaporation applications. It consists of a shell and a large number of parallel tubes housing in it. The heat transfer takes place as one fluid flows through the tubes and other fluid flows outside the tubes through the shell. The baffles are commonly used on the shell to create turbulence and to keep the uniform spacing between the tubes and thus to enhance the heat transfer rate. They are having large surface area in small volume. A typical shell and tube type heat exchanger is shown in Figure 6.4. The shell and tube type heat exchangers are
further classified according to number of shell and tube passes involved. A heat exchanger with all tubes make one U turn in a shell is called **one shell pass and two tube pass** heat exchanger. Similarly, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a **two shell pass and four tube pass** heat exchanger as shown in Figure 6.5.

![Figure 6.4: Shell and tube type heat exchanger: one shell and one tube pass](image)

![Figure 6.5: Multipass flow arrangement in shell and tube type heat exchanger](image)

(iii) **Finned tube type:**
When a high operating pressure or an enhanced heat transfer rate is required, the extended surfaces are used on one side of the heat exchanger. These heat exchangers are used for liquid to gas heat exchange. Fins are always added on gas side. The finned tubes are used in gas turbines, automobiles, aero planes, heat pumps, refrigeration, electronics, cryogenics, air-conditioning systems etc. The radiator of an automobile is an example of such heat exchanger.
(iv) Compact heat exchanger:
These are special class of heat exchangers in which the heat transfer surface area per unit volume is very large. The ratio of heat transfer surface area to the volume is called area density. A heat exchanger with an area density greater than 700 \( m^2/m^3 \) is called compact heat exchanger. The compact heat exchangers are usually cross flow, in which the two fluids usually flow perpendicular to each other. These heat exchangers have dense arrays of finned tubes or plates, where at least one of the fluid used is gas. For example, automobile radiators have an area density in order of 1100 \( m^2/m^3 \).

According to Flow Arrangement:

(i) Parallel flow:
The hot and cold fluids enter at same end of the heat exchanger, flow through in same direction and leave at other end. It is also called the concurrent heat exchanger Figure 6.6.

(ii) Counter flow:
The hot and cold fluids enter at the opposite ends of heat exchangers, flow through in opposite direction and leave at opposite ends Figure 6.6.

![Figure 6.6: Concentric tube heat exchanger](image)

(iii) Cross flow: The two fluids flow at right angle to each other. The cross flow heat exchanger is further classified as unmixed flow and mixed flow depending on the flow configuration. If both the fluids flow through individual channels and are not free to move in transverse direction, the arrangement is called unmixed as shown in Figure 6.7a. If any fluid flows on the surface and free to move in transverse direction, then this fluid stream is said to be mixed as shown in Figure 6.7b.
Figure 6.7: Different flow configurations in cross-flow heat exchangers.

6.3 Fouling factor:

Material deposits on the surfaces of the heat exchanger tube may add further resistance to heat transfer in addition to those listed below. Such deposits are termed fouling and may significantly affect heat exchanger performance.

We know, the surfaces of heat exchangers do not remain clean after it has been in use for some time. The surfaces become fouled with scaling or deposits. The effect of these deposits affecting the value of overall heat transfer co-efficient. This effect is taken care of by introducing an additional thermal resistance called the fouling resistance.

- **Scaling** is the most common form of fouling and is associated with inverse solubility salts. Examples of such salts are CaCO3, CaSO4, Ca3(PO4)2, CaSiO3, Ca(OH)2, Mg(OH)2, MgSiO3, Na2SO4, LiSO4, and Li2CO3.

- **Corrosion fouling** is classified as a chemical reaction which involves the heat exchanger tubes. Many metals, copper and aluminum being specific examples, form adherent oxide coatings which serve to passivate the surface and prevent further corrosion.

- **Chemical reaction fouling** involves chemical reactions in the process stream which results in deposition of material on the heat exchanger tubes. When food products are involved this may be termed scorching but a wide range of organic materials are subject to similar problems.
- **Freezing fouling** is said to occur when a portion of the hot stream is cooled to near the freezing point for one of its components. This is most notable in refineries where paraffin frequently solidifies from petroleum products at various stages in the refining process, obstructing both flow and heat transfer.

