AC VOLTAGE CONTROLLER CIRCUITS  
(RMS VOLTAGE CONTROLLERS)

AC voltage controllers (ac line voltage controllers) are employed to vary the RMS value of the alternating voltage applied to a load circuit by introducing Thyristors between the load and a constant voltage ac source. The RMS value of alternating voltage applied to a load circuit is controlled by controlling the triggering angle of the Thyristors in the ac voltage controller circuits.

In brief, an ac voltage controller is a type of thyristor power converter which is used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output. The RMS value of the ac output voltage and the ac power flow to the load is controlled by varying (adjusting) the trigger angle ‘α’

There are two different types of thyristor control used in practice to control the ac power flow:

- On-Off control
- Phase control

These are the two ac output voltage control techniques.

In On-Off control technique Thyristors are used as switches to connect the load circuit to the ac supply (source) for a few cycles of the input ac supply and then to disconnect it for few input cycles. The Thyristors thus act as a high speed contactor (or high speed ac switch).

PHASE CONTROL

In phase control the Thyristors are used as switches to connect the load circuit to the input ac supply, for a part of every input cycle. That is the ac supply voltage is chopped using Thyristors during a part of each input cycle.

The thyristor switch is turned on for a part of every half cycle, so that input supply voltage appears across the load and then turned off during the remaining part of input half cycle to disconnect the ac supply from the load.

By controlling the phase angle or the trigger angle ‘α’ (delay angle), the output RMS voltage across the load can be controlled.

The trigger delay angle ‘α’ is defined as the phase angle (the value of ωt) at which the thyristor turns on and the load current begins to flow.

Thyristor ac voltage controllers use ac line commutation or ac phase commutation. Thyristors in ac voltage controllers are line commutated (phase commutated) since the input supply is ac. When the input ac voltage reverses and becomes negative during the negative half cycle the current flowing through the conducting thyristor decreases and
falls to zero. Thus the ON thyristor naturally turns off, when the device current falls to zero.

Phase control Thyristors which are relatively inexpensive, converter grade Thyristors which are slower than fast switching inverter grade Thyristors are normally used.

For applications upto 400Hz, if Triacs are available to meet the voltage and current ratings of a particular application, Triacs are more commonly used.

Due to ac line commutation or natural commutation, there is no need of extra commutation circuitry or components and the circuits for ac voltage controllers are very simple.

Due to the nature of the output waveforms, the analysis, derivations of expressions for performance parameters are not simple, especially for the phase controlled ac voltage controllers with RL load. But however most of the practical loads are of the RL type and hence RL load should be considered in the analysis and design of ac voltage controller circuits.

**TYPE OF AC VOLTAGE CONTROLLERS**

The ac voltage controllers are classified into two types based on the type of input ac supply applied to the circuit.

- Single Phase AC Controllers.
- Three Phase AC Controllers.

Single phase ac controllers operate with single phase ac supply voltage of 230V RMS at 50Hz in our country. Three phase ac controllers operate with 3 phase ac supply of 400V RMS at 50Hz supply frequency.

Each type of controller may be sub divided into

- Uni-directional or half wave ac controller.
- Bi-directional or full wave ac controller.

In brief different types of ac voltage controllers are

- Single phase half wave ac voltage controller (uni-directional controller).
- Single phase full wave ac voltage controller (bi-directional controller).
- Three phase half wave ac voltage controller (uni-directional controller).
- Three phase full wave ac voltage controller (bi-directional controller).

**APPLICATIONS OF AC VOLTAGE CONTROLLERS**

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Industrial heating & Domestic heating.
- Transformer tap changing (on load transformer tap changing).
- Speed control of induction motors (single phase and poly phase ac induction motor control).
- AC magnet controls.

**PRINCIPLE OF ON-OFF CONTROL TECHNIQUE (INTEGRAL CYCLE CONTROL)**

The basic principle of on-off control technique is explained with reference to a single phase full wave ac voltage controller circuit shown below. The thyristor switches $T_1$ and $T_2$ are turned on by applying appropriate gate trigger pulses to connect the input ac supply to the load for ‘$n$’ number of input cycles during the time interval $t_{on}$. The
Thyristor switches \( T_1 \) and \( T_2 \) are turned off by blocking the gate trigger pulses for ‘m’ number of input cycles during the time interval \( t_{OFF} \). The ac controller ON time \( t_{ON} \) usually consists of an integral number of input cycles.

\[
R = R_I = \text{Load Resistance}
\]

Fig.: Single phase full wave AC voltage controller circuit

**Example**

Referring to the waveforms of ON-OFF control technique in the above diagram,

\( n = \) Two input cycles. Thyristors are turned ON during \( t_{ON} \) for two input cycles.
$m =$ One input cycle. Thyristors are turned OFF during $t_{off}$ for one input cycle.

Thyristors are turned ON precisely at the zero voltage crossings of the input supply. The thyristor $T_1$ is turned on at the beginning of each positive half cycle by applying the gate trigger pulses to $T_1$ as shown, during the ON time $t_{on}$. The load current flows in the positive direction, which is the downward direction as shown in the circuit diagram when $T_1$ conducts. The thyristor $T_2$ is turned on at the beginning of each negative half cycle, by applying gating signal to the gate of $T_2$, during $t_{on}$. The load current flows in the reverse direction, which is the upward direction when $T_2$ conducts. Thus we obtain a bi-directional load current flow (alternating load current flow) in a dc voltage controller circuit, by triggering the thyristors alternately.

This type of control is used in applications which have high mechanical inertia and high thermal time constant (Industrial heating and speed control of ac motors). Due to zero voltage and zero current switching of Thyristors, the harmonics generated by switching actions are reduced.

For a sine wave input supply voltage,

$$v = V_m \sin \omega t = \sqrt{2} V \sin \omega t$$

$V_\text{rms}$ = RMS value of input ac supply = $V_m / \sqrt{2}$ = RMS phase supply voltage.

If the input ac supply is connected to load for ‘n’ number of input cycles and disconnected for ‘m’ number of input cycles, then

$$t_{on} = n \times T, \quad t_{off} = m \times T$$

Where $T = \frac{1}{f}$ = input cycle time (time period) and $f =$ input supply frequency.

$t_{on} =$ controller on time = $n \times T$.

$t_{off} =$ controller off time = $m \times T$.

$T_0 =$ Output time period = $(t_{on} + t_{off}) = (nT + mT)$.
We can show that,

Output RMS voltage \( V_{O,(RMS)} = V_{i,(RMS)} \sqrt{\frac{t_{ON}}{T_O}} = V_S \sqrt{\frac{t_{ON}}{T_O}} \)

Where \( V_{i,(RMS)} \) is the RMS input supply voltage = \( V_S \).

**TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE, FOR ON-OFF CONTROL METHOD.**

Output RMS voltage

\[
V_{O,(RMS)} = \sqrt{\frac{1}{\omega T_O} \int_{0}^{t_{ON}} V_m^2 \sin^2 \omega t \, dt}
\]

Substituting for

\[
\sin^2 \theta = \frac{1 - \cos 2\theta}{2}
\]

\[
V_{O,(RMS)} = \sqrt{\frac{V_m^2}{\omega T_O} \int_{0}^{t_{ON}} \left( \frac{1 - \cos 2\omega t}{2} \right) \, d (\omega t)}
\]

\[
V_{O,(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_O} \left[ \int_{0}^{t_{ON}} d (\omega t) - \int_{0}^{t_{ON}} \cos 2\omega t \, d (\omega t) \right]}
\]

\[
V_{O,(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_O} \left[ \sin 2\omega t - 0 \right]_{0}^{t_{ON}}}
\]

\[
V_{O,(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_O} \left[ \sin 2\omega t_{ON} - 0 \right]}
\]

Now \( t_{ON} = \) An integral number of input cycles; Hence

\( t_{ON} = T, 2T, 3T, 4T, 5T, \ldots \) & \( \omega t_{ON} = 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, \ldots \)

Where \( T \) is the input supply time period (\( T = \) input cycle time period). Thus we note that \( \sin 2\omega t_{ON} = 0 \)

\[
V_{O,(RMS)} = \sqrt{\frac{V_m^2 \omega t_{ON}}{2\omega T_O}} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{t_{ON}}{T_O}}
\]
\[ V_{O(RMS)} = \frac{V_{i(RMS)}}{T_O} = \frac{V_S}{\sqrt{2}} \]

Where \( V_{i(RMS)} = \frac{V_m}{\sqrt{2}} = V_S \) = RMS value of input supply voltage;

\[ \frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON} + t_{OFF}} = \frac{nT}{nT + mT} = \frac{n}{n + m} = k \] = duty cycle (d).

\[ V_{O(RMS)} = V_S \sqrt{\frac{n}{m + n}} = V_S \sqrt{k} \]

**PERFORMANCE PARAMETERS OF AC VOLTAGE CONTROLLERS**

- **RMS Output (Load) Voltage**
  \[ V_{O(RMS)} = \left[ \frac{n}{2\pi (n+m)} \right]^{\frac{1}{2}} \int_0^{2\pi} V_m^2 \sin^2(\omega t) d(\omega t) \]

\[ V_{O(RMS)} = V_m \sqrt{\frac{n}{2\pi (m+n)}} = V_{i(RMS)} \sqrt{k} = V_S \sqrt{k} \]

Where \( V_S = V_{i(RMS)} \) = RMS value of input supply voltage.

- **Duty Cycle**
  \[ k = \frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON} + t_{OFF}} = \frac{nT}{(m+n)T} \]

Where, \( k = \frac{n}{m+n} \) = duty cycle (d).

- **RMS Load Current**
  \[ I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L} \] for a resistive load \( Z = R_L \).

- **Output AC (Load) Power**
  \[ P_O = I_{O(RMS)}^2 \times R_L \]
• Input Power Factor

\[ PF = \frac{P_O}{VA} = \frac{\text{output load power}}{\text{input supply volt amperes}} = \frac{P_O}{V S I _S} \]

\[ PF = \frac{I_{O(RMS)}^2 \times R_L}{V_{d(RMS)} \times I_{in(RMS)}} ; \quad I_S = I_{in(RMS)} = \text{RMS input supply current}. \]

The input supply current is same as the load current \( I_m = I_O = I_L \)

Hence, RMS supply current = RMS load current; \( I_{in(RMS)} = I_{O(RMS)} \)

\[ PF = \frac{I_{O(RMS)}^2 \times R_L}{V_{d(RMS)} \times I_{in(RMS)}} = \frac{V_{O(RMS)}}{V_{d(RMS)}} \sqrt{k} = \sqrt{k} \]

\[ PF = \sqrt{k} = \sqrt{\frac{n}{m+n}} \]

• The Average Current of Thyristor \( I_{T(Avg)} \)

**Waveform of Thyristor Current**

\[ I_{T(Avg)} = \frac{n}{2\pi(m+n)} \int _0 ^{\pi} I_m \sin \omega t . d(\omega t) \]

\[ I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \int _0 ^{\pi} \sin \omega t . d(\omega t) \]

\[ I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left\{ -\cos \omega t \right\}_0 ^{\pi} \]

\[ I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left\{ -\cos \pi + \cos 0 \right\} \]
\[ I_{T(Avg)} = \frac{nI_m}{2\pi (m+n)} \left[ (0)^2 + (1)^2 \right] \]

\[ I_{T(Avg)} = \frac{n}{2\pi (m+n)} \left[ 2I_m \right] \]

\[ I_{T(Avg)} = \frac{I_m n}{\pi (m+n)} = \frac{kI_m}{\pi} \]

\[ k = \text{duty cycle} = \frac{t_{ON}}{t_{ON} + t_{OFF}} = \frac{n}{(n+m)} \]

\[ I_{T(Avg)} = \frac{I_m n}{\pi (m+n)} = \frac{kI_m}{\pi} \]

Where \( I_m = \frac{V_m}{R_L} \) is maximum or peak thyristor current.

- **RMS Current of Thyristor** \( I_{T(RMS)} \)

\[ I_{T(RMS)} = \left[ \frac{n}{2\pi (n+m)} \int_0^\pi I_m^2 \sin^2 \omega t \, d(\omega t) \right]^{1/2} \]

\[ I_{T(RMS)} = \left[ \frac{nI_m^2}{2\pi (n+m)} \int_0^\pi \sin^2 \omega t \, d(\omega t) \right]^{1/2} \]

\[ I_{T(RMS)} = \left[ \frac{nI_m^2}{2\pi (n+m)} \int_0^\pi \frac{1 - \cos 2\omega t}{2} \, d(\omega t) \right]^{1/2} \]

\[ I_{T(RMS)} = \left[ \frac{nI_m^2}{4\pi (n+m)} \left\{ d(\omega t) - \left( \frac{\sin 2\omega t}{2} \right) \right\}_0^\pi \right]^{1/2} \]

\[ I_{T(RMS)} = \left[ \frac{nI_m^2}{4\pi (n+m)} \left\{ (\omega t) - \left( \frac{\sin 2\omega t}{2} \right) \right\}_0^\pi \right]^{1/2} \]

\[ I_{T(RMS)} = \left[ \frac{nI_m^2}{4\pi (n+m)} \left\{ (\pi - 0) - \left( \frac{\sin 2\pi - \sin 0}{2} \right) \right\} \right]^{1/2} \]
PROBLEM

1. A single phase full wave AC voltage controller working on ON-OFF control technique has supply voltage of 230V, RMS 50Hz, load = 50Ω. The controller is ON for 30 cycles and off for 40 cycles. Calculate
   - ON & OFF time intervals.
   - RMS output voltage.
   - Input P.F.
   - Average and RMS thyristor currents.

\[ V_{in(RMS)} = 230V, \quad V_m = \sqrt{2} \times 230 = 325.269V, \quad V_m = 325.269V, \]

\[ T = \frac{1}{f} = \frac{1}{50Hz} = 0.02\text{sec}, \quad T = 20\text{ms}. \]

\( n = \) number of input cycles during which controller is ON; \( n = 30 \).

\( m = \) number of input cycles during which controller is OFF; \( m = 40 \).

\[ t_{ON} = n \times T = 30 \times 20\text{ms} = 600\text{ms} = 0.6\text{sec} \]

\[ t_{ON} = n \times T = 0.6\text{sec} = \text{controller ON time.} \]

\[ t_{OFF} = m \times T = 40 \times 20\text{ms} = 800\text{ms} = 0.8\text{sec} \]

\[ t_{OFF} = m \times T = 0.8\text{sec} = \text{controller OFF time.} \]

Duty cycle \( k = \frac{n}{m+n} = \frac{30}{40+30} = 0.4285 \)

**RMS output voltage**

\[ V_{O(RMS)} = V_{in(RMS)} \times \frac{\sqrt{n}}{\sqrt{m+n}} \]
\[ V_{o(RMS)} = 230V \times \sqrt{\frac{3}{7}} = 230 \times \sqrt{\frac{3}{7}} \]

\[ V_{o(RMS)} = 230V \times 0.42857 = 230 \times 0.65465 \]

\[ V_{o(RMS)} = 150.570V \]

\[ I_{o(RMS)} = \frac{V_{o(RMS)}}{Z} = \frac{V_{o(RMS)}}{R_L} = \frac{150.570V}{50\Omega} = 3.0114A \]

\[ P_D = I_{o(RMS)}^2 \times R_L = 3.0114^2 \times 50 = 453.426498 \]

**Input Power Factor**  
\[ P.F = \sqrt{k} \]

\[ PF = \sqrt{\frac{n}{m+n}} = \sqrt{\frac{30}{70}} = \sqrt{0.4285} \]

\[ PF = 0.654653 \]

**Average Thyristor Current Rating**

\[ I_{T(avg)} = \frac{I_n}{\pi} \times \left(\frac{m+n}{n}\right) = \frac{k \times I_m}{\pi} \]

where

\[ I_m = \frac{V}{R_L} = \frac{\sqrt{2} \times 230}{50} = 325.269 \]

\[ I_m = 6.505382 \text{A} = \text{Peak (maximum) thyristor current.} \]

\[ I_{T(avg)} = \frac{6.505382}{\pi} \times \left(\frac{3}{7}\right) \]

\[ I_{T(avg)} = 0.88745 \text{A} \]

**RMS Current Rating of Thyristor**

\[ I_{T(RMS)} = \frac{I_m}{2} \sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2} \sqrt{k} = \frac{6.505382}{2} \times \sqrt{\frac{3}{7}} \]

\[ I_{T(RMS)} = 2.129386 \text{A} \]
**PRINCIPLE OF AC PHASE CONTROL**

The basic principle of ac phase control technique is explained with reference to a single phase half wave ac voltage controller (unidirectional controller) circuit shown in the below figure.