- **Biological fouling** is common where untreated water is used as a coolant stream. Problems range from algae or other microbes to barnacles.

\[\text{ appliying the energy balance to differentials elements in hot and cold fluids.} \]

\[\Delta Q = m_h C_p_h \cdot dT_h \quad \text{and for cold fluid, } \Delta Q = m_c C_p_c \cdot dT_c \quad \text{[1]}\]

The temperature difference at hot fluid is a negative quantity, and so the sign is added to eqn. [1] to make the heat transfer rate \(\Delta Q\) a positive quantity. Let \(\Delta T\) be the differential surface area \(dA\), the temperature difference \(\Delta T\) at hot \& cold fluid is expressed as \(\Delta T = T_h - T_c \quad \text{[2]}\).

In differential form, \(d(\Delta T) = d(T_h - T_c) \quad \text{[3]}\).

Solving the eqns. [1], [2], [3], and [4] as

\[dT_h = \frac{-\Delta Q}{m_h C_p_h} \quad \text{[5]}\]

\[dT_c = \frac{\Delta Q}{m_c C_p_c} \quad \text{[6]}\]


\[d(\Delta T) = \frac{-\Delta Q}{m_h C_p_h} + \frac{1}{m_c C_p_c} \quad \text{[7]}\]

The heat transfer rate across the differential surface area \(dA\) is

\[\Delta Q = U \Delta T \cdot dA \quad \text{[8]}\]
d(ΔT) = -UA \left[ \frac{1}{m_i c_{p_i}} + \frac{1}{m_e c_{p_e}} \right] \Delta T \quad \text{(8)}

Re-arranging \( \frac{d(ΔT)}{ΔT} = -UBA B \)

Integrating Eqn (8) from inlet to outlet conditions of the heat exchanger

\( \ln \left[ \frac{ΔT_i}{ΔT} \right] = -UBA \)

Substituting \( B \) gives \( \ln \left[ \frac{ΔT_i}{ΔT} \right] = -UA \left[ \frac{1}{m_i c_{p_i}} + \frac{1}{m_e c_{p_e}} \right] \) \( \text{(9)} \)

Where, \( ΔT_i = T_{i2} - T_{i1} \) & \( ΔT_e = T_{e2} - T_{e1} \)

It may be expressed in terms of inlet, outlet temperatures & heat transfer

Capacity ratio as \( \ln \left[ \frac{T_{e2} - T_{e1}}{T_{i2} - T_{i1}} \right] = -UA \left[ \frac{1}{C_{hi}} \right] \)

Rearranging we get \( \frac{T_{e2} - T_{e1}}{T_{i2} - T_{i1}} = \exp \left[ -UA \left( \frac{1}{C_{hi}} \right) \right] \) \( \text{(10)} \)

Substituting \( T_{e1}, T_{e2} \) & \( T_{i1}, T_{i2} \) we get

\( T_{i1} - E \frac{C_{min}}{C_{hi}} \left( T_{i2} - T_{e1} \right) \) - \( E \frac{C_{max}}{C_{hi}} \left( T_{e2} - T_{e1} \right) \)

Which is simplified as

\( 1 - E \frac{C_{min}}{C_{hi}} \left( \frac{1 + \frac{C_{hi}}{C_{e}} \right) = \exp \left[ -UA \left( \frac{1 + \frac{C_{hi}}{C_{e}} \right) \right] \)

\( E \frac{C_{max}}{C_{hi}} \left( \frac{1 + \frac{C_{hi}}{C_{e}} \right) = 1 - \exp \left[ -UA \left( \frac{1 + \frac{C_{hi}}{C_{e}} \right) \right] \)

Rearranging, we get relation for effectiveness of a parallel flow heat exchanger

\( \frac{E_{\text{parallel flow}}}{E_{\text{series flow}}} = \frac{1 - \exp \left[ -UA \left( \frac{1 + \frac{C_{hi}}{C_{e}} \right) \right]}{1 - \exp \left[ -NTU \left( \frac{1}{C_{e}} \right) \right]} \)

Introducing NTU \( C \), we get the relation for effectiveness in the form

\( \frac{E_{\text{parallel flow}}}{E_{\text{series flow}}} = \frac{1 - \exp \left[ -NTU \left( \frac{1}{C_{e}} \right) \right]}{1 + C} \)
For a Countercurrent flow heat exchanger, from heat exchanger
with usual notations

In Countercurrent flow arrangement, the hot and cold fluids enter the heat exchanger from opposite ends. Two fluid flow in opposite directions. Thus for the countercurrent temp. & cold fluid may cross the outlet temp. By hot fluid. The temp distribution for countercurrent flow heat exchanger is shown in fig.

Here, \( \Delta T_h = T_h - T_c \)
\( \Delta T_c = T_c - T_h \)

Using these values & \( \Delta T_h & \Delta T_c \) we may derive for the Countercurrent flow heat exchanger. The derivation can also take as

Applying the energy balance to different elements in hot & cold fluid.

The rate of heat transfer \( dq \) from hot fluid:
\( dq_h = -m_hC_p\Delta T_h \)
For cold fluid:
\( dq_c = -m_cC_p\Delta T_c \)

The temp. difference \( \Delta T \) between hot & cold fluids will be in \( dq \), since

\( d\Delta T = dq_h - dq_c \)

Substituting values & \( dq_h \) & \( dq_c \) from eq. (1)

\( d\Delta T = -U \left( \frac{1}{m_hC_p} - \frac{1}{m_cC_p} \right) \)

Substituting \( \Delta T = \frac{\Delta T_h}{\Delta T_c} \) differentiated element, we get

\( d\Delta T = -U \left( \frac{1}{m_hC_p} - \frac{1}{m_cC_p} \right) \)

Rearranging & integrating, we get

\[ \ln \left[ \frac{\Delta T_h}{\Delta T_c} \right] = -UA \left[ \frac{1}{m_hC_p} - \frac{1}{m_cC_p} \right] \]

Substituting \( m_cC_p = \alpha m_hC_p \), we get

\[ \ln \left[ \frac{\Delta T_h}{\Delta T_c} \right] = -UA \left[ \frac{T_h - T_c}{T_h - T_c} \right] \]

(4) \[ Q = \frac{UA}{\ln \left[ \frac{\Delta T_h}{\Delta T_c} \right]} \left[ (T_h - T_c) - (T_h - T_c) \right] \]

(5) \[ Q = UA \Delta T_c = U_b A_r \Delta T_m \]
The $\Delta T_{m}$ is called log mean temperature difference for counter flow in a heat exchanger and is expressed as:

$$\Delta T_{m, \text{compl}} = \frac{(T_{hi} - T_{lo}) - (T_{lo} - T_{ci})}{\ln \left(\frac{T_{hi} - T_{lo}}{T_{lo} - T_{ci}}\right)}$$

A shell and tube condenser is constructed with 2.5 cm OD, single pass horizontal tube with steam condensing at $T_{hi} = 54^\circ C$. The cooling water enters the tubes at $T_{ci} = 18^\circ C$, 15mH, a flow rate of $m_{c} = 0.7 \text{ kg/s}$ and leaves at $T_{lo} = 36^\circ C$. The overall heat transfer coefficient is calculated on the outer surface of tube. $U_{o} = 3509 \text{ W/m}^2 \text{K}$.

Calculate the tube length and heat transfer rate by NTU method.