The half wave ac controller uses one thyristor and one diode connected in parallel across each other in opposite direction that is anode of thyristor $T_1$ is connected to the cathode of diode $D_1$ and the cathode of $T_1$ is connected to the anode of $D_1$. The output voltage across the load resistor ‘R’ and hence the ac power flow to the load is controlled by varying the trigger angle ‘$\alpha$’.

The trigger angle or the delay angle ‘$\alpha$’ refers to the value of $\omega t$ or the instant at which the thyristor $T_1$ is triggered to turn it ON, by applying a suitable gate trigger pulse between the gate and cathode lead.

The thyristor $T_1$ is forward biased during the positive half cycle of input ac supply. It can be triggered and made to conduct by applying a suitable gate trigger pulse only during the positive half cycle of input supply. When $T_1$ is triggered it conducts and the load current flows through the thyristor $T_1$, the load and through the transformer secondary winding.

By assuming $T_1$ as an ideal thyristor switch it can be considered as a closed switch when it is ON during the period $\omega t = \alpha$ to $\pi$ radians. The output voltage across the load follows the input supply voltage when the thyristor $T_1$ is turned-on and when it conducts from $\omega t = \alpha$ to $\pi$ radians. When the input supply voltage decreases to zero at $\omega t = \pi$, for a resistive load the load current also falls to zero at $\omega t = \pi$ and hence the thyristor $T_1$ turns off at $\omega t = \pi$. Between the time period $\omega t = \pi$ to $2\pi$, when the supply voltage reverses and becomes negative the diode $D_1$ becomes forward biased and hence turns ON and conducts. The load current flows in the opposite direction during $\omega t = \pi$ to $2\pi$ radians when $D_1$ is ON and the output voltage follows the negative half cycle of input supply.

![Fig.: Halfwave AC phase controller (Unidirectional Controller)](image-url)
Equations

Input AC Supply Voltage across the Transformer Secondary Winding.

\[ v_s = V_m \sin \omega t \]

\[ V_S = V_{in(RMS)} = \frac{V_m}{\sqrt{2}} = \text{RMS value of secondary supply voltage.} \]

Output Load Voltage

\[ v_o = v_L = 0; \text{ for } \omega t = 0 \text{ to } \alpha \]

\[ v_o = v_L = V_o \sin \omega t; \text{ for } \omega t = \alpha \text{ to } 2\pi. \]

Output Load Current

\[ i_o = i_L = \frac{v_o}{R_L} = \frac{V_m \sin \omega t}{R_L}; \text{ for } \omega t = \alpha \text{ to } 2\pi. \]

\[ i_o = i_L = 0; \text{ for } \omega t = 0 \text{ to } \alpha. \]

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE \( V_{o(RMS)} \)

\[ V_{o(RMS)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{2\pi} v_o^2 \sin^2 \omega t \, d(\omega t)} \]

\[ V_{o(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[ \int_{\alpha}^{2\pi} \frac{1 - \cos 2\omega t}{2} \, d(\omega t) \right]} \]
\[ V_{O_{RMS}} = \frac{V_m}{4\pi} \left[ \int_a^{2\pi} (1 - \cos 2\omega t) \cdot d(\omega t) \right] \]

\[ V_{O_{RMS}} = \frac{V_m}{2\sqrt{\pi}} \left[ \int_a^{2\pi} d(\omega t) - \int_a^{2\pi} \cos 2\omega t \cdot d\omega t \right] \]

\[ V_{O_{RMS}} = \frac{V_m}{2\sqrt{\pi}} \left[ \int_a^{2\pi} (\omega t) - \left( \sin 2\omega t \cdot \frac{2}{\alpha} \right) \right] \]

\[ V_{O_{RMS}} = \frac{V_m}{2\sqrt{\pi}} \left[ (2\pi - \alpha) - \left( \sin 2\omega t \cdot \frac{2}{\alpha} \right) \right] \quad ; \sin 4\pi = 0 \]

\[ V_{O_{RMS}} = \frac{V_m}{2\sqrt{\pi}} \left[ (2\pi - \alpha) + \sin 2\omega t \cdot \frac{2}{\alpha} \right] \]

\[ V_{O_{RMS}} = \frac{V_m}{\sqrt{2}2\pi} \left[ (2\pi - \alpha) + \sin 2\omega t \cdot 2 \right] \]

\[ V_{O_{RMS}} = \frac{V_m}{\sqrt{2}2\pi} \left[ (2\pi - \alpha) + \frac{\sin 2\alpha}{2} \right] \]

\[ V_{O_{RMS}} = \frac{V_m}{\sqrt{2}2\pi} \left[ \frac{1}{2\pi} \left( (2\pi - \alpha) + \frac{\sin 2\alpha}{2} \right) \right] \]

Where, \[ V_{i_{RMS}} = V_s = \frac{V_m}{\sqrt{2}} \] = RMS value of input supply voltage (across the transformer secondary winding).

**Note:** Output RMS voltage across the load is controlled by changing '\( \alpha \)' as indicated by the expression for \( V_{O_{RMS}} \)
PLOT OF $V_{O(RMS)}$ VERSUS TRIGGER ANGLE $\alpha$ FOR A SINGLE PHASE HALF-WAVE AC VOLTAGE CONTROLLER (UNIDIRECTIONAL CONTROLLER)

$$V_{O(RMS)} = V_{\text{in}} \sqrt{\frac{1}{2\pi}} \left[ (2\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]$$

$$V_{O(RMS)} = V_s \sqrt{\frac{1}{2\pi}} \left[ (2\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]$$

By using the expression for $V_{O(RMS)}$, we can obtain the control characteristics, which is the plot of RMS output voltage $V_{O(RMS)}$ versus the trigger angle $\alpha$. A typical control characteristic of single phase half-wave phase controlled ac voltage controller is as shown below.

<table>
<thead>
<tr>
<th>Trigger angle $\alpha$ in degrees</th>
<th>Trigger angle $\alpha$ in radians</th>
<th>$V_{O(RMS)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\pi/6$</td>
<td>$V_{\text{in}} \sqrt{2}$</td>
</tr>
<tr>
<td>30°</td>
<td>$\pi/6$</td>
<td>0.992765 $V_s$</td>
</tr>
<tr>
<td>60°</td>
<td>$\pi/3$</td>
<td>0.949868 $V_s$</td>
</tr>
<tr>
<td>90°</td>
<td>$\pi/2$</td>
<td>0.866025 $V_s$</td>
</tr>
<tr>
<td>120°</td>
<td>$2\pi/3$</td>
<td>0.77314 $V_s$</td>
</tr>
<tr>
<td>150°</td>
<td>$5\pi/6$</td>
<td>0.717228 $V_s$</td>
</tr>
<tr>
<td>180°</td>
<td>$\pi$</td>
<td>0.707106 $V_s$</td>
</tr>
</tbody>
</table>

-Volution.in
Fig.: Control characteristics of single phase half-wave phase controlled ac voltage controller

Note: We can observe from the control characteristics and the table given above that the range of RMS output voltage control is from 100% of $V_S$ to 70.7% of $V_S$ when we vary the trigger angle $\alpha$ from zero to 180 degrees. Thus the half wave ac controller has the draw back of limited range RMS output voltage control.

TO CALCULATE THE AVERAGE VALUE (DC VALUE) OF OUTPUT VOLTAGE

$$V_{O_{(dc)}} = \frac{1}{2\pi} \int_{0}^{2\pi} V_m \sin \omega t \cdot d(\omega t)$$

$$V_{O_{(dc)}} = \frac{V_m}{2\pi} \int_{a}^{2\pi} \sin \omega t \cdot d(\omega t)$$

$$V_{O_{(dc)}} = \frac{V_m}{2\pi} \left[ -\cos \omega t \right]_{a}^{2\pi}$$

$$V_{O_{(dc)}} = \frac{V_m}{2\pi} \left[ -\cos 2\pi + \cos \alpha \right] \cdot \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi} \left[ \cos \alpha - 1 \right] ; \ V_s = \sqrt{2}V_m$$

Hence $V_{dc} = \frac{\sqrt{2}V_s}{2\pi} \left( \cos \alpha - 1 \right)$

When 'α' is varied from 0 to $\pi$. $V_{dc}$ varies from 0 to $\frac{-V_m}{\pi}$.

DISADVANTAGES OF SINGLE PHASE HALF WAVE AC VOLTAGE CONTROLLER:

- The output load voltage has a DC component because the two halves of the output voltage waveform are not symmetrical with respect to ‘0’ level. The input supply current waveform also has a DC component (average value) which can result in the problem of core saturation of the input supply transformer.
- The half wave ac voltage controller using a single thyristor and a single diode provides control on the thyristor only in one half cycle of the input supply. Hence ac power flow to the load can be controlled only in one half cycle.
- Half wave ac voltage controller gives limited range of RMS output voltage control. Because the RMS value of ac output voltage can be varied from a maximum of 100% of $V_S$ at a trigger angle $\alpha = 0$ to a low of 70.7% of $V_S$ at $\alpha = \pi$ Radians.

These drawbacks of single phase half wave ac voltage controller can be overcome by using a single phase full wave ac voltage controller.
APPLICATIONS OF RMS VOLTAGE CONTROLLER

- Speed control of induction motor (polyphase ac induction motor).
- Heater control circuits (industrial heating).
- Welding power control.
- Induction heating.
- On load transformer tap changing.
- Lighting control in ac circuits.
- Ac magnet controls.

Problem

1. A single phase half-wave ac voltage controller has a load resistance \( R = 50\, \Omega \), input ac supply voltage is 230V RMS at 50Hz. The input supply transformer has a turns ratio of 1:1. If the thyristor \( T_1 \) is triggered at \( \alpha = 60^\circ \). Calculate

- RMS output voltage.
- Output power.
- RMS load current and average load current.
- Input power factor.
- Average and RMS thyristor current.

Given,

\[ V_p = 230\, \text{V}\, \text{RMS} \]  
\[ f = 50\, \text{Hz} \]  
\[ R_L = 50\, \Omega \]  
\[ \alpha = 60^\circ = \frac{\pi}{3} \text{ radians} \]  
\[ V_s = \text{RMS secondary voltage} \]

\[ \frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{1}{1} = 1 \]

Therefore \[ V_p = V_s = 230\, \text{V} \]

Where, \( N_p = \text{Number of turns in the primary winding} \)  
\( N_s = \text{Number of turns in the secondary winding} \)
• RMS Value of Output (Load) Voltage \( V_{O_{\text{RMS}}} \)

\[
V_{O_{\text{RMS}}} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\alpha + \pi} V_0^2 \sin^2 \omega t \, d(\omega t)}
\]

We have obtained the expression for \( V_{O_{\text{RMS}}} \) as

\[
V_{O_{\text{RMS}}} = V_S \sqrt{\frac{1}{2\pi} \left[ (2\pi - \alpha) + \sin2\alpha \right] + \frac{\sin120^\circ}{2}}
\]

\[
V_{O_{\text{RMS}}} = 230 \sqrt{\frac{1}{2\pi} \left[ \frac{2\pi - \frac{\pi}{3}}{3} \right] + \frac{\sin120^\circ}{2}}
\]

\[
V_{O_{\text{RMS}}} = 230 \sqrt{\frac{1}{2\pi} \left[ 5.669 \right]} = 230 \times 0.94986
\]

\[
V_{O_{\text{RMS}}} = 218.4696 \, V \approx 218.47 \, V
\]

• RMS Load Current \( I_{O_{\text{RMS}}} \)

\[
I_{O_{\text{RMS}}} = \frac{V_{O_{\text{RMS}}}}{R_L} = \frac{218.46966}{50} = 4.36939 \,\text{Amps}
\]

• Output Load Power \( P_{O} \)

\[
P_{O} = I_{O_{\text{RMS}}}^2 \times R_L = (4.36939)^2 \times 50 = 954.5799 \,\text{Watts}
\]

\[
P_{O} = 0.9545799 \,\text{KW}
\]

• Input Power Factor

\[
PF = \frac{P_{O}}{V_S \times I_S}
\]

\[
V_S = \text{RMS secondary supply voltage} = 230\,\text{V}.
\]

\[
I_S = \text{RMS secondary supply current} = \text{RMS load current}.
\]

\[
\therefore I_S = I_{O_{\text{RMS}}} = 4.36939 \,\text{Amps}
\]

\[
\therefore PF = \frac{954.5799 \,\text{W}}{(230 \times 4.36939) \,\text{W}} = 0.9498
\]
• **Average Output (Load) Voltage**

\[ V_{O\,(dc)} = \frac{1}{2\pi} \left[ \int_{0}^{2\pi} V_m \sin \omega t \, d\omega t \right] \]

We have obtained the expression for the average / DC output voltage as,

\[ V_{O\,(dc)} = \frac{V_m}{2\pi} \left[ \cos \alpha - 1 \right] \]

\[ V_{O\,(dc)} = \frac{\sqrt{2} \times 230}{2\pi} \left[ \cos \left(60^\circ\right) - 1 \right] = \frac{325.2691193}{2\pi} \left[ 0.5 - 1 \right] \]

\[ V_{O\,(dc)} = \frac{325.2691193}{2\pi} \left[ -0.5 \right] = -25.88409 \text{ Volts} \]

• **Average DC Load Current**

\[ I_{O\,(dc)} = \frac{V_{O\,(dc)}}{R_L} = \frac{-25.884094}{50} = -0.51768 \text{ Amps} \]

• **Average & RMS Thyristor Currents**

Referring to the thyristor current waveform of a single phase half-wave ac voltage controller circuit, we can calculate the average thyristor current \( I_{T\,(Avg)} \) as

\[ I_{T\,(Avg)} = \frac{1}{2\pi} \int_{0}^{2\pi} I_m \sin \omega t \, d\omega t \]

\[ I_{T\,(Avg)} = \frac{I_m}{2\pi} \int_{0}^{\alpha} \sin \omega t \, d\omega t \]
\[ I_{T(Avg)} = \frac{I_m}{2\pi} \left[ -\cos \omega t \right]_{0}^{\alpha} \]

\[ I_{T(Avg)} = \frac{I_m}{2\pi} \left[ -\cos \pi + \cos \alpha \right] \]

\[ I_{T(Avg)} = \frac{I_m}{2\pi} \left[ 1 + \cos \alpha \right] \]

Where, \( I_m = \frac{V_m}{R_L} \) = Peak thyristor current = Peak load current.

\[ I_m = \frac{\sqrt{2} \times 230}{50} \]

\[ I_m = 6.505382 \text{ Amps} \]

\[ I_{T(Avg)} = \frac{V_m}{2\pi R_L} \left[ 1 + \cos \alpha \right] \]

\[ I_{T(Avg)} = \frac{\sqrt{2} \times 230}{2\pi \times 50} \left[ 1 + \cos (60') \right] \]

\[ I_{T(Avg)} = \frac{\sqrt{2} \times 230}{100\pi} \left[ 1 + 0.5 \right] \]

\[ I_{T(Avg)} = 1.5530 \text{ Amps} \]

- RMS thyristor current \( I_{T(RMS)} \) can be calculated by using the expression

\[ I_{T(RMS)} = \sqrt{\frac{1}{2\pi} \int_{0}^{\frac{\pi}{\omega}} I_m^2 \sin^2 \omega t \cdot d(\omega t)} \]

\[ I_{T(RMS)} = \sqrt{\frac{I_m^2}{2\pi} \int_{0}^{\frac{\pi}{\omega}} (1 - \cos 2\omega t) \cdot d(\omega t)} \]

\[ I_{T(RMS)} = \sqrt{\frac{I_m^2}{4\pi} \int_{0}^{\frac{\pi}{\omega}} d(\omega t) - \int_{0}^{\frac{\pi}{\omega}} \cos 2\omega t \cdot d(\omega t)} \]

\[ I_{T(RMS)} = I_m \sqrt{\frac{1}{4\pi} \left[ \left( \frac{\omega t}{\omega} \right) \left( \frac{\sin 2\omega t}{2} \right) \right]_{0}^{\frac{\pi}{\alpha}}} \]
SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (AC REGULATOR) OR RMS VOLTAGE CONTROLLER WITH RESISTIVE LOAD

Single phase full wave ac voltage controller circuit using two SCRs or a single triac is generally used in most of the ac control applications. The ac power flow to the load can be controlled in both the half cycles by varying the trigger angle \( \alpha \).

The RMS value of load voltage can be varied by varying the trigger angle \( \alpha \). The input supply current is alternating in the case of a full wave ac voltage controller and due to the symmetrical nature of the input supply current waveform there is no dc component of input supply current i.e., the average value of the input supply current is zero.

A single phase full wave ac voltage controller with a resistive load is shown in the figure below. It is possible to control the ac power flow to the load in both the half cycles by adjusting the trigger angle \( \alpha \). Hence the full wave ac voltage controller is also referred to as a bi-directional controller.
The thyristor \( T_1 \) is forward biased during the positive half cycle of the input supply voltage. The thyristor \( T_1 \) is triggered at a delay angle of \( \alpha \) (\( 0 \leq \alpha \leq \pi \) radians). Considering the ON thyristor \( T_1 \) as an ideal closed switch the input supply voltage appears across the load resistor \( R_L \) and the output voltage \( v_o = v_s \) during \( \omega t = \alpha \) to \( \pi \) radians. The load current flows through the ON thyristor \( T_1 \) and through the load resistor \( R_L \) in the downward direction during the conduction time of \( T_1 \) from \( \omega t = \alpha \) to \( \pi \) radians.

At \( \omega t = \pi \), when the input voltage falls to zero the thyristor current (which is flowing through the load resistor \( R_L \)) falls to zero and hence \( T_1 \) naturally turns off. No current flows in the circuit during \( \omega t = \pi \) to \( \pi + \alpha \).

The thyristor \( T_2 \) is forward biased during the negative cycle of input supply and when thyristor \( T_2 \) is triggered at a delay angle \( (\pi + \alpha) \), the output voltage follows the negative halfcycle of input from \( \omega t = (\pi + \alpha) \) to \( 2\pi \). When \( T_2 \) is ON, the load current flows in the reverse direction (upward direction) through \( T_2 \) during \( \omega t = (\pi + \alpha) \) to \( 2\pi \) radians. The time interval (spacing) between the gate trigger pulses of \( T_1 \) and \( T_2 \) is kept at \( \pi \) radians or 180°. At \( \omega t = 2\pi \) the input supply voltage falls to zero and hence the load current also falls to zero and thyristor \( T_2 \) turn off naturally.

Instead of using two SCR’s in parallel, a Triac can be used for full wave ac voltage control.

**Diagram:** Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC.

**Diagram:** Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC.
EQUATIONS

Input supply voltage
\[ v_s = V_m \sin \omega t = \sqrt{2} V_s \sin \omega t; \]

Output voltage across the load resistor \( R_L \);
\[ v_o = v_s = V_m \sin \omega t; \]
for \( \omega t = \alpha \) to \( \pi \) and \( \omega t = (\pi + \alpha) \) to \( 2\pi \)

Output load current
\[ i_o = \frac{v_o}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t; \]
for \( \omega t = \alpha \) to \( \pi \) and \( \omega t = (\pi + \alpha) \) to \( 2\pi \)

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT (LOAD) VOLTAGE

The RMS value of output voltage (load voltage) can be found using the expression
\[ V_{o(RMS)}^2 = V_{L(RMS)}^2 = \frac{1}{2\pi} \int_{2\pi}^{2\pi} v_L^2 d(\omega t); \]
For a full wave ac voltage controller, we can see that the two half cycles of output voltage waveforms are symmetrical and the output pulse time period (or output pulse repetition time) is \( \pi \) radians. Hence we can also calculate the RMS output voltage by using the expression given below.

\[
V_{L_{(RMS)}}^2 = \frac{1}{\pi} \int_0^\pi V_m^2 \sin^2 \omega t \, d\omega t
\]

\[
V_{L_{(RMS)}}^2 = \frac{1}{2\pi} \int_0^{2\pi} v_L^2 \, d(\omega t);
\]

\[v_L = v_O = V_m \sin \omega t; \text{ For } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi\]

Hence,

\[
V_{L_{(RMS)}}^2 = \frac{1}{2\pi} \left[ \int_0^\pi (V_m \sin \omega t)^2 \, d(\omega t) + \int_{\pi+\alpha}^{2\pi} (V_m \sin \omega t)^2 \, d(\omega t) \right]
\]

\[
= \frac{1}{2\pi} \left[ V_m^2 \int_0^\pi \sin^2 \omega t \, d(\omega t) + V_m^2 \int_{\pi+\alpha}^{2\pi} \sin^2 \omega t \, d(\omega t) \right]
\]

\[
= \frac{V_m^2}{2\pi} \left[ \frac{1 - \cos 2\omega t}{2} \right]_0^\pi + \frac{1 - \cos 2\omega t}{2} \int_{\pi+\alpha}^{2\pi} \, d(\omega t)
\]

\[
= \frac{V_m^2}{2\pi} \left[ \frac{1 - \cos 2\omega t}{2} \right]_0^\pi + \frac{1 - \cos 2\omega t}{2} \int_{\pi+\alpha}^{2\pi} \, d(\omega t)
\]

\[
= \frac{V_m^2}{4\pi} \left[ \frac{(\pi - \alpha) + (\pi - \alpha) - \frac{1}{2}(\sin 2\pi - \sin 2\alpha) - \frac{1}{2}(\sin 4\pi - \sin 2(\pi + \alpha))}{2} \right]
\]

\[
= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) - \frac{1}{2}(0 - \sin 2\alpha) - \frac{1}{2}(0 - \sin 2(\pi + \alpha)) \right]
\]

\[
= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \sin 2\alpha \frac{2}{2} + \frac{\sin 2(\pi + \alpha)}{2} \right]
\]

\[
= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right]
\]
\[
V_L^{2}_{(RMS)} = \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \sin \frac{2\alpha}{2} + \sin \frac{2\alpha}{2} \right]
\]

\[
\sin 2\pi = 0 \quad \text{&} \quad \cos 2\pi = 1
\]

Therefore,

\[
V_L^{2}_{(RMS)} = \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \sin 2\alpha \right]
\]

\[
V_L^{2}_{(RMS)} = \frac{V_m^2}{4\pi} \left[ (2\pi - 2\alpha) + \sin 2\alpha \right]
\]

Taking the square root, we get

\[
V_L^{(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{\left[(2\pi - 2\alpha) + \sin 2\alpha\right]}
\]

\[
V_L^{(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{\pi}} \sqrt{\left[(2\pi - 2\alpha) + \sin 2\alpha\right]}
\]

\[
V_L^{(RMS)} = \frac{V_m}{\sqrt{2} \sqrt{\pi}} \sqrt{\left[(2\pi - 2\alpha) + \sin \frac{2\alpha}{2}\right]}
\]

\[
V_L^{(RMS)} = \frac{V_m}{\sqrt{2} \sqrt{\pi}} \sqrt{\left[\frac{2\pi}{2}(\pi - \alpha) + \sin \frac{2\alpha}{2}\right]}
\]

\[
V_L^{(RMS)} = \frac{V_m}{\sqrt{2} \sqrt{\pi}} \sqrt{\left[\frac{2\pi}{2}(\pi - \alpha) + \sin \frac{2\alpha}{2}\right]}
\]

\[
V_L^{(RMS)} = V_L^{(RMS)} \sqrt{\left[\frac{1}{\pi}(\pi - \alpha) + \sin \frac{2\alpha}{2}\right]}
\]

\[
V_L^{(RMS)} = V_L^{(RMS)} \sqrt{\left[\frac{1}{\pi}(\pi - \alpha) + \sin \frac{2\alpha}{2}\right]}
\]

\[
V_L^{(RMS)} = V_L^{(RMS)} \sqrt{\left[\frac{1}{\pi}(\pi - \alpha) + \sin \frac{2\alpha}{2}\right]}
\]

Maximum RMS voltage will be applied to the load when \( \alpha = 0 \), in that case the full sine wave appears across the load. RMS load voltage will be the same as the RMS supply voltage \( \frac{V_m}{\sqrt{2}} \). When \( \alpha \) is increased the RMS load voltage decreases.
The output control characteristic for a single phase full wave ac voltage controller with resistive load can be obtained by plotting the equation for \(V_{o(RMS)}\).

**CONTROL CHARACTERISTIC OF SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD**

The control characteristic is the plot of RMS output voltage \(V_{o(RMS)}\) versus the trigger angle \(\alpha\); which can be obtained by using the expression for the RMS output voltage of a full-wave ac controller with resistive load:

\[
V_{o(RMS)} = V_s \sqrt{\frac{1}{\pi} \left[ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]};
\]

Where \(V_s = \frac{V_m}{\sqrt{2}}\) = RMS value of input supply voltage.

<table>
<thead>
<tr>
<th>Trigger angle (\alpha) in degrees</th>
<th>Trigger angle (\alpha) in radians</th>
<th>(V_{o(RMS)})</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(^\circ)</td>
<td>0(\pi)</td>
<td>(V_s)</td>
<td>100% (V_s)</td>
</tr>
<tr>
<td>30(^\circ)</td>
<td>(\pi/6)</td>
<td>(0.985477\ \ V_s)</td>
<td>98.54% (V_s)</td>
</tr>
<tr>
<td>60(^\circ)</td>
<td>(\pi/3)</td>
<td>(0.896938\ \ V_s)</td>
<td>89.69% (V_s)</td>
</tr>
<tr>
<td>90(^\circ)</td>
<td>(\pi/2)</td>
<td>(0.7071\ \ V_s)</td>
<td>70.7% (V_s)</td>
</tr>
<tr>
<td>120(^\circ)</td>
<td>(2\pi/3)</td>
<td>(0.44215\ \ V_s)</td>
<td>44.21% (V_s)</td>
</tr>
<tr>
<td>150(^\circ)</td>
<td>(5\pi/6)</td>
<td>(0.1698\ \ V_s)</td>
<td>16.98% (V_s)</td>
</tr>
<tr>
<td>180(^\circ)</td>
<td>(\pi)</td>
<td>(0\ \ V_s)</td>
<td>0% (V_s)</td>
</tr>
</tbody>
</table>

\[V_{L(RMS)}|_{\alpha=0} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]};\]

\[V_{L(RMS)}|_{\alpha=0} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[ (\pi + \alpha) + \frac{\sin 2\alpha}{2} \right]};\]

\[V_{L(RMS)}|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i(RMS)} = V_S;\]
We can notice from the figure, that we obtain a much better output control characteristic by using a single phase full wave ac voltage controller. The RMS output voltage can be varied from a maximum of 100% \( V_S \) at \( \alpha = 0^\circ \) to a minimum of ‘0’ at \( \alpha = 180^\circ \). Thus we get a full range output voltage control by using a single phase full wave ac voltage controller.

**Need For Isolation**

In the single phase full wave ac voltage controller circuit using two SCRs or Thyristors \( T_1 \) and \( T_2 \) in parallel, the gating circuits (gate trigger pulse generating circuits) of Thyristors \( T_1 \) and \( T_2 \) must be isolated. Figure shows a pulse transformer with two separate windings to provide isolation between the gating signals of \( T_1 \) and \( T_2 \).

**Fig.: Pulse Transformer**

**SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH COMMON CATHODE**

It is possible to design a single phase full wave ac controller with a common cathode configuration by having a common cathode point for \( T_1 \) and \( T_2 \) & by adding two diodes in a full wave ac controller circuit as shown in the figure below
Fig.: Single phase full wave ac controller with common cathode (Bidirectional controller in common cathode configuration)

Thyristor $T_1$ and diode $D_1$ are forward biased during the positive half cycle of input supply. When thyristor $T_1$ is triggered at a delay angle $\alpha$, Thyristor $T_1$ and diode $D_1$ conduct together from $\omega t = \alpha$ to $\pi$ during the positive half cycle.

The thyristor $T_2$ and diode $D_2$ are forward biased during the negative half cycle of input supply, when triggered at a delay angle $\alpha$, thyristor $T_2$ and diode $D_2$ conduct together during the negative half cycle from $\omega t = (\pi + \alpha)$ to $2\pi$.

In this circuit as there is one single common cathode point, routing of the gate trigger pulses to the thyristor gates of $T_1$ and $T_2$ is simpler and only one isolation circuit is required.

But due to the need of two power diodes the costs of the devices increase. As there are two power devices conducting at the same time the voltage drop across the ON devices increases and the ON state conducting losses of devices increase and hence the efficiency decreases.

**SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER USING A SINGLE THYRISTOR**
A single phase full wave ac controller can also be implemented with one thyristor and four diodes connected in a full wave bridge configuration as shown in the above figure. The four diodes act as a bridge full wave rectifier. The voltage across the thyristor $T_1$ and current through thyristor $T_1$ are always unidirectional. When $T_1$ is triggered at $\omega t = \alpha$, during the positive half cycle ($0 \leq \alpha \leq \pi$), the load current flows through $D_1$, $T_1$, diode $D_2$ and through the load. With a resistive load, the thyristor current (flowing through the ON thyristor $T_1$), the load current falls to zero at $\omega t = \pi$, when the input supply voltage decreases to zero.

In the negative half cycle, diodes $D_3$ & $D_4$ are forward biased during $\omega t = \pi$ to $2\pi$ radians. When $T_1$ is triggered at $\omega t = (\pi + \alpha)$, the load current flows in the opposite direction (upward direction) through the load, through $D_3$, $T_1$ and $D_4$. Thus $D_3$, $D_4$ and $T_1$ conduct together during the negative half cycle to supply the load power. When the input supply voltage becomes zero at $\omega t = 2\pi$, the thyristor current (load current) falls to zero at $\omega t = 2\pi$ and the thyristor $T_1$ naturally turns OFF. The waveforms and the expression for the RMS output voltage are the same as discussed earlier for the single phase full wave ac controller.

But however if there is a large inductance in the load circuit, thyristor $T_1$ may not be turned OFF at the zero crossing points, in every half cycle of input voltage and this may result in a loss of output control. This would require detection of the zero crossing of the load current waveform in order to ensure guaranteed turn off of the conducting thyristor before triggering the thyristor in the next half cycle, so that we gain control on the output voltage.

In this full wave ac controller circuit using a single thyristor, as there are three power devices conducting together at the same time there is more conduction voltage drop and an increase in the ON state conduction losses and hence efficiency is also reduced.

The diode bridge rectifier and thyristor (or a power transistor) act together as a bidirectional switch which is commercially available as a single device module and it has relatively low ON state conduction loss. It can be used for bidirectional load current control and for controlling the RMS output voltage.

**SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (BIDIRECTIONAL CONTROLLER) WITH RL LOAD**

In this section we will discuss the operation and performance of a single phase full wave ac voltage controller with RL load. In practice most of the loads are of RL type. For example if we consider a single phase full wave ac voltage controller controlling the speed of a single phase ac induction motor, the load which is the induction motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.
A single phase full wave ac voltage controller circuit (bidirectional controller) with an RL load using two thyristors $T_1$ and $T_2$ ($T_i$ and $T_j$ are two SCRs) connected in parallel is shown in the figure below. In place of two thyristors a single Triac can be used to implement a full wave ac controller, if a suitable Triac is available for the desired RMS load current and the RMS output voltage ratings.

**Fig: Single phase full wave ac voltage controller with RL load**

The thyristor $T_1$ is forward biased during the positive half cycle of input supply. Let us assume that $T_1$ is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to $T_1$ during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when $T_1$ is ON. The load current $i_o$ flows through the thyristor $T_1$ and through the load in the downward direction. This load current pulse flowing through $T_1$ can be considered as the positive current pulse. Due to the inductance in the load, the load current $i_o$ flowing through $T_1$ would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative.