\[d_{t} = 2.5 \text{ cm} = 0.025 \text{ m}\]

\[T_{hi} = 54^\circ C, T_{ci} = 18^\circ C, T_{lo} = 36^\circ C\]

\[m_{c} = 0.7 \text{ kg/s}, U_{o} = 3509 \text{ W/m}^2 \text{K}\]

\[\Delta T_{m} = \frac{\Delta T_{hi} - \Delta T_{lo}}{\ln \left(\frac{\Delta T_{hi}}{\Delta T_{lo}}\right)} = \frac{36 - 18}{\ln \left(\frac{36}{18}\right)} = \Delta T_{m} = 25.9^\circ C\]

\[Q = m_{c} C_{p} \Delta T_{m} = 0.7 \times 4180 \times (36 - 18) = Q = 52668 \text{ W}\]

\[h_{y} = 2373 \text{ W/m}^2 \text{K} \text{ at } 54^\circ C\]

\[m_{h} C_{p} \Delta T = m_{c} C_{p} \Delta T\]

\[m_{h} \times h_{y} = \frac{Q}{\Delta T} \Rightarrow m_{h} = \frac{52668}{2373.2 \times 10^3} = \frac{52668}{2373} = 0.022 \text{ kg/s}\]

\[A = \frac{Q}{U_{o} \Delta T_{m}} = \frac{52668}{3509 \times 0.573} = \frac{52668}{2926} = 1.81 \text{ m}^2\]

\[\text{NTU} = \frac{UA}{\text{min}} = \frac{3509 \times 0.573}{2926} = 0.693\]

\[E = 1 - e^{-\text{NTU}} = 1 - e^{-0.673} = 0.5\]

\[A_{0} = \pi d L N = \pi 	imes 0.025 \times 6 \times 1\]
Effectiveness of a Counter flow Heat Exchanger:

In counter flow arrangement, the hot and cold fluids enter the heat exchanger from opposite ends, thus fluids flow in opposite directions. Therefore, the outlet temperature of the cold fluid may exceed the outlet temperature of hot fluid. The temperature distribution for counter flow heat exchanger is shown in Fig.

Here, \( \Delta T_i = T_{hi} - T_{ci} \)

\[ \Delta T_e = T_{he} - T_{ce} \]

Using these values of \( \Delta T_i \) & \( \Delta T_e \), we get for counter flow heat exchanger. In derivation can also taken.

Applying the energy balance to differential elements in hot & cold fluid.

The rate of heat transfer \( SQ \) from hot fluid,

\[ SQ_i = \dot{m}_h C_p h \Delta T_i \]

For cold fluid, \( SQ_c = -\dot{m}_c C_p c \Delta T_c \)

The temperature difference \( \Delta T_i \) between hot & cold fluid within the differential area \( dA \) can be expressed as \( \Delta T = T_{hi} - T_{ci} \)

In differential form, \( d(SQ) = dQ_i - dQ_c \)

Substituting values of \( dT_h\) & \( dT_c \) from Eqn (1) & (2) we get

\[ d(SQ) = -SQ_i \left( \frac{1}{m_h C_p h} - \frac{1}{m_c C_p c} \right) \]

Substituting \( SQ = U(\Delta T) \, dA \) for differential element, we get

\[ d(SQ) = -U(\Delta T) \, dA \left( \frac{1}{m_h C_p h} - \frac{1}{m_c C_p c} \right) \]

Re-arranging & integrating, we get

\[ \ln \left[ \frac{T_{he} - T_{ce}}{T_{hi} - T_{ci}} \right] = -U A \left( \frac{1}{C_p h} - \frac{1}{C_p c} \right) \]

Thus,

\[ \Delta T_i = T_{hi} - T_{ci} \quad \text{and} \quad \Delta T_e = T_{he} - T_{ce} \]

It may be expressed in terms of inlet, outlet temperature and heat capacity rates.
Rearranging, we get
\[
\frac{T_h - T_c}{T_i - T_o} = \exp \left[ -\frac{UA}{C_a} \left( 1 - \frac{C_h}{C_e} \right) \right]
\]

Continuing for \( T_{i1} \) and \( T_{i2} \), we get
\[
\begin{bmatrix}
\frac{T_{i1} - E \frac{C_{i1}}{C_e} \left( T_{i1} - T_{o1} \right)}{T_{i1} - T_{o1}} \\
\frac{T_{i2} - E \frac{C_{i2}}{C_e} \left( T_{i2} - T_{o2} \right)}{T_{i2} - T_{o2}}
\end{bmatrix}
= \exp \left\{ -\frac{UA}{C_a} \left[ 1 - \frac{C_h}{C_e} \right] \right\}
\]

which is simplified to
\[
\begin{bmatrix}
\frac{T_{i1} - T_{o1}}{T_{i1} - T_{o1}} \left[ 1 - \frac{E \frac{C_{i1}}{C_e} \left( T_{i1} - T_{o1} \right)}{T_{i1} - T_{o1}} \right] \\
\frac{T_{i2} - T_{o2}}{T_{i2} - T_{o2}} \left[ 1 - \frac{E \frac{C_{i2}}{C_e} \left( T_{i2} - T_{o2} \right)}{T_{i2} - T_{o2}} \right]
\end{bmatrix}
= \exp \left\{ -\frac{UA}{C_a} \left[ 1 - \frac{C_h}{C_e} \right] \right\}
\]

Taking either \( C_h = C_e \) or \( C_h = C_{min} \), both approaches to same result.
Let \( C_h = C_{min} \) and \( C_e = C_{max} \), where relation yields to
\[
\frac{1 - E}{1 - E \frac{C_{min}}{C_{max}}} = \exp \left\{ -\frac{UA}{C_{min}} \left[ 1 - \frac{C_{min}}{C_{max}} \right] \right\}
\]
\[
\frac{1 - E}{1 - E \frac{C_{min}}{C_{max}}} = \exp \left\{ -\text{NTU} \left( 1 - C \right) \right\}
\]

Rearranging, we get relation for effective mean of a counter-flow heat exchanger.
\[
\text{Effective heat exchanger} = \frac{1 - \exp \left\{ -\text{NTU} \left( 1 - C \right) \right\}}{1 - C \exp \left\{ -\text{NTU} \left( 1 - C \right) \right\}}
\]
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UNIT-7

CONDENSATION AND BOILING
CONDENSATION

7.1 Introduction:

The condensation sets in, whenever saturation vapour comes in contact with surface whose temp is lower than saturation temp corresponding to vapour pressure. It is the reverse of boiling process.

This process occurs whenever saturation vapour comes in contact with surface whose temp is lower than saturation temp corresponding to vapour pressure. As the vapour condenses, the latent heat is liberated and there is flow of heat to the surface. The liquid condensate may get sub cooled by contact with the cooled surface and that may eventually cause more vapour to condensate on the exposed surface or upon the previously formed condensate.

7.2 Types of condensation:

- Film wise condensation
- Drop wise condensation

Film wise condensation:

If the condensate tends to wet the surface and thereby forms a liquid film, then process is known as film condensation. The heat transferred from vapour to condensate formed on surface by convection and further from film to cooled surface by conduction. This combined mode of heat transfer reduces the rate of heat transfer and hence it’s heat transfer rates are lower.

Drop wise condensation:

In this, vapour condenses into small liquid droplets of various sizes and which fall down surface in random fashion. A large portion of surface exposed to vapour without an insulating film of condensate liquid; hence higher rates of heat transfer (order of 750 kW/m2) are achieved. Coefficient of heat transfer is 5 to 10 times larger than with film condensation. Yet this type is extremely difficult to maintain or achieve.
7.3 Laminar film condensation on a vertical wall:

prove that the heat transfer coefficient during film wise
condensation is given by

\[ h_x = \frac{k^3 \rho^2 g \kappa y}{4 \mu M x (T_s - T_w)} \]

Consider a cold vertical plate at a surface temperature, \(T_s\) in
exposed to saturated vapours at temperature, \(T_w\) (\(T_s < T_w\)).
The condensate film starts
at the top of the plate and
flows downward under the
influence of gravity and
its thickness \(\delta x\) and condensate
mass rate \(m_c\) increase with
increasing the distance \(x\), along
the surface. The velocity and temperature profile as shown in figure.