The thyristor $T_1$ will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through $T_1$ falls to zero at $\omega t = \beta$, where $\beta$ is referred to as the Extinction angle, (the value of $\omega t$) at which the load current falls to zero. The extinction angle $\beta$ is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.
The thyristor $T_1$ conducts from $\omega \alpha = \omega \beta$. The conduction angle of $T_1$ is $\delta = (\beta - \alpha)$, which depends on the delay angle $\alpha$ and the load impedance angle $\phi$. The waveforms of the input supply voltage, the gate trigger pulses of $T_1$ and $T_2$, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.

**Fig.: Input supply voltage & Thyristor current waveforms**

$\beta$ is the extinction angle which depends upon the load inductance value.

**Fig.: Gating Signals**
Waveforms of single phase full wave AC voltage controller with RL load for $\alpha > \phi$. Discontinuous load current operation occurs for $\alpha > \phi$ and $\beta < (\pi + \alpha)$; i.e., $(\beta - \alpha) < \pi$, conduction angle $< \pi$.

Fig.: Waveforms of Input supply voltage, Load Current, Load Voltage and Thyristor Voltage across $T_1$

Note
- The RMS value of the output voltage and the load current may be varied by varying the trigger angle $\alpha$.
- This circuit, AC RMS voltage controller can be used to regulate the RMS voltage across the terminals of an ac motor (induction motor). It can be used to control the temperature of a furnace by varying the RMS output voltage.
For very large load inductance, the SCR may fail to commutate, after it is triggered and the load voltage will be a full sine wave (similar to the applied input supply voltage and the output control will be lost) as long as the gating signals are applied to the thyristors $T_1$ and $T_2$. The load current waveform will appear as a full continuous sine wave and the load current waveform lags behind the output sine wave by the load power factor angle $\phi$.

**TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t = \alpha$ to $\beta$ WHEN THYRISTOR $T_1$ CONDUCTS**

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$$v_s = V_m \sin \omega t = \text{instantaneous value of the input supply voltage.}$$

Let us assume that the thyristor $T_1$ is triggered by applying the gating signal to $T_1$ at $\omega t = \alpha$. The load current which flows through the thyristor $T_1$ during $\omega t = \alpha$ to $\beta$ can be found from the equation

$$L \left( \frac{di_o}{dt} \right) + R_i o = V_m \sin \omega t ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_o = \frac{V_m}{Z} \sin (\omega t - \phi) + Ae^{\frac{-\omega}{\tau}},$$

Where $V_m = \sqrt{2}V_s = \text{maximum or peak value of input supply voltage.}$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle (power factor angle of load).}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

Therefore the general expression for the output load current is given by the equation

$$i_o = \frac{V_m}{Z} \sin (\omega t - \phi) + Ae^{\frac{-\omega}{\tau}} ;$$
The value of the constant $A_1$ can be determined from the initial condition. i.e. initial value of load current $i_o = 0$, at $\omega t = \alpha$. Hence from the equation for $i_o$ equating $i_o$ to zero and substituting $\omega t = \alpha$, we get

$$i_o = 0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

Therefore

$$A_1 e^{\frac{-R}{L}t} = -\frac{V_m}{Z} \sin(\alpha - \phi)$$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right]$$

$$A_1 = e^{\frac{R(\omega t)}{\omega t L}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right]$$

By substituting $\omega t = \alpha$, we get the value of constant $A_1$ as

$$A_1 = e^{\frac{R(\omega t)}{\omega t L}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right]$$

Substituting the value of constant $A_1$ from the above equation into the expression for $i_o$, we obtain

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{L}t} e^{\frac{R(\omega t)}{\omega t L}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right]$$

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R(\omega t)}{\omega t L}} e^{\frac{R(\omega t)}{\omega t L}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right]$$

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R(\omega t - \phi)}{\omega t L}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right]$$

Therefore we obtain the final expression for the inductive load current of a single phase full wave ac voltage controller with RL load as

$$i_o = \frac{V_m}{Z} \left[ \sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R(\omega t - \phi)}{\omega t L}} \right]$$

; Where $\alpha \leq \omega t \leq \beta$. 
The above expression also represents the thyristor current $i_{T1}$, during the conduction time interval of thyristor $T_1$ from $\omega t = \alpha$ to $\beta$.

**To Calculate Extinction Angle $\beta$**

The extinction angle $\beta$, which is the value of $\omega t$ at which the load current $i_o$ falls to zero and $T_1$ is turned off can be estimated by using the condition that $i_o = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_o = 0 = \frac{V_m}{Z} \left[ \sin(\beta - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta - \alpha)} \right]$$

As $\frac{V_m}{Z} \neq 0$ we can write

$$\left[ \sin(\beta - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta - \alpha)} \right] = 0$$

Therefore we obtain the expression

$$\sin(\beta - \phi) = \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

The extinction angle $\beta$ can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After $\beta$ is calculated, we can determine the thyristor conduction angle $\delta = (\beta - \alpha)$.

$\beta$ is the extinction angle which depends upon the load inductance value. Conduction angle $\delta$ increases as $\alpha$ is decreased for a known value of $\beta$.

For $\delta < \pi$ radians, i.e., for $(\beta - \alpha) < \pi$ radians, for $(\pi + \alpha) > \beta$ the load current waveform appears as a discontinuous current waveform as shown in the figure. The output load current remains at zero during $\omega t = \beta$ to $(\pi + \alpha)$. This is referred to as discontinuous load current operation which occurs for $\beta < (\pi + \alpha)$.

When the trigger angle $\alpha$ is decreased and made equal to the load impedance angle $\phi$ i.e., when $\alpha = \phi$ we obtain from the expression for $\sin(\beta - \phi)$,

$$\sin(\beta - \phi) = 0$$; Therefore $(\beta - \phi) = \pi$ radians.

**Extinction angle** $\beta = (\pi + \phi) = (\pi + \alpha)$; for the case when $\alpha = \phi$

**Conduction angle** $\delta = (\beta - \alpha) = \pi$ radians = $180^\circ$; for the case when $\alpha = \phi$

Each thyristor conducts for $180^\circ$ ($\pi$ radians). $T_1$ conducts from $\omega t = \phi$ to $(\pi + \phi)$ and provides a positive load current. $T_2$ conducts from $(\pi + \phi)$ to $(2\pi + \phi)$ and provides a negative load current. Hence we obtain a continuous load current and the
output voltage waveform appears as a continuous sine wave identical to the input supply voltage waveform for trigger angle $\alpha \leq \phi$ and the control on the output is lost.

![Diagram of voltage and current waveforms](image)

**Fig.:** Output voltage and output current waveforms for a single phase full wave ac voltage controller with RL load for $\alpha \leq \phi$

Thus we observe that for trigger angle $\alpha \leq \phi$, the load current tends to flow continuously and we have continuous load current operation, without any break in the load current waveform and we obtain output voltage waveform which is a continuous sinusoidal waveform identical to the input supply voltage waveform. We loose the control on the output voltage for $\alpha \leq \phi$ as the output voltage becomes equal to the input supply voltage and thus we obtain

$$V_{o(rms)} = \frac{V_m}{\sqrt{2}} = V_s$$, for $\alpha \leq \phi$

Hence,

RMS output voltage = RMS input supply voltage for $\alpha \leq \phi$

**TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE $V_{o(rms)}$ OF A SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RL LOAD.**
When $\alpha > \phi$, the load voltage and output voltage waveforms become discontinuous as shown in the figure above.

$$V_{o_{\text{RMS}}} = \left[ \frac{1}{\pi} \int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^\frac{1}{2}$$

Output $v_o = V_m \sin \omega t$, for $\omega t = \alpha$ to $\beta$, when $T_i$ is ON.

$$V_{o_{\text{RMS}}} = \left[ \frac{1}{\pi} \int_{\alpha}^{\beta} \left( 1 - \cos 2\omega t \right) d(\omega t) \right]^\frac{1}{2}$$

$$V_{o_{\text{RMS}}} = \left[ \frac{V_m^2}{2\pi} \left( \beta - \int_{\alpha}^{\beta} \cos 2\omega t \cdot d(\omega t) \right) \right]^\frac{1}{2}$$

$$V_{o_{\text{RMS}}} = \left[ \frac{V_m^2}{2\pi} \left( \sin 2\omega t \right) \right]_{\alpha}^{\beta} \left[ \frac{1}{\pi} \int_{\alpha}^{\beta} \left( \sin 2\omega t \right) \cdot d(\omega t) \right]^\frac{1}{2}$$

The RMS output voltage across the load can be varied by changing the trigger angle $\alpha$.

For a purely resistive load $L = 0$, therefore load power factor angle $\phi = 0$.

$$\phi = \tan^{-1} \left( \frac{\omega L}{R} \right) = 0 ;$$

Extinction angle $\beta = \pi$ radians $= 180^\circ$
PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD

- **RMS Output Voltage**
  \[ V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left( (\pi - \alpha) + \frac{\sin 2\alpha}{2} \right)} \] ; \[ V = V_s = \text{RMS input supply voltage.} \]

- **Load Current**
  \[ I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L} = \text{RMS value of load current.} \]

- **Input Current**
  \[ I_s = I_{O(RMS)} = \text{RMS value of input supply current.} \]

- **Output load power**
  \[ P_o = I_{O(RMS)}^2 \times R_L \]

- **Input Power Factor**
  \[ PF = \frac{P_o}{V_s \times I_s} = \frac{I_{O(RMS)}^2 \times R_L}{V_s \times I_{O(RMS)}} = \frac{I_{O(RMS)}^2 \times R_L}{V_s} \]

- **Average Thyristor Current**
  \[ I_{T1} = \frac{1}{2\pi} \int_\alpha^{2\pi} I_m \sin \omega t.d(\omega t) \]

Fig.: Thyristor Current Waveform

- **Average Thyristor Current**
  \[ I_{T(\text{avg})} = \frac{I_m}{2\pi} \int_\alpha^{2\pi} \sin \omega t.d(\omega t) = \frac{I_m}{2\pi} \left[ -\cos \omega t \right]_{\alpha}^{2\pi} \]

  \[ I_{T(\text{avg})} = \frac{I_m}{2\pi} \left[ -\cos \pi + \cos \alpha \right] = \frac{I_m}{2\pi} \left[ 1 + \cos \alpha \right] \]
- Maximum Average Thyristor Current, for \( \alpha = 0 \),
  \[
  I_{T(\text{avg})} = \frac{I_m}{\pi}
  \]

- RMS Thyristor Current
  \[
  I_{T(\text{RMS})} = \sqrt{\frac{1}{2\pi} \int_0^{\frac{\pi}{2}} I_m^2 \sin^2 \omega t \, d(\omega t)}
  \]
  \[
  I_{T(\text{RMS})} = \frac{I_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi} \left( \frac{\pi}{2} - \alpha \right) + \frac{\sin 2\alpha}{2}}
  \]

- Maximum RMS Thyristor Current, for \( \alpha = 0 \),
  \[
  I_{T(\text{RMS})} = \frac{I_m}{2}
  \]

In the case of a single phase full wave ac voltage controller circuit using a Triac with resistive load, the average thyristor current \( I_{T(\text{avg})} = 0 \). Because the Triac conducts in both the half cycles and the thyristor current is alternating and we obtain a symmetrical thyristor current waveform which gives an average value of zero on integration.

**PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH R-L LOAD**

**The Expression for the Output (Load) Current**

The expression for the output (load) current which flows through the thyristor, during \( \omega t = \alpha \) to \( \beta \) is given by

\[
i_O = i_i = \frac{V_m}{Z} \left[ \sin(\omega t - \phi) - \sin(\alpha - \phi) e^{-R(\omega t - \alpha)} \right] ; \text{ for } \alpha \leq \omega t \leq \beta
\]

Where,

\[
V_m = \sqrt{2}V_s = \text{Maximum or peak value of input ac supply voltage.}
\]

\[
Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}
\]

\[
\phi = \tan^{-1}\left( \frac{\omega L}{R} \right) = \text{Load impedance angle (load power factor angle).}
\]

\[
\alpha = \text{Thyristor trigger angle = Delay angle.}
\]

\[
\beta = \text{Extinction angle of thyristor, (value of } \omega t \text{) at which the thyristor (load) current falls to zero.}
\]

\[
\beta \text{ is calculated by solving the equation}
\]

\[
\sin(\beta - \phi) = \sin(\alpha - \phi) e^{-R(\beta - \alpha)}
\]
Thyristor Conduction Angle \( \delta = (\beta - \alpha) \)

Maximum thyristor conduction angle \( \delta = (\beta - \alpha) = \pi \) radians = 180° for \( \alpha \leq \phi \).

RMS Output Voltage

\[
V_{o(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left( \frac{\cos \alpha - \sin \alpha \sin \beta}{2} - \frac{\sin \beta}{2} \right)}
\]

The Average Thyristor Current

\[
I_{T(Avg)} = \frac{1}{2\pi} \int_{\alpha}^{\beta} i_d(\omega t) \, d(\omega t)
\]

\[
I_{T(Avg)} = \frac{1}{2\pi} \int_{\alpha}^{\beta} \frac{V_m}{Z} \left[ \sin(\omega t - \phi) - \sin(\omega t - \phi) e^{-\frac{R}{Z}(\omega t - \alpha)} \right] \, d(\omega t)
\]

\[
I_{T(Avg)} = \frac{V_m}{2\pi Z} \left[ \int_{\alpha}^{\beta} \sin(\omega t - \phi) \, d(\omega t) - \int_{\alpha}^{\beta} \sin(\omega t - \phi) e^{-\frac{R}{Z}(\omega t - \alpha)} \, d(\omega t) \right]
\]

Maximum value of \( I_{T(Avg)} \) occur at \( \alpha = 0 \). The thyristors should be rated for maximum \( I_{T(Avg)} = \left( \frac{I_m}{\pi} \right) \), where \( I = \frac{V_m}{Z} \).

RMS Thyristor Current \( I_{T(RMS)} \)

\[
I_{T(RMS)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\beta} i_d^2(\omega t) \, d(\omega t)}
\]

Maximum value of \( I_{T(RMS)} \) occurs at \( \alpha = 0 \). Thyristors should be rated for maximum \( I_{T(RMS)} = \left( \frac{I_m}{2} \right) \).

When a Triac is used in a single phase full wave ac voltage controller with RL type of load, then \( I_{T(Avg)} = 0 \) and maximum \( I_{T(RMS)} = \frac{I_m}{\sqrt{2}} \).
PROBLEMS

1. A single phase full wave ac voltage controller supplies an RL load. The input supply voltage is 230V, RMS at 50Hz. The load has \( L = 10 \text{mH} \), \( R = 10 \Omega \), the delay angle of thyristors \( T_1 \) and \( T_2 \) are equal, where \( \alpha_1 = \alpha_2 = \frac{\pi}{3} \). Determine
   
   a. Conduction angle of the thyristor \( T_1 \).
   
   b. RMS output voltage.
   
   c. The input power factor.

Comment on the type of operation.