The weight \(\delta y\) the fluid element of thickness \(dx\) to be
y and \(\delta y\) balanced by viscous shear force at \(y\) and buoyancy
force due to displaced vapour. Thus,

\[ \rho g \delta y (\delta y - \delta y) = \frac{\delta y}{\delta x} + \mu \frac{dy}{dx} \]

\[ \text{where,} \quad \delta = \text{density of condensate (liquid)} \]
[\( \rho \) = density of vapour]
[\( \mu \) = viscosity of the condensate (liquid)]
[\( \delta \) = thickness of boundary layer at any \(x\)]
[\( U \) = local velocity of condensate]

\[ (\rho - \rho_c) g (\delta y - \delta y) = \mu \frac{dy}{dx} \]

Integrating both sides, we get:

\[ (\rho - \rho_c) g (\delta y - \delta y) = \mu \frac{dy}{dx} \]

\[ \text{where \(\rho\) and \(\rho_c\) \(\in\) boundary conditions at \(y = 0, U = 0\), we get, \(\rho_c = 0\)} \]

\[ \frac{(\rho - \rho_c) g (\delta y - \delta y)}{\mu} = \mu U \]

\[ U = \frac{(\rho - \rho_c) g (\delta y - \delta y)}{\mu} \]
The main flow war and condensate per unit length position is given by

\[ m = \int \rho \, du \, dy = \int \frac{\rho}{\mu} \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

\[ m = \int \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

Since liquid temp distribution is linear, the elemental flow at the wall

\[ q_x = K_x \left( \frac{x}{\delta} \right) \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \]

The \(-\) sign is omitted, since heat flows in opposite direction.

The heat transfer rate

\[ Q = \int K_x \, dx \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

The amount of condensate added by \( \delta w \times \delta x \, dx \) can be obtained by differentiating eq. 8 at \( x \)

\[ \frac{dm}{dx} = \int p \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

\[ \frac{dm}{dx} = \frac{d}{dx} \left( \int p \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \right) \]

\[ = p \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

For the element,

The heat transfer rate \( Q \) at the wall = Heat transfer rate during condensation = energy

\[ K_x \, dx \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

\[ \frac{dm}{dx} = \int K_x \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

Integrating both sides

\[ \int \frac{dm}{dx} \, dx = \int K_x \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

Treatig \( K_x \), \( h_x \), \( \frac{y_1 - y_2}{y_1} \) as constant quantities,

Then integration leads to

\[ \int \frac{dm}{dx} \, dx = \frac{K_x \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy}{h_x} \]

\[ \frac{dt}{dt} = K_x \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

The surface heat transfer rate per unit length of the plate by convection can be expressed as:

\[ Q = h \, dx \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \]

Equating both sides, we get

\[ h = \frac{K_x}{\frac{y_1 - y_2}{y_1} \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}}} \]

Substituting \( c \), we get

\[ h_x = \int p \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy \]

Generally \( p << \delta \), therefore \( \delta_x \) is neglected from above expression, and

\[ h_x = \left[ \frac{K_x \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \, dy}{h_x} \left( \frac{y_1 - y_2}{y_1} \right)^{\frac{3}{2}} \right] \]
7.4 Introduction:

Boiling is a convection process involving a change in phase from liquid to vapor. Boiling may occur when a liquid is in contact with a surface maintained at a temperature higher than the saturation temperature of the liquid. If heat is added to a liquid from a submerged solid surface, the boiling process is referred to as pool boiling. In this process the vapor produced may form bubbles, which grow and subsequently detach themselves from the surface, rising to the free surface due to buoyancy effects. A common example of pool boiling is the boiling of water in a vessel on a stove. In contrast, flow boiling or forced convection boiling occurs in a flowing stream and the boiling surface may itself be an portion of the flow passage. This phenomenon is generally associated with two phase flows through confined passages.

A necessary condition for the occurrence of pool boiling is that the temperature of the heating surface exceeds the saturation temperature of the liquid. The type of boiling is determined by the temperature of the liquid. If the temperature of the liquid is below the saturation temperature, the process is called sub cooled or local boiling. In local boiling, the bubbles formed at the surface eventually condense in the liquid. If the liquid is maintained at saturation temperature, the process is called saturated or bulk boiling.

There are various distinct regimes of pool boiling in which the heat transfer mechanism differs radically. The temperature distribution in saturated pool boiling with a liquid vapor interface is shown in the Figure.
7.4 Regimes of pool boiling:

The different regimes of boiling are indicated in Figure 2. This specific curve has been obtained from an electrically heated platinum wire submerged in water by varying its surface temperature and measuring the surface heat flux $q_s$. The six regimes of Figure 2 will now be described briefly.

In region I, called the free convection zone, the excess temperature, $T$ is very small and $\leq 5^\circ C$. Here the liquid near the surface is superheated slightly, the convection currents circulate the liquid and evaporation takes place at the liquid surface.

Nucleate boiling exists in regions II and III. As the excess temperature, $T$ is increased, bubbles begin to form on the surface of the wire at certain localized spots. The bubbles condense in the liquid without reaching the liquid surface. Region II is in fact the beginning of nucleate boiling. As the excess temperating is further increased bubbles are formed more rapidly and rise to the surface of the liquid resulting in rapid evaporation.

This is indicated in region III. Nucleate boiling exists up to $T \leq 50^\circ C$. The maximum heat flux, known as the critical heat flux, occurs at point A and is of the order of 1MW/m².
The trend of increase of heat flux with increase in excess temperature observed up to region III is reversed in region IV, called the film boiling region. This is due to the fact that bubbles now form so rapidly that they blanket the heating surface with a vapor film preventing the inflow of fresh liquid from taking their place. Now the heat must be transferred through this vapor film (by conduction) to the liquid to effect any further boiling. Since the thermal conductivity of the vapor film is much less than that of the liquid, the value of $q.$ must then decrease with increase of $T$. In region IV the vapor film is not stable and collapses and reforms rapidly. With further increase in $T$ the vapor film is stabilized and the heating surface is completely covered by a vapor blanket and the heat flux is the lowest as shown in region V. The surface temperatures required to maintain a stable film are high and under these conditions a sizeable amount of heat is lost by the surface due to radiation, as indicated in region VI.

The phenomenon of stable film boiling can be observed when a drop of water falls on a red hot stove. The drop does not evaporate immediately but dances a few times on the stove. This is due to the formation of a stable steam film at the interface between the hot surface and the liquid droplet. From Fig.2 it is clear that high heat transfer rates are associated with small values of the excess temperature in the nucleate boiling regime. The equipment used for boiling should be designed to operate in this region only. The critical heat flux point A in Fig.2 is also called the boiling crisis because the boiling process beyond that point is unstable unless of course, point B is reached. The temperature at point B is extremely high and normally above the melting point of the solid. So if the heating of the metallic surface is not limited to point A, the metal may be damaged or it may even melt. That is why the peak heat flux point is called the burnout point and an accurate knowledge of this point is very important. Our aim should be to operate the equipment close to this value but never beyond it.

7.5 Mass transfer:

Mass transfer is the movement of molecules of one material into another due to the concentration difference in a system. Mass transfer occurs in the direction of negative concentration gradient, similar to heat transfer in the direction of negative temperature gradient.
7.6 Fick's first law of diffusion:

The Fick’s law for the rate of transfer of species A in x-direction in a binary mixture of A and B can be expressed as:

\[ \dot{m} = A \Phi_A \]

Where,

\[ \dot{m} = \text{mass flow rate of species A by diffusion, kg/s} \]
\[ A = \text{area through which mass is flowing, m}^2 \]
\[ \Phi_A = \text{mass flux of species A i.e. amount of species A that is transferred per unit time and per unit area perpendicular to the direction of transfer, kg/s-m}^2 \]
\[ D = \text{diffusion coefficient or mass diffusivity for binary mixture of species A and B, m}^2/\text{s}. \]

The –ve sign indicates that diffusion takes place in the direction opposite to that of increasing concentration.
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UNIT-8
RADIATION HEAT TRANSFER
8.1 Introduction:

Radiation, energy transfer across a system boundary due to a T, by the mechanism of photon emission or electromagnetic wave emission.

Because the mechanism of transmission is photon emission, unlike conduction and convection, there need be no intermediate matter to enable transmission.