Given

\[
V_s = 230V, \quad f = 50Hz, \quad L = 10\text{mH}, \quad R = 10\Omega, \quad \alpha = 60^\circ, \\
\alpha_1 = \alpha_2 = \frac{\pi}{3} \text{ radians}
\]

\[
V_m = \sqrt{2} V_s = \sqrt{2} \times 230 = 325.2691193 \text{ V}
\]

\[
Z = \text{Load Impedance} = \sqrt{R^2 + (\omega L)^2} = \sqrt{(10)^2 + (\omega L)^2}
\]

\[
\omega L = (2\pi fL) = (2\pi \times 50 \times 10 \times 10^{-3}) = \pi = 3.14159\Omega
\]

\[
Z = \sqrt{(10)^2 + (3.14159)^2} = \sqrt{109.8696} = 10.4818\Omega
\]

\[
I_m = \frac{V_m}{Z} = \frac{\sqrt{2} \times 230}{10.4818} = 31.03179 \text{ A}
\]

Load Impedance Angle \( \phi = \tan^{-1} \left( \frac{\omega L}{R} \right) \)

\[
\phi = \tan^{-1} \left( \frac{\pi}{10} \right) = \tan^{-1} (0.314159) = 17.44059^\circ
\]

Trigger Angle \( \alpha > \phi \). Hence the type of operation will be discontinuous load current operation, we get

\[
\beta < (\pi + \alpha)
\]

\[
\beta < (180 + 60) ; \beta < 240^\circ
\]

Therefore the range of \( \beta \) is from 180 degrees to 240 degrees. (\( 180^\circ < \beta < 240^\circ \))
Extinction Angle $\beta$ is calculated by using the equation

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-\pi}{\beta \omega (\beta - \alpha)}}$$

In the exponential term the value of $\alpha$ and $\beta$ should be substituted in radians. Hence

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-\pi}{\beta \omega (\beta - \alpha)}}; \quad \alpha_{\text{Rad}} = \frac{\pi}{3}$$

$$\left(\alpha - \phi\right) = \left(60 - 17.44059\right) = 42.559^\circ$$

$$\sin(\beta - 17.44^\circ) = \sin(42.559^\circ)e^{-\frac{\pi}{10}}(\beta - \alpha)$$

$$\sin(\beta - 17.44^\circ) = 0.676354e^{-3.183(\beta - \alpha)}$$

$180^\circ \rightarrow \pi$ radians, $\beta_{\text{Rad}} = \frac{\beta^\circ \times \pi}{180^\circ}$

Assuming $\beta = 190^\circ$;

$$\beta_{\text{Rad}} = \frac{190^\circ \times \pi}{180^\circ} = 3.1161$$

L.H.S: $\sin(190 - 17.44) = \sin(172.56) = 0.129487$

R.H.S: $0.676354e^{-3.183(\beta - \alpha)} = 4.94 \times 10^{-4}$

Assuming $\beta = 183^\circ$;

$$\beta_{\text{Rad}} = \frac{183^\circ \times \pi}{180^\circ} = 3.19395$$

$$\left(\beta - \alpha\right) = \left(3.19395 - \frac{\pi}{3}\right) = 2.14675$$

L.H.S: $\sin(\beta - \phi) = \sin(183 - 17.44) = \sin(165.56^\circ) = 0.24936$

R.H.S: $0.676354e^{-3.183(2.14675)} = 7.2876 \times 10^{-4}$

Assuming $\beta \approx 180^\circ$

$$\beta_{\text{Rad}} = \frac{\beta^\circ \times \pi}{180^\circ} = \frac{180^\circ \times \pi}{180} = \pi$$

$$\left(\beta - \alpha\right) = \left(\pi - \frac{\pi}{3}\right) = \left(\frac{2\pi}{3}\right)$$
L.H.S: \( \sin(\beta - \phi) = \sin(196 - 17.44) \approx 0.02513 \)

R.H.S: \( 0.676354 e^{-3.183 \left( \frac{430.676354 - \pi}{3} \right)} = 3.5394 \times 10^{-4} \)

Assuming \( \beta = 196^\circ \)

\[ \beta_{\text{rad}} = \frac{\beta^\circ \times \pi}{180^\circ} = \frac{196^\circ \times \pi}{180^\circ} = 3.420845 \]

L.H.S: \( \sin(\beta - \phi) = \sin(196 - 17.44) = 0.02513 \)

R.H.S: \( 0.676354 e^{-3.183 \left( \frac{430.676354 - \pi}{3} \right)} = 3.5394 \times 10^{-4} \)

Assuming \( \beta = 197^\circ \)

\[ \beta_{\text{rad}} = \frac{\beta^\circ \times \pi}{180^\circ} = \frac{197^\circ \times \pi}{180^\circ} = 3.43829 \]

L.H.S: \( \sin(\beta - \phi) = \sin(197 - 17.44) \approx 7.69 = 7.67937 \times 10^{-3} \)

R.H.S: \( 0.676354 e^{-3.183 \left( \frac{3.43829 - \pi}{3} \right)} = 4.950386476 \times 10^{-4} \)

Assuming \( \beta = 197.42^\circ \)

\[ \beta_{\text{rad}} = \frac{\beta^\circ \times \pi}{180^\circ} = \frac{197.42^\circ \times \pi}{180^\circ} = 3.4456 \]

L.H.S: \( \sin(\beta - \phi) = \sin(197.42 - 17.44) = 3.4906 \times 10^{-4} \)

R.H.S: \( 0.676354 e^{-3.183 \left( \frac{3.4456 - \pi}{3} \right)} = 3.2709 \times 10^{-4} \)

Conduction Angle \( \delta = (\beta - \alpha) = (197.42^\circ - 60^\circ) = 137.42^\circ \)

RMS Output Voltage

\[
V_{O(RMS)} = V_x \sqrt{\frac{1}{\pi} \left[ (\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right]} 
\]

\[
V_{O(RMS)} = 230 \sqrt{\frac{1}{\pi} \left[ \left(3.4456 - \frac{\pi}{3}\right) + \frac{\sin 2(60^\circ)}{2} - \frac{\sin 2(197.42^\circ)}{2} \right]} 
\]

\[
V_{O(RMS)} = 230 \sqrt{\frac{1}{\pi} \left[(2.39843) + 0.4330 - 0.285640 \right]} 
\]

\[
V_{O(RMS)} = 230 \times 0.9 = 207.0445 \text{ V} 
\]
Input Power Factor

\[ PF = \frac{P_o}{V_s \times I_s} \]

\[ I_{o(RMS)} = \frac{V_{o(RMS)}}{Z} = \frac{207.0445}{10.4818} = 19.7527 \text{ A} \]

\[ P_o = I_{o(RMS)}^2 \times R_L = (19.7527)^2 \times 10 = 3901.716 \text{ W} \]

\[ V_s = 230V, \quad I_s = I_{o(RMS)} = 19.7527 \]

\[ PF = \frac{P_o}{V_s \times I_s} = \frac{3901.716}{230 \times 19.7527} = 0.8588 \]

2. A single phase full wave controller has an input voltage of 120V (RMS) and a load resistance of 6 ohm. The firing angle of thyristor is \( \frac{\pi}{2} \). Find
   a. RMS output voltage
   b. Power output
   c. Input power factor
   d. Average and RMS thyristor current.

Solution

\[ \alpha = \frac{\pi}{2} = 90^0, \quad V_s = 120 \text{ V}, \quad R = 6\Omega \]

RMS Value of Output Voltage

\[ V_o = V_s \left[ \frac{1}{\pi} \left( \frac{\pi - \alpha + \sin 2\alpha}{2} \right) \right]^{\frac{1}{2}} \]

\[ V_o = 120 \left[ \frac{1}{\pi} \left( \frac{\pi - \pi}{2} + \sin 180 \right) \right]^{\frac{1}{2}} \]

\[ V_o = 84.85 \text{ Volts} \]

RMS Output Current

\[ I_o = \frac{V_o}{R} = \frac{84.85}{6} = 14.14 \text{ A} \]

Load Power

\[ P_o = I_o^2 \times R \]

\[ P_o = (14.14)^2 \times 6 = 1200 \text{ watts} \]
Input Current is same as Load Current
Therefore \( I_S = I_o = 14.14 \text{ Amps} \)
Input Supply Volt-Amp = \( V_S I_S = 120 \times 14.14 = 1696.8 \text{ VA} \)

Therefore
\[
\text{Input Power Factor} = \frac{\text{Load Power}}{\text{Input Volt-Amp}} = \frac{1200}{1696.8} = 0.707 \text{ (lag)}
\]

Each Thyristor Conducts only for half a cycle

Average thyristor current \( I_{T(Avg)} \)
\[
I_{T(Avg)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t. d(\omega t)
\]
\[
= \frac{V_m}{2\pi R} (1 + \cos \alpha) ; \quad V_m = \sqrt{2} V_S
\]
\[
= \frac{\sqrt{2} \times 120}{2\pi \times 6}[1 + \cos 90] = 4.5 \text{ A}
\]

RMS thyristor current \( I_{T(RMS)} \)
\[
I_{T(RMS)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m^2}{R^2} \sin^2 \omega t. d(\omega t)}
\]
\[
= \sqrt{\frac{V_m^2}{2\pi R^2} \frac{1}{2} \frac{1 - \cos 2\omega t}{2} d(\omega t)}
\]
\[
= \frac{V_m}{2R} \left[ \frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}
\]
\[
= \frac{\sqrt{2} V_S}{2R} \left[ \frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}
\]
\[
= \frac{\sqrt{2} \times 120}{2 \times 6} \left[ \frac{1}{\pi} \left( \pi - \frac{\pi}{2} + \frac{\sin 180}{2} \right) \right]^{\frac{1}{2}} = 10 \text{ Amps}
\]
3. A single phase half wave rectifier, using one SCR in anti-parallel with a diode feeds 1 kW, 230 V heater. Find load power for a firing angle of $45^0$.

**Solution**

$$\alpha = 45^0 = \frac{\pi}{4}, \quad V_s = 230 \, \text{V} \ ; \quad P_o = 1 \, \text{KW} = 1000 \, \text{W}$$

At standard rms supply voltage of 230V, the heater dissipates 1KW of output power.

Therefore

$$P_o = V_o \times I_o = \frac{V_o \times V_o}{R} = \frac{V_o^2}{R}$$

Resistance of heater

$$R = \frac{V_o^2}{P_o} = \frac{(230)^2}{1000} = 52.9 \, \Omega$$

RMS value of output voltage

$$V_o = V_s \left[ \frac{1}{2\pi} \left( 2\pi - \alpha + \frac{\sin2\alpha}{2} \right) \right]^{1/2} : \text{for firing angle } \alpha = 45^0$$

$$V_o = 230 \left[ \frac{1}{2\pi} \left( 2\pi - \frac{\pi}{4} + \frac{\sin90}{2} \right) \right]^{1/2} = 224.7157 \, \text{Volts}$$

RMS value of output current

$$I_o = \frac{V_o}{R} = \frac{224.7}{52.9} = 4.2479 \, \text{Amps}$$

Load Power

$$P_o = I_o^2 \times R = (4.25)^2 \times 52.9 = 954.56 \, \text{Watts}$$

4. Find the RMS and average current flowing through the heater shown in figure. The delay angle of both the SCRs is $45^0$. 

![Diagram of SCR circuit](https://example.com/diagram.png)
Solution

\[ \alpha = 45^\circ = \frac{\pi}{4}, \quad V_s = 220 \text{ V} \]

**Resistance of heater**

\[ R = \frac{V^2}{R} = \frac{(220)^2}{1000} = 48.4 \Omega \]

**Resistance value of output voltage**

\[ V_o = V_s \frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right) \]

\[ V_o = 220 \sqrt{\frac{1}{\pi} \left( \pi - \frac{\pi}{4} + \frac{\sin 90^\circ}{2} \right)} \]

\[ V_o = 220 \sqrt{\frac{1}{\pi} \left( \pi - \frac{\pi}{4} + \frac{1}{2} \right)} = 209.769 \text{ Volts} \]

**RMS current flowing through heater**

\[ I_{RMS} = \frac{V_o}{R} = \frac{209.769}{48.4} = 4.334 \text{ Amps} \]

**Average current flowing through the heater**

\[ I_{AVG} = 0 \]

5. A single phase voltage controller is employed for controlling the power flow from a 220 V, 50 Hz source into a load circuit consisting of \( R = 4 \Omega \) and \( \omega L = 6 \Omega \). Calculate the following:
   a. Control range of firing angle
   b. Maximum value of RMS load current
   c. Maximum power and power factor
   d. Maximum value of average and RMS thyristor current.

**Solution**

For control of output power, minimum angle of firing angle \( \alpha \) is equal to the load impedance angle \( \theta \)

\[ \alpha = \theta, \quad \text{load angle} \]

\[ \theta = \tan^{-1}\left( \frac{\omega L}{R} \right) = \tan^{-1}\left( \frac{6}{4} \right) = 56.3^\circ \]

Maximum possible value of \( \alpha \) is 180\(^\circ\)
Therefore control range of firing angle is 56.3\(^\circ\) < \alpha < 180\(^\circ\)
Maximum value of RMS load current occurs when $\alpha = \theta = 56.3^\circ$. At this value of $\alpha$ the Maximum value of RMS load current

$$I_o = \frac{V_s}{Z} = \frac{220}{\sqrt{4^2 + 6^2}} = 30.5085 \text{ Amps}$$

**Maximum Power**

$$P_o = I_o^2 R = \left(30.5085\right)^2 \times 4 = 3723.077 \text{ W}$$

**Input Volt-Amp**

$$V_sI_o = 220 \times 30.5085 = 6711.87 \text{ W}$$

Power Factor

$$\text{Power Factor} = \frac{P_o}{\text{Input VA}} = \frac{3723.077}{6711.87} = 0.5547$$

**Average thyristor current** will be maximum when $\alpha = \theta$ and conduction angle $\gamma = 180^\circ$.

Therefore maximum value of average thyristor current

$$I_{T(avg)} = \frac{1}{2\pi} \int_{a}^{\pi+a} \frac{V_m}{Z} \sin(\omega t - \theta) d(\omega t)$$

**Note:**

$$i_o = i_i = \frac{V_m}{Z} \left[ \sin(\omega t - \theta) - \sin(\alpha - \theta) e^{j\theta} \right]$$

At $\alpha = 0$,

$$i_i = i_o = \frac{V_m}{Z} \sin(\omega t - \theta)$$

$$I_{T(avg)} = \frac{V_m}{2\pi Z} \left[ \cos(\omega t - \theta) \right]_{a}^{\pi+a}$$

But $\alpha = \theta$,

$$I_{T(avg)} = \frac{V_m}{2\pi Z} \left[ -\cos(\pi + \alpha - \theta) + \cos(\alpha - \theta) \right]$$

$$\therefore I_{T(avg)} = \frac{V_m}{\pi Z} \left[ -\cos(\pi) + \cos(0) \right] = \frac{V_m}{\pi Z} \left[ 2 \right] = \frac{V_m}{\pi Z} \times 220 = \frac{\sqrt{2} \times 220}{\pi \sqrt{4^2 + 6^2}} = 13.7336 \text{ Amps}$$

Similarly, maximum RMS value occurs when $\alpha = 0$ and $\gamma = \pi$.

Therefore maximum value of RMS thyristor current

$$I_{TM} = \sqrt{\frac{1}{2\pi} \int_{a}^{\pi+a} \left\{ \frac{V_m}{Z} \sin(\omega t - \theta) \right\}^2 d(\omega t)}$$
\[ I_{TM} = \sqrt{\frac{v_m}{2\pi Z^2}} \int_{a}^{\pi+a} \left[ \cos\left(\frac{2ot-2\theta}{2}\right) \right] d(\omega t) \]

\[ I_{TM} = \sqrt{\frac{V_m^2}{4\pi Z^2}} \left[ \omega t - \frac{\sin\left(2\omega t - 2\theta\right)}{2} \right]_{a}^{\pi+a} \]

\[ I_{TM} = \sqrt{\frac{V_m^2}{4\pi Z^2}} \left[ \pi + \alpha - \alpha - 0 \right] \]

\[ I_{TM} = \frac{V_m}{2Z} = \frac{\sqrt{2 \times 220}}{2\sqrt{4^2+6^2}} = 21.57277 \text{ Amps} \]
CONTROLLED RECTIFIERS  
(*Line Commutated AC to DC converters*)

INTRODUCTION TO CONTROLLED RECTIFIERS

Controlled rectifiers are line commutated ac to dc power converters which are used to convert a fixed voltage, fixed frequency ac power supply into variable dc output voltage.

Type of input: Fixed voltage, fixed frequency ac power supply.
Type of output: Variable dc output voltage

The input supply fed to a controlled rectifier is ac supply at a fixed rms voltage and at a fixed frequency. We can obtain variable dc output voltage by using controlled rectifiers. By employing phase controlled thyristors in the controlled rectifier circuits we can obtain variable dc output voltage and variable dc (average) output current by varying the trigger angle (phase angle) at which the thyristors are triggered. We obtain a unidirectional and pulsating load current waveform which has a specific average value.