The significance of this is that radiation will be the only mechanism for heat transfer whenever a vacuum is present. Thermal energy emitted by matter as a result of vibrational and rotational movements of molecules, atoms and electrons. The energy is transported by electromagnetic waves (or photons). Radiation requires no medium for its propagation, therefore, can take place also in vacuum. All matters emit radiation as long as they have a finite (greater than absolute zero) temperature. The rate at which radiation energy is emitted is usually quantified by the modified Stefan-Boltzmann law:

8.2 Definitions of various terms used in radiation heat transfer:

• Stefan-Boltzmann law:

  In 1884, Boltzman showed that heat flux energy emitted by radiation from an ideal surface called black is proportional to its absolute temperature of fourth power.

  Where:

  = Emissive Power, the gross energy emitted from an ideal surface per unit area, time.

  Stefan Boltzman constant, $5.67 	imes 10^{-8}$ W/m2K4

  Absolute temperature of the emitting surface, K.

• Kirchoff’s law:

  It states that at any temperature the ratio of total emissive power E to the total absorptive $\alpha$ is a constant for all substances which are in thermal equilibrium with their environment.
• Planck’s law:

While the Stefan-Boltzmann law is useful for studying overall energy emissions, it does not allow us to treat those interactions, which deal specifically with wavelength, \( \lambda \). This problem was overcome by another of the modern physicists, Max Plank, who developed a relationship for wave-based emissions.

\[
E_{\lambda} = \frac{C_1}{\lambda^5 \left( e^{\frac{C_2}{\lambda T}} - 1 \right)}
\]

• Wein’s displacement law:

Radiation heat exchange between two parallel infinite black surfaces:

View factor and View factor Algebra:

Radiation analysis must take account of the fact that not all of one surface ‘sees’ all of another. This is characterised by the view factor (sometimes called the radiation configuration factor or shape factor).

The view factor, \( F \), is defined as the fraction of radiation emitted from one surface that is incident upon another. It is usually given two subscripts, \( F_{ij} \), \( F_{12} \), \( F_{ab} \) etc. The first subscript refers to the emitting surface the second the receiving surface. The mathematical definition of the view factors \( F_{ij} \) and \( F_{ji} \) are given by the expressions:

\[
F_{ij} = \frac{1}{\pi A_i \int \frac{\cos \theta_i \cos \theta_j dA_i dA_j}{r^2}}
\]

\[
F_{ji} = \frac{1}{\pi A_j \int \frac{\cos \theta_i \cos \theta_j dA_i dA_j}{r^2}}
\]
The above equations may be integrated to calculate view factors directly. In some cases, the integration can be simplified. View factors are also available for a large number of configurations in tabular, parametric or graphical form for a wide range of geometries. The Catalogue by Howell (1982) provides a comprehensive and useful source of view factor data.

From inspection of the symmetry between equations

\[ A_i F_{ij} = A_j F_{ji} \]  
(Reciprocity rule)

Also for an enclosure of \( n \) surfaces:

\[ \sum_{j=1}^{n} F_{ij} = 1 \]  
(Summation rule)

For a convex or flat surface \( F_{ii} = 0 \) (it does not ‘see’ any part of itself)
For a concave surface \( F_{ii} > 0 \) (it does ‘see’ part of itself)

**Solar Irradiation:**

![Solar Irradiation Graph](image-url)
Angles and Arc Length:
We are well accustomed to thinking of an angle as a two dimensional object. It may be used to find an arc length.

\[ L = r\alpha \]

Solid Angle:
We generalize the idea of an angle and an arc length to three dimensions and define a solid angle, \( \Omega \), which like the standard angle has no dimensions. The solid angle, when multiplied by the radius squared will have dimensions of length squared, or area, and will have the magnitude of the encompassed area.

\[ A = r^2 \cdot d\Omega \]

Projected Area:
The area, \( dA_1 \), as seen from the prospective of a viewer, situated at an angle \( \theta \) from the normal to the surface, will appear somewhat smaller, as \( \cos \theta \cdot dA_1 \). This smaller area is termed the projected area.

\[ A_{\text{projected}} = \cos \theta \cdot A_{\text{normal}} \]