The thyristors are forward biased during the positive half cycle of input supply and can be turned ON by applying suitable gate trigger pulses at the thyristor gate leads. The thyristor current and the load current begin to flow once the thyristors are triggered (turned ON) say at $ot = \alpha$. The load current flows when the thyristors conduct from $ot = \alpha$ to $\beta$. The output voltage across the load follows the input supply voltage through the conducting thyristor. At $ot = \beta$, when the load current falls to zero, the thyristors turn off due to AC line (natural) commutation.

In some bridge controlled rectifier circuits the conducting thyristor turns off, when the other thyristor is (other group of thyristors are) turned ON.

The thyristor remains reverse biased during the negative half cycle of input supply. The type of commutation used in controlled rectifier circuits is referred to AC line commutation or Natural commutation or AC phase commutation.

When the input ac supply voltage reverses and becomes negative during the negative half cycle, the thyristor becomes reverse biased and hence turns off. There are several types of power converters which use ac line commutation. These are referred to as line commutated converters.

Different types of line commutated converters are

- Phase controlled rectifiers which are AC to DC converters.
- AC to AC converters
  - AC voltage controllers, which convert input ac voltage into variable ac output voltage at the same frequency.
  - Cyclo converters, which give low output frequencies.
All these power converters operate from ac power supply at a fixed rms input supply voltage and at a fixed input supply frequency. Hence they use ac line commutation for turning off the thyristors after they have been triggered ON by the gating signals.

DIFFERENCES BETWEEN DIODE RECTIFIERS AND PHASE CONTROLLED RECTIFIERS

The diode rectifiers are referred to as uncontrolled rectifiers which make use of power semiconductor diodes to carry the load current. The diode rectifiers give a fixed dc output voltage (fixed average output voltage) and each diode rectifying element conducts for one half cycle duration (T/2 seconds), that is the diode conduction angle = $180^0$ or $\pi$ radians.

A single phase half wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{o(dc)} = \frac{V_m}{\pi}$ and single phase full wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{o(dc)} = \frac{2V_m}{\pi}$, where $V_m$ is maximum value of the available ac supply voltage.

Thus we note that we can not control (we can not vary) the dc output voltage or the average dc load current in a diode rectifier circuit.

In a phase controlled rectifier circuit we use a high current and a high power thyristor device (silicon controlled rectifier; SCR) for conversion of ac input power into dc output power.

Phase controlled rectifier circuits are used to provide a variable voltage output dc and a variable dc (average) load current.

We can control (we can vary) the average value (dc value) of the output load voltage (and hence the average dc load current) by varying the thyristor trigger angle.

We can control the thyristor conduction angle $\delta$ from $180^0$ to $0^0$ by varying the trigger angle $\alpha$ from $0^0$ to $180^0$, where thyristor conduction angle $\delta = (\pi - \alpha)$

APPLICATIONS OF PHASE CONTROLLED RECTIFIERS

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Reactor controls.
- Portable hand tool drives.
- Variable speed industrial drives.
- Battery charges.
- High voltage DC transmission.
- Uninterruptible power supply systems (UPS).

Some years back ac to dc power conversion was achieved using motor generator sets, mercury arc rectifiers, and thyratrons tubes. The modern ac to dc power converters are designed using high power, high current thyristors and presently most of the ac-dc power converters are thyristorised power converters. The thyristor devices are phase controlled to obtain a variable dc output voltage across the output load terminals. The
Phase controlled thyristor converter uses ac line commutation (natural commutation) for commutating (turning off) the thyristors that have been turned ON.

The phase controlled converters are simple and less expensive and are widely used in industrial applications for industrial dc drives. These converters are classified as two quadrant converters if the output voltage can be made either positive or negative for a given polarity of output load current. There are also single quadrant ac-dc converters where the output voltage is only positive and cannot be made negative for a given polarity of output current. Of course single quadrant converters can also be designed to provide only negative dc output voltage.

The two quadrant converter operation can be achieved by using fully controlled bridge converter circuit and for single quadrant operation we use a half controlled bridge converter.

**CLASSIFICATION OF PHASE CONTROLLED RECTIFIERS**

The phase controlled rectifiers can be classified based on the type of input power supply as

- **Single Phase Controlled Rectifiers** which operate from single phase ac input power supply.
- **Three Phase Controlled Rectifiers** which operate from three phase ac input power supply.

**DIFFERENT TYPES OF SINGLE PHASE CONTROLLED RECTIFIERS**

*Single Phase Controlled Rectifiers* are further subdivided into different types

- **Half wave controlled rectifier** which uses a single thyristor device (which provides output control only in one half cycle of input ac supply, and it provides low dc output).
- **Full wave controlled rectifiers** (which provide higher dc output)
  - Full wave controlled rectifier using a center tapped transformer (which requires two thyristors).
  - Full wave bridge controlled rectifiers (which do not require a center tapped transformer)
    - *Single phase semi-converter* (half controlled bridge converter, using two SCR’s and two diodes, to provide single quadrant operation).
    - *Single phase full converter* (fully controlled bridge converter which requires four SCR’s, to provide two quadrant operation).

*Three Phase Controlled Rectifiers* are of different types

- Three phase half wave controlled rectifiers.
- Three phase full wave controlled rectifiers.
  - Semi converter (half controlled bridge converter).
  - Full converter (fully controlled bridge converter).

**PRINCIPLE OF PHASE CONTROLLED RECTIFIER OPERATION**

The basic principle of operation of a phase controlled rectifier circuit is explained with reference to a single phase half wave phase controlled rectifier circuit with a resistive load shown in the figure.
A single phase half wave thyristor converter which is used for ac-dc power conversion is shown in the above figure. The input ac supply is obtained from a main supply transformer to provide the desired ac supply voltage to the thyristor converter depending on the output dc voltage required. $v_p$ represents the primary input ac supply voltage. $v_s$ represents the secondary ac supply voltage which is the output of the transformer secondary.

During the positive half cycle of input supply when the upper end of the transformer secondary is at a positive potential with respect to the lower end, the thyristor anode is positive with respect to its cathode and the thyristor is in a forward biased state. The thyristor is triggered at a delay angle of $\omega t = \alpha$, by applying a suitable gate trigger pulse to the gate lead of thyristor. When the thyristor is triggered at a delay angle of $\omega t = \alpha$, the thyristor conducts and assuming an ideal thyristor, the thyristor behaves as a closed switch and the input supply voltage appears across the load when the thyristor conducts from $\alpha \omega = \alpha$ to $\pi$ radians. Output voltage $v_o = v_s$, when the thyristor conducts from $\omega t = \alpha$ to $\pi$.

For a purely resistive load, the load current $i_o$ (output current) that flows when the thyristor $T_1$ is on, is given by the expression

$$i_o = \frac{v_o}{R_L}, \text{ for } \alpha \leq \omega t \leq \pi$$

The output load current waveform is similar to the output load voltage waveform during the thyristor conduction time from $\alpha$ to $\pi$. The output current and the output voltage waveform are in phase for a resistive load. The load current increases as the input supply voltage increases and the maximum load current flows at $\omega t = \frac{\pi}{2}$, when the input supply voltage is at its maximum value.

The maximum value (peak value) of the load current is calculated as

$$i_{o(max)} = I_m = \frac{V_m}{R_L}.$$
Note that when the thyristor conducts \((t_1 < t < \alpha \pi)\) during \(\omega t = \alpha\) to \(\pi\), the thyristor current \(i_{r1}\), the load current \(i_o\) through \(R_L\) and the source current \(i_s\) flowing through the transformer secondary winding are all one and the same.

Hence we can write

\[
i_s = i_{r1} = i_o = \frac{V_o}{R} = \frac{V_o \sin \omega t}{R} \quad ; \quad \text{for} \quad \alpha \leq \omega t \leq \pi
\]

\(I_m\) is the maximum (peak) value of the load current that flows through the transformer secondary winding, through \(T_i\) and through the load resistor \(R_L\) at the instant \(\omega t = \frac{\pi}{2}\), when the input supply voltage reaches its maximum value.

When the input supply voltage decreases the load current decreases. When the supply voltage falls to zero at \(\omega t = \pi\), the thyristor and the load current also falls to zero at \(\omega t = \pi\). Thus the thyristor naturally turns off when the current flowing through it falls to zero at \(\omega t = \pi\).

During the negative half cycle of input supply when the supply voltage reverses and becomes negative during \(\omega t = \pi\) to \(2\pi\) radians, the anode of thyristor is at a negative potential with respect to its cathode and as a result the thyristor is reverse biased and hence it remains cut-off (in the reverse blocking mode). The thyristor cannot conduct during its reverse biased state between \(\omega t = \pi\) to \(2\pi\). An ideal thyristor under reverse biased condition behaves as an open switch and hence the load current and load voltage are zero during \(\omega t = \pi\) to \(2\pi\). The maximum or peak reverse voltage that appears across the thyristor anode and cathode terminals is \(V_o\).

The trigger angle \(\alpha\) (delay angle or the phase angle \(\alpha\)) is measured from the beginning of each positive half cycle to the time instant when the gate trigger pulse is applied. The thyristor conduction angle is from \(\alpha\) to \(\pi\), hence the conduction angle \(\delta = (\pi - \alpha)\). The maximum conduction angle is \(\pi\) radians (180°) when the trigger angle \(\alpha = 0\).

The waveforms shows the input ac supply voltage across the secondary winding of the transformer which is represented as \(v_s\), the output voltage across the load, the output (load) current, and the thyristor voltage waveform that appears across the anode and cathode terminals.

![Image of Quadrant Diagram](image-url)
Fig: Waveforms of single phase half-wave controlled rectifier with resistive load

EQUATIONS

\[ v_s = V_m \sin \omega t \]  
the ac supply voltage across the transformer secondary.

\[ V_m = \text{max. (peak) value of input ac supply voltage across transformer secondary.} \]

\[ V_s = \frac{V_m}{\sqrt{2}} \]  
RMS value of input ac supply voltage across transformer secondary.

\[ v_o = v_L = \text{the output voltage across the load} \; ; \; i_o = i_L = \text{output (load) current.} \]
When the thyristor is triggered at \( t = \omega_\alpha \) (an ideal thyristor behaves as a closed switch) and hence the output voltage follows the input supply voltage.

\[
V_o = V_L = V_m \sin \omega t ; \quad \text{for } \omega t = \alpha \text{ to } \pi , \text{ when the thyristor is on.}
\]

\[
i_o = i_L = \frac{V_o}{R} = \text{Load current for } \omega t = \alpha \text{ to } \pi , \text{ when the thyristor is on.}
\]

**TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE ACROSS THE LOAD**

If \( V_m \) is the peak input supply voltage, the average output voltage \( V_{dc} \) can be found from

\[
V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_0^{\alpha} V_o.d(\omega t)
\]

\[
V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_0^{\alpha} V_m \sin \omega t.d(\omega t)
\]

\[
V_{O(dc)} = \frac{1}{2\pi} \int_0^{\alpha} V_m \sin \omega t.d(\omega t)
\]

\[
V_{O(dc)} = \frac{V_m}{2\pi} \int_0^{\alpha} \sin \omega t.d(\omega t)
\]

\[
V_{O(dc)} = \frac{V_m}{2\pi} \left[ -\cos \omega t \right]_0^{\alpha}
\]

\[
V_{O(dc)} = \frac{V_m}{2\pi} \left[ -\cos \pi + \cos \alpha \right] \quad ; \quad \cos \pi = -1
\]

\[
V_{O(dc)} = \frac{V_m}{2\pi} \left[ 1 + \cos \alpha \right] \quad ; \quad V_m = \sqrt{2}V_s
\]

The maximum average (dc) output voltage is obtained when \( \alpha = 0 \) and the maximum dc output voltage \( V_{dc(max)} = V_{dc} = \frac{V_m}{\pi} \).

The average dc output voltage can be varied by varying the trigger angle \( \alpha \) from 0 to a maximum of \( 180^\circ (\pi \text{ radians}) \).

We can plot the control characteristic, which is a plot of dc output voltage versus the trigger angle \( \alpha \) by using the equation for \( V_{O(dc)} \).
CONTROL CHARACTERISTIC OF SINGLE PHASE HALF WAVE PHASE CONTROLED RECTIFIER WITH RESISTIVE LOAD

The average dc output voltage is given by the expression

\[ V_{O(d)} = \frac{V_m}{2\pi} [1 + \cos \alpha] \]

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle \( \alpha \)

<table>
<thead>
<tr>
<th>Trigger angle ( \alpha ) in degrees</th>
<th>( V_{O(d)} )</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( V_{dm} = \frac{V_m}{\pi} )</td>
<td>100% ( V_{dm} )</td>
</tr>
<tr>
<td>30°</td>
<td>0.933 ( V_{dm} )</td>
<td>93.3% ( V_{dm} )</td>
</tr>
<tr>
<td>60°</td>
<td>0.75 ( V_{dm} )</td>
<td>75% ( V_{dm} )</td>
</tr>
<tr>
<td>90°</td>
<td>0.5 ( V_{dm} )</td>
<td>50% ( V_{dm} )</td>
</tr>
<tr>
<td>120°</td>
<td>0.25 ( V_{dm} )</td>
<td>25% ( V_{dm} )</td>
</tr>
<tr>
<td>150°</td>
<td>0.06698 ( V_{dm} )</td>
<td>6.69% ( V_{dm} )</td>
</tr>
<tr>
<td>180°</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig.: Control characteristic

Normalizing the dc output voltage with respect to \( V_{dm} \), the normalized output voltage

\[ V_{dcn} = \frac{V_{O(d)}}{V_{dc(max)}} = \frac{V_{dc}}{V_{dm}} \]
TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE OF A SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The rms output voltage is given by

\[ V_{o\,(RMS)} = \frac{1}{2\pi} \int_{0}^{\frac{2\pi}{\omega}} \sqrt{V_{o}^2} \, d(\omega t) \]

Output voltage \( v_o = V_m \sin \omega t \); for \( \omega t = \alpha \) to \( \pi \)

\[ V_{o\,(RMS)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t) \]

By substituting \( \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \), we get

\[ V_{o\,(RMS)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \frac{1 - \cos 2\omega t}{2} \, d(\omega t) \]

\[ V_{o\,(RMS)} = \frac{V_m^2}{4\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \, d(\omega t) \]

\[ V_{o\,(RMS)} = \frac{V_m^2}{4\pi} \left\{ \left[ d(\omega t) \right]_{\alpha}^{\pi} - \left[ \cos 2\omega t \, d(\omega t) \right]_{\alpha}^{\pi} \right\} \]

\[ V_{o\,(RMS)} = \frac{V_m}{2} \left\{ \frac{1}{\pi} \left[ (\omega t) \right]_{\alpha}^{\pi} - \left[ \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} \right\} \]

\[ V_{o\,(RMS)} = \frac{V_m}{2} \left\{ \frac{1}{\pi} \left[ \frac{\sin 2\pi - \sin 2\alpha}{2} \right] \right\} ; \quad \sin 2\pi = 0 \]
Hence we get,

\[
V_{O(RMS)} = \frac{V_m}{2} \left[ \frac{1}{\pi} \left( \pi - \alpha \right) + \sin \frac{2\alpha}{2} \right]^{\frac{1}{2}}
\]

\[
V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \left( \pi - \alpha + \sin \frac{2\alpha}{2} \right)^{\frac{1}{2}}
\]

PERFORMANCE PARAMETERS OF PHASE CONTROLLED RECTIFIERS

Output dc power (average or dc output power delivered to the load)

\[
P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \; ; \; \text{i.e.,} \; P_{dc} = V_{dc} \times I_{dc}
\]

Where

\[V_{O(dc)} = V_{dc} = \text{average or dc value of output (load) voltage.}\]

\[I_{O(dc)} = I_{dc} = \text{average or dc value of output (load) current.}\]

Output ac power

\[
P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}
\]

Efficiency of Rectification (Rectification Ratio)

\[
\eta = \frac{P_{O(dc)}}{P_{O(ac)}} \; ; \; \% \; \text{Efficiency} \; \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100
\]

The output voltage can be considered as being composed of two components

- The dc component \(V_{O(dc)} = \text{DC or average value of output voltage.}\)
- The ac component or the ripple component \(V_{ac} = V_{r(rms)} = \text{RMS value of all the ac ripple components.}\)

The total RMS value of output voltage is given by

\[
V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}
\]

Therefore

\[
V_{ac} = V_{O(rms)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}
\]
**Form Factor (FF)**  which is a measure of the shape of the output voltage is given by
\[
FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC output (load) voltage}}
\]

**The Ripple Factor (RF)**  which is a measure of the ac ripple content in the output voltage waveform. The output voltage ripple factor defined for the output voltage waveform is given by
\[
r_v = RF = \frac{V_{r(ma)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}
\]
\[
r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left(\frac{V_{O(RMS)}}{V_{O(dc)}}\right)^2 - 1}
\]
Therefore
\[
r_v = \sqrt{FF^2 - 1}
\]

**Current Ripple Factor**  defined for the output (load) current waveform is given by
\[
r_i = \frac{I_{r(ma)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}
\]
\[
I_{r(ma)} = I_{ac} - \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}
\]

Some times the peak to peak output ripple voltage is also considered to express the peak to peak output ripple voltage as
\[
V_{r(pp)} = \text{peak to peak ac ripple output voltage}
\]

The peak to peak ac ripple load current is the difference between the maximum and the minimum values of the output load current.
\[
I_{i(pp)} = I_{O(max)} - I_{O(min)}
\]

**Transformer Utilization Factor (TUF)**
\[
TUF = \frac{P_{O(dc)}}{V_s \times I_s}
\]
\[
V_s = \text{RMS value of transformer secondary output voltage (RMS supply voltage at the secondary)}
\]
\[ I_s = \text{RMS value of transformer secondary current (RMS line or supply current).} \]

\[ V_s = \text{Supply voltage at the transformer secondary side.} \]
\[ I_i = \text{Input supply current (transformer secondary winding current).} \]
\[ I_{s1} = \text{Fundamental component of the input supply current.} \]
\[ I_p = \text{Peak value of the input supply current.} \]
\[ \phi = \text{Phase angle difference between (sine wave components) the fundamental components of input supply current and the input supply voltage.} \]
\[ \phi = \text{Displacement angle (phase angle)} \]

For an RL load \( \phi = \text{Displacement angle} = \text{Load impedance angle} \)

\[ \therefore \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load} \]

**Displacement Factor (DF) or Fundamental Power Factor**

\[ DF = \cos\phi \]

**Harmonic Factor (HF) or Total Harmonic Distortion Factor (THD)**

The harmonic factor is a measure of the distortion in the output waveform and is also referred to as the total harmonic distortion (THD)

\[ HF = \left[ \frac{I_s^2 - I_{s1}^2}{I_{s1}^2} \right]^{\frac{1}{2}} = \left[ \left( \frac{I_s}{I_{s1}} \right)^2 - 1 \right]^{\frac{1}{2}} \]

Where
\[ I_s = \text{RMS value of input supply current.} \]
\[ I_{s1} = \text{RMS value of fundamental component of the input supply current.} \]

**Input Power Factor (PF)**
The Crest Factor (CF)

\[ CF = \frac{I_{S(\text{peak})}}{I_S} \]  

Peak input supply current  
RMS input supply current

For an Ideal Controlled Rectifier

\[ FF = 1 \]  
which means that \( V_{O(RMS)} = V_{O(\text{dc})} \).

Efficiency \( \eta = 100\% \); which means that \( P_{O(\text{dc})} = P_{O(ac)} \).

\( V_{ac} = V_{r(mv)} = 0 \); so that RF = r_r = 0 ; Ripple factor = 0 (ripple free converter).

\[ TUF = 1 \]  
which means that \( P_{O(\text{dc})} = V_s \times I_s \)

\[ HF = THD = 0 \]  
which means that \( I_s = I_s \)

\[ PF = DPF = 1 \]  
which means that \( \phi = 0 \)

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH AN RL LOAD

In this section we will discuss the operation and performance of a single phase half wave controlled rectifier with RL load. In practice most of the loads are of RL type. For example if we consider a single phase controlled rectifier controlling the speed of a dc motor, the load which is the dc motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

A single phase half wave controlled rectifier circuit with an RL load using a thyristor \( T_1 \) (\( T_1 \) is an SCR) is shown in the figure below.
The thyristor $T_i$ is forward biased during the positive half cycle of input supply. Let us assume that $T_i$ is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to $T_i$ during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when $T_i$ is ON. The load current $i_o$ flows through the thyristor $T_i$ and through the load in the downward direction. This load current pulse flowing through $T_i$ can be considered as the positive current pulse. Due to the inductance in the load, the load current $i_o$ flowing through $T_i$ would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative. A phase shift appears between the load voltage and the load current waveforms, due to the load inductance.

The thyristor $T_i$ will continue to conduct the load current until all the inductive energy stored in the load inductor $L$ is completely utilized and the load current through $T_i$ falls to zero at $\omega t = \beta$, where $\beta$ is referred to as the Extinction angle, (the value of $\omega t$) at which the load current falls to zero. The extinction angle $\beta$ is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor $T_i$ conducts from $\omega t = \alpha$ to $\beta$. The conduction angle of $T_i$ is $\delta = (\beta - \alpha)$, which depends on the delay angle $\alpha$ and the load impedance angle $\phi$. The waveforms of the input supply voltage, the gate trigger pulse of $T_i$, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.

![Fig.: Input supply voltage & Thyristor current waveforms](image-url)

\[ i_1 = i_o = i_s \]
\( \beta \) is the extinction angle which depends upon the load inductance value.

![Output (load) voltage waveform of a single phase half wave controlled rectifier with RL load](image)

From \( \beta \) to \( 2\pi \), the thyristor remains cut-off as it is reverse biased and behaves as an open switch. The thyristor current and the load current are zero and the output voltage also remains at zero during the non conduction time interval between \( \beta \) to \( 2\pi \). In the next cycle the thyristor is triggered again at a phase angle of \( (2\pi + \alpha) \), and the same operation repeats.

**TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING \( \omega t = \alpha \) TO \( \beta \) WHEN THYRISTOR \( T_i \) CONDUCTS**

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

\[
V_s = V_m \sin \omega t = \text{instantaneous value of the input supply voltage.}
\]

Let us assume that the thyristor \( T_i \) is triggered by applying the gating signal to \( T_i \) at \( \omega t = \alpha \). The load current which flows through the thyristor \( T_i \) during \( \omega t = \alpha \) to \( \beta \) can be found from the equation

\[
L \left( \frac{di_L}{dt} \right) + R_iO = V_m \sin \omega t ;
\]

The solution of the above differential equation gives the general expression for the output load current which is of the form

\[
i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + A_i e^{\frac{-s}{Z}} ;
\]

Where \( V_m = \sqrt{2}V_s = \text{maximum or peak value of input supply voltage.} \)

\[
Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}
\]
\[ \phi = \tan^{-1}\left( \frac{\omega L}{R} \right) = \text{Load impedance angle (power factor angle of load).} \]

\[ \tau = \frac{L}{R} = \text{Load circuit time constant.} \]

Therefore the general expression for the output load current is given by the equation

\[ i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + A_i e^{\frac{-R}{L} t} ; \]

The value of the constant \( A_i \) can be determined from the initial condition, i.e. initial value of load current \( i_o = 0 \), at \( \omega t = \alpha \). Hence from the equation for \( i_o \) equating \( i_o \) to zero and substituting \( \omega t = \alpha \), we get

\[ i_o = 0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A_i e^{\frac{-R}{L} \alpha} ; \]

Therefore

\[ A_i e^{\frac{-R}{L} \alpha} = -\frac{V_m}{Z} \sin(\alpha - \phi) \]

\[ A_i = \frac{1}{e^{\frac{-R}{L} \alpha}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right] \]

\[ A_i = e^{\frac{R(\alpha)}{L}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right] \]

By substituting \( \omega t = \alpha \), we get the value of constant \( A_i \) as

\[ A_i = e^{\frac{R(\alpha)}{2zL}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right] \]

Substituting the value of constant \( A_i \) from the above equation into the expression for \( i_o \), we obtain

\[ i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{L} t} e^{\frac{R(\alpha)}{2zL}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right] ; \]

\[ i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R(\alpha)}{2zL}} e^{\frac{R(\alpha)}{2zL}} \left[ -\frac{V_m}{Z} \sin(\alpha - \phi) \right] \]
Therefore we obtain the final expression for the inductive load current of a single phase half wave controlled rectifier with RL load as

\[ i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\omega L} \left[ \frac{R}{Z} \sin(\alpha - \phi) \right] \]

The above expression also represents the thyristor current \( i_{T1} \), during the conduction time interval of thyristor \( T_1 \) from \( \omega t = \alpha \) to \( \beta \).

**TO CALCULATE EXTINCTION ANGLE \( \beta \)**

The extinction angle \( \beta \), which is the value of \( \omega t \) at which the load current \( i_o \) falls to zero and \( T_1 \) is turned off can be estimated by using the condition that \( i_o = 0 \), at \( \omega t = \beta \).

By using the above expression for the output load current, we can write

\[ i_o = \frac{V_m}{Z} \left[ \sin(\beta - \phi) - \sin(\alpha - \phi) e^{\omega L (\beta - \alpha)} \right] \]

As \( \frac{V_m}{Z} \neq 0 \), we can write

\[ \left[ \sin(\beta - \phi) - \sin(\alpha - \phi) e^{\omega L (\beta - \alpha)} \right] = 0 \]

Therefore we obtain the expression

\[ \sin(\beta - \phi) = \sin(\alpha - \phi) e^{\omega L (\beta - \alpha)} \]

The extinction angle \( \beta \) can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After \( \beta \) is calculated, we can determine the thyristor conduction angle \( \delta = (\beta - \alpha) \).

\( \beta \) is the extinction angle which depends upon the load inductance value. Conduction angle \( \delta \) increases as \( \alpha \) is decreased for a specific value of \( \beta \).

**Conduction angle** \( \delta = (\beta - \alpha) \): for a purely resistive load or for an RL load when the load inductance \( L \) is negligible the extinction angle \( \beta = \pi \) and the conduction angle \( \delta = (\pi - \alpha) \)
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Equations

\[ v_s = V_m \sin \omega t = \text{Input supply voltage} \]

\[ v_o = v_l = V_m \sin \omega t = \text{Output load voltage for } \omega t = \alpha \text{ to } \beta \],

when the thyristor \( T \) conducts \( (T \text{ is on}) \).

**Expression for the load current (thyristor current):** for \( \omega t = \alpha \) to \( \beta \)

\[ i_o = \frac{V_m}{Z} \left[ \sin (\omega t - \phi) - \sin (\alpha - \phi) e^{\frac{\omega t}{\omega(t-\alpha)}} \right]; \quad \text{Where } \alpha \leq \omega t \leq \beta. \]

**Extinction angle** \( \beta \) can be calculated using the equation

\[ \sin (\beta - \phi) = \sin (\alpha - \phi) e^{\frac{\omega(\beta-\alpha)}{\pi}} \]

**TO DERIVE AN EXPRESSION FOR AVERAGE (DC) LOAD VOLTAGE**

\[ V_{O(\text{dc})} = V_L = \frac{1}{2\pi} \int_0^{2\pi} v_o.d(\omega t) \]

\[ V_{O(\text{dc})} = V_L = \frac{1}{2\pi} \left[ \int_0^{\omega t} v_o.d(\omega t) + \int_{\omega t}^{\beta} v_o.d(\omega t) + \int_{\beta}^{2\pi} v_o.d(\omega t) \right]; \]

\[ v_o = 0 \text{ for } \omega t = 0 \text{ to } \alpha \text{ & for } \omega t = \beta \text{ to } 2\pi; \]

\[ \therefore V_{O(\text{dc})} = V_L = \frac{1}{2\pi} \int_0^{\beta} v_o.d(\omega t) ; v_o = V_m \sin \omega t \text{ for } \omega t = \alpha \text{ to } \beta \]

\[ V_{O(\text{dc})} = V_L = \frac{1}{2\pi} \left[ \int_0^{\omega t} V_m \sin \omega t.d(\omega t) \right] \]

\[ V_{O(\text{dc})} = V_L = \frac{V_m}{2\pi} \left[ -\cos \omega t \right]_{\alpha}^{\beta} = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta) \]

\[ \therefore V_{O(\text{dc})} = V_L = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta) \]

**Note:** During the period \( \omega t = \pi \) to \( \beta \), we can see from the output load voltage waveform that the instantaneous output voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.
Average DC Load Current

\[ I_{O(d)} = I_{L(avg)} = \frac{V_{O(d)}}{R_L} = \frac{V_m}{2\pi R_L} (\cos \alpha - \cos \beta) \]

**SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FREE WHEELING DIODE**

![Circuit Diagram](image)

**Fig. : Single Phase Half Wave Controlled Rectifier with RL Load and Free Wheeling Diode (FWD)**

With a RL load it was observed that the average output voltage reduces. This disadvantage can be overcome by connecting a diode across the load as shown in figure. The diode is called as a Free Wheeling Diode (FWD). The waveforms are shown below.
At $\omega t = \pi$, the source voltage $v_s(t)$ falls to zero and as $v_s$ becomes negative, the free wheeling diode is forward biased. The stored energy in the inductance maintains the load current flow through R, L, and the FWD. Also, as soon as the FWD is forward biased, at $\omega t = \pi$, the SCR becomes reverse biased, the current through it becomes zero and the SCR turns off. During the period $\omega t = \pi$ to $\beta$, the load current flows through FWD (free wheeling load current) and decreases exponentially towards zero at $\omega t = \beta$.

Also during this free wheeling time period the load is shorted by the conducting FWD and the load voltage is almost zero, if the forward voltage drop across the conducting FWD is neglected. Thus there is no negative region in the load voltage waveform. This improves the average output voltage.

The average output voltage $V_{dc} = \frac{V_m}{2\pi}[1 + \cos \alpha]$, which is the same as that of a purely resistive load. The output voltage across the load appears similar to the output voltage of a purely resistive load.

The following points are to be noted.

- If the inductance value is not very large, the energy stored in the inductance is able to maintain the load current only up to $\omega t = \beta$, where $\pi < \beta < 2\pi$, well before the next gate pulse and the load current tends to become discontinuous.
- During the conduction period $\alpha$ to $\pi$, the load current is carried by the SCR and during the free wheeling period $\pi$ to $\beta$, the load current is carried by the free wheeling diode.
- The value of $\beta$ depends on the value of R and L and the forward resistance of the FWD. Generally $\pi < \beta < 2\pi$.

If the value of the inductance is very large, the load current does not decrease to zero during the free wheeling time interval and the load current waveform appears as shown in the figure.

**Fig.**: Waveform of Load Current in Single Phase Half Wave Controlled Rectifier with a Large Inductance and FWD
During the periods \( t_1, t_3, \ldots \), the SCR carries the load current and during the periods \( t_2, t_4, \ldots \) the FWD carries the load current.

It is to be noted that

- The load current becomes continuous and the load current does not fall to zero for large value of load inductance.
- The ripple in the load current waveform (the amount of variation in the output load current) decreases.

**SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH A GENERAL LOAD**

A general load consists of \( R, L \) and a DC source ‘\( E \)’ in the load circuit.

In the half wave controlled rectifier circuit shown in the figure, the load circuit consists of a dc source ‘\( E \)’ in addition to resistance and inductance. When the thyristor is in the cut-off state, the current in the circuit is zero and the cathode will be at a voltage equal to the dc voltage in the load circuit i.e. the cathode potential will be equal to ‘\( E \)’. The thyristor will be forward biased for anode supply voltage greater than the load dc voltage.

When the supply voltage is less than the dc voltage ‘\( E \)’ in the circuit the thyristor is reverse biased and hence the thyristor cannot conduct for supply voltage less than the load circuit dc voltage.

The value of \( \omega t \) at which the supply voltage increases and becomes equal to the load circuit dc voltage can be calculated by using the equation \( V_m \sin \omega t = E \). If we assume the value of \( \omega t \) is equal to \( \gamma \) then we can write \( V_m \sin \gamma = E \). Therefore \( \gamma \) is calculated as \( \gamma = \sin^{-1} \left( \frac{E}{V_m} \right) \).

For trigger angle \( \alpha < \gamma \), the thyristor conducts only from \( \omega t = \gamma \) to \( \beta \).

For trigger angle \( \alpha > \gamma \), the thyristor conducts from \( \omega t = \alpha \) to \( \beta \).

The waveforms appear as shown in the figure.
Equations

\[ v_s = V_m \sin \omega t = \text{Input supply voltage} \]
\[ v_o = V_m \sin \omega t = \text{Output load voltage for } \omega t = \alpha \text{ to } \beta \]
\[ v_o = E \text{ for } \omega t = 0 \text{ to } \alpha \text{ & for } \omega t = \beta \text{ to } 2\pi \]

Expression for the Load Current

When the thyristor is triggered at a delay angle of \( \alpha \), the equation for the circuit can be written as
\[ V_m \sin \omega t = i_L \times R + L \left( \frac{di_o}{dt} \right) + E ; \alpha \leq \omega t \leq \beta \]

The general expression for the output load current can be written as
\[ i_o = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{E}{R} + A e^{-\tau} \]

Where
\[ Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance} \]
\[ \phi = \tan^{-1} \left( \frac{\omega L}{R} \right) = \text{Load impedance angle} \]
\[ \tau = \frac{L}{R} = \text{Load circuit time constant} \]

The general expression for the output load current can be written as
\[
i_o = \frac{V_m}{Z} \sin(\omega t - \phi) \frac{E}{R} + Ae^{\frac{-R}{2\alpha t}}
\]

To find the value of the constant ‘A’ apply the initial condition at \( \omega t = \alpha \), load current \( i_o = 0 \). Equating the general expression for the load current to zero at \( \omega t = \alpha \), we get

\[
i_o = 0 = \frac{V_m}{Z} \sin(\alpha - \phi) \frac{E}{R} + Ae^{\frac{-R}{2\alpha}}
\]

We obtain the value of constant ‘A’ as

\[
A = \left[ \frac{E}{R} - \frac{V_m}{Z} \sin(\alpha - \phi) \right] \frac{R}{e^{\alpha L}}
\]

Substituting the value of the constant ‘A’ in the expression for the load current, we get the complete expression for the output load current as

\[
i_o = \frac{V_m}{Z} \sin(\omega t - \phi) \frac{E}{R} + \left[ \frac{E}{R} - \frac{V_m}{Z} \sin(\alpha - \phi) \right] \frac{R}{e^{\alpha L}}\left(\alpha - \phi\right)
\]

The Extinction angle \( \beta \) can be calculated from the final condition that the output current \( i_o = 0 \) at \( \omega t = \beta \). By using the above expression we get,

\[
i_o = 0 = \frac{V_m}{Z} \sin(\beta - \phi) \frac{E}{R} + \left[ \frac{E}{R} - \frac{V_m}{Z} \sin(\alpha - \phi) \right] \frac{R}{e^{\alpha L}}\left(\beta - \alpha\right)
\]

To derive an expression for the average or dc load voltage

\[
V_{o(d)} = \frac{1}{2\pi} \int_0^{2\pi} v_o d(\omega t)
\]

\[
V_{o(d)} = \frac{1}{2\pi} \left[ \int_0^\alpha v_o d(\omega t) + \int_\alpha^\beta v_o d(\omega t) + \int_\beta^{2\pi} v_o d(\omega t) \right]
\]

\( v_o = V_m \sin \omega t = \) Output load voltage for \( \omega t = \alpha \) to \( \beta \)

\( v_o = E \) for \( \omega t = 0 \) to \( \alpha \) & for \( \omega t = \beta \) to \( 2\pi \)

\[
V_{o(d)} = \frac{1}{2\pi} \left[ \int_0^\alpha E d(\omega t) + \int_\alpha^\beta V_m \sin \omega t + \int_\beta^{2\pi} E d(\omega t) \right]
\]

\[
V_{o(d)} = \frac{1}{2\pi} \left[ E(\omega t) \right]_0^\alpha + V_m \left( -\cos \omega t \right) \left. \right|_\alpha^\beta + E(\omega t) \left. \right|_\beta^{2\pi}
\]
\[ V_{O(dc)} = \frac{1}{2\pi} \left[ E(\alpha - 0) - V_m (\cos \beta - \cos \alpha) + E(2\pi - \beta) \right] \]

\[ V_{O(dc)} = \frac{V_m}{2\pi} \left[ (\cos \alpha - \cos \beta) + \frac{E}{2\pi} (2\pi - \beta + \alpha) \right] \]

\[ V_{O(dc)} = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta) + \left[ \frac{2\pi - (\beta - \alpha)}{2\pi} \right] E \]

Conduction angle of thyristor \( \delta = (\beta - \alpha) \)

**RMS Output Voltage** can be calculated by using the expression:

\[ V_{O(RMS)} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v_o^2 d(\omega t)} \]

**DISADVANTAGES OF SINGLE PHASE HALF WAVE CONTROLLED RECTIFIERS**

Single phase half wave controlled rectifier gives:
- Low dc output voltage.
- Low dc output power and lower efficiency.
- Higher ripple voltage & ripple current.
- Higher ripple factor.
- Low transformer utilization factor.
- The input supply current waveform has a dc component which can result in dc saturation of the transformer core.

Single phase half wave controlled rectifiers are rarely used in practice as they give low dc output and low dc output power. They are only of theoretical interest.

The above disadvantages of a single phase half wave controlled rectifier can be overcome by using a full wave controlled rectifier circuit. Most of the practical converter circuits use full wave controlled rectifiers.

**SINGLE PHASE FULL WAVE CONTROLLED RECTIFIERS**

Single phase full wave controlled rectifier circuit combines two half wave controlled rectifiers in one single circuit so as to provide two pulse output across the load. Both the half cycles of the input supply are utilized and converted into a uni-directional output current through the load so as to produce a two pulse output waveform. Hence a full wave controlled rectifier circuit is also referred to as a two pulse converter.

Single phase full wave controlled rectifiers are of various types:
- Single phase full wave controlled rectifier using a center tapped transformer (two pulse converter with mid point configuration).
- Single phase full wave bridge controlled rectifier
  - Half controlled bridge converter (semi converter).
  - Fully controlled bridge converter (full converter).
SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER USING A CENTER TAPPED TRANSFORMER

\[ v_s = \text{Supply Voltage across the upper half of the transformer secondary winding} \]

\[ v_s = v_{AO} = V_m \sin \omega t \]

\[ v_{BO} = -v_{AO} = -V_m \sin \omega t = \text{supply voltage across the lower half of the transformer secondary winding.} \]

This type of full wave controlled rectifier requires a center tapped transformer and two thyristors \( T_1 \) and \( T_2 \). The input supply is fed through the mains supply transformer, the primary side of the transformer is connected to the ac line voltage which is available (normally the primary supply voltage is 230V RMS ac supply voltage at 50Hz supply frequency in India). The secondary side of the transformer has three lines and the center point of the transformer (center line) is used as the reference point to measure the input and output voltages.

The upper half of the secondary winding and the thyristor \( T_1 \) along with the load act as a half wave controlled rectifier, the lower half of the secondary winding and the thyristor \( T_2 \) with the common load act as the second half wave controlled rectifier so as to produce a full wave load voltage waveform.

There are two types of operations possible.

- **Discontinuous load current operation**, which occurs for a purely resistive load or an RL load with low inductance value.
- **Continuous load current operation** which occurs for an RL type of load with large load inductance.

**Discontinuous Load Current Operation** (for low value of load inductance)

Generally the load current is discontinuous when the load is purely resistive or when the RL load has a low value of inductance.

During the positive half cycle of input supply, when the upper line of the secondary winding is at a positive potential with respect to the center point ‘O’ the thyristor \( T_1 \) is forward biased and it is triggered at a delay angle of \( \alpha \). The load current
flows through the thyristor \( T_1 \), through the load, and through the upper part of the secondary winding, during the period \( \alpha \) to \( \beta \), when the thyristor \( T_1 \) conducts.

The output voltage across the load follows the input supply voltage that appears across the upper part of the secondary winding from \( \omega t = \alpha \) to \( \beta \). The load current through the thyristor \( T_1 \) decreases and drops to zero at \( \omega t = \beta \), where \( \beta > \pi \) for RL type of load and the thyristor \( T_1 \) naturally turns off at \( \omega t = \beta \).

\[
\begin{align*}
V_O &= V_m \\
\alpha &< \beta \\
\alpha + \beta &< 2\pi \\
\beta \pi &> \alpha
\end{align*}
\]

**Fig.: Waveform for Discontinuous Load Current Operation without FWD**

During the negative half cycle of the input supply the voltage at the supply line ‘A’ becomes negative whereas the voltage at line ‘B’ (at the lower side of the secondary winding) becomes positive with respect to the center point ‘O’. The thyristor \( T_2 \) is forward biased during the negative half cycle and it is triggered at a delay angle of \( \pi + \alpha \). The current flows through the thyristor \( T_2 \), through the load, and through the lower part of the secondary winding when \( T_2 \) conducts during the negative half cycle the load is connected to the lower half of the secondary winding when \( T_2 \) conducts.

For purely resistive loads when \( L = 0 \), the extinction angle \( \beta = \pi \). The load current falls to zero at \( \omega t = \beta = \pi \), when the input supply voltage falls to zero at \( \omega t = \pi \). The load current and the load voltage waveforms are in phase and there is no phase shift between the load voltage and the load current waveform in the case of a purely resistive load.

For low values of load inductance the load current would be discontinuous and the extinction angle \( \beta > \pi \) but \( \beta < (\pi + \alpha) \).

For large values of load inductance the load current would be continuous and does not fall to zero. The thyristor \( T_1 \) conducts from \( \alpha \) to \( \pi + \alpha \), until the next thyristor \( T_2 \) is triggered. When \( T_2 \) is triggered at \( \omega t = (\pi + \alpha) \), the thyristor \( T_1 \) will be reverse biased and hence \( T_1 \) turns off.
TO DERIVE AN EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD (WITHOUT FREE WHEELING DIODE (FWD))

The average or dc output voltage of a full-wave controlled rectifier can be calculated by finding the average value of the output voltage waveform over one output cycle (i.e., $\pi$ radians) and note that the output pulse repetition time is $T/2$ seconds where $T$ represents the input supply time period and $T = \frac{1}{f}$; where $f$ = input supply frequency.

Assuming the load inductance to be small so that $\beta \pi > \alpha \pi$ we obtain discontinuous load current operation. The load current flows through $T_1$ form $t = \alpha$ to $t = \beta$, where $\alpha$ is the trigger angle of thyristor $T_1$ and $\beta$ is the extinction angle where the load current through $T_1$ falls to zero at $t = \beta$. Therefore the average or dc output voltage can be obtained by using the expression

$$V_{O_{dc}} = V_{dc} = \frac{2}{2\pi} \int_{\alpha}^{\beta} v_o \cdot d(\omega t)$$

$$V_{O_{dc}} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\beta} v_o \cdot d(\omega t)$$

$$V_{O_{dc}} = V_{dc} = \frac{1}{\pi} \left[ V_m \sin(\omega t) \cdot d(\omega t) \right]_{\alpha}^{\beta}$$

$$V_{O_{dc}} = V_{dc} = \frac{V_m}{\pi} \cos(\omega t) \int_{\alpha}^{\beta}$$

$$V_{O_{dc}} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

Therefore $V_{O_{dc}} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$, for discontinuous load current operation, $\pi < \beta < (\pi + \alpha)$.

When the load inductance is small and negligible that is $L \approx 0$, the extinction angle $\beta = \pi$ radians. Hence the average or dc output voltage for resistive load is obtained as

$$V_{O_{dc}} = \frac{V_m}{\pi} (\cos \alpha - \cos \pi) \quad ; \cos \pi = -1$$

$$V_{O_{dc}} = \frac{V_m}{\pi} (\cos \alpha - (-1))$$
THE EFFECT OF LOAD INDUCTANCE

Due to the presence of load inductance the output voltage reverses and becomes negative during the time period $\omega t = \pi$ to $\beta$. This reduces the dc output voltage. To prevent this reduction of dc output voltage due to the negative region in the output load voltage waveform, we can connect a free wheeling diode across the load. The output voltage waveform and the dc output voltage obtained would be the same as that for a full wave controlled rectifier with resistive load.

When the Free wheeling diode (FWD) is connected across the load

When $T_1$ is triggered at $\omega t = \alpha$, during the positive half cycle of the input supply the FWD is reverse biased during the time period $\omega t = \alpha$ to $\pi$. FWD remains reverse biased and cut-off from $\omega t = \alpha$ to $\pi$. The load current flows through the conducting thyristor $T_1$, through the RL load and through upper half of the transformer secondary winding during the time period $\alpha$ to $\pi$.

At $\omega t = \pi$, when the input supply voltage across the upper half of the secondary winding reverses and becomes negative the FWD turns-on. The load current continues to flow through the FWD from $\omega t = \pi$ to $\beta$.

Fig.: Waveform for Discontinuous Load Current Operation with FWD

EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FWD

$$V_{O(\text{dc})} = V_m \left( 1 + \cos \alpha \right), \text{ for resistive load, when } L \approx 0$$

$$V_{O(\text{dc})} = V_{dc} = \frac{1}{\pi} \int_{\omega t = 0}^{\pi} v_O d(\omega t)$$

Thyristor $T_1$ is triggered at $\omega t = \alpha$. $T_1$ conducts from $\omega t = \alpha$ to $\pi$.
Output voltage \( v_o = V_m \sin(\omega t + \alpha) \) for \( \omega t = \pi \) to \( \beta \) and \( v_o \approx 0 \) during discontinuous load current.

Therefore

\[
V_{O(dc)} = V_{dc} = \frac{1}{\pi} V_m \sin(\omega t) \sin(\omega t) dt
\]

\[
V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[ -\cos \omega t \right]_\alpha^\pi
\]

\[
V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[ -\cos \pi + \cos \alpha \right]; \quad \cos \pi = -1
\]

Therefore

\[
V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)
\]

The DC output voltage \( V_{dc} \) is same as the DC output voltage of a single phase full wave controlled rectifier with resistive load. Note that the dc output voltage of a single phase full wave controlled rectifier is two times the dc output voltage of a half wave controlled rectifier.

**CONTROL CHARACTERISTICS OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH R LOAD OR RL LOAD WITH FWD**

The control characteristic can be obtained by plotting the dc output voltage \( V_{dc} \) versus the trigger angle \( \alpha \).

The average or dc output voltage of a single phase full wave controlled rectifier circuit with R load or RL load with FWD is calculated by using the equation

\[
V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)
\]

\( V_{dc} \) can be varied by varying the trigger angle \( \alpha \) from 0 to 180°. (i.e., the range of trigger angle \( \alpha \) is from 0 to \( \pi \) radians).

Maximum dc output voltage is obtained when \( \alpha = 0 \)

\[
V_{dc(\text{max})} = V_{dc} = \frac{V_m}{\pi} (1 + \cos 0) = \frac{2V_m}{\pi}
\]

Therefore

\[
V_{dc(\text{max})} = V_{dc} = \frac{2V_m}{\pi} \quad \text{for a single phase full wave controlled rectifier.}
\]

Normalizing the dc output voltage with respect to its maximum value, we can write the normalized dc output voltage as
\[
V_{dc\text{n}} = V_n = \frac{V_{dc}}{V_{dc(\text{max})}} = \frac{a}{\lambda}
\]

\[
V_{dc\text{n}} = V_n = \frac{V_m (1 + \cos \alpha)}{\pi} = \frac{1}{2} (1 + \cos \alpha)
\]

Therefore
\[
V_{dc\text{n}} = V_n = \frac{1}{2} (1 + \cos \alpha) \frac{V_{dc}}{V_{dm}}
\]

\[
V_{dc} = \frac{1}{2} (1 + \cos \alpha) V_{dm}
\]

<table>
<thead>
<tr>
<th>Trigger angle ( \alpha ) in degrees</th>
<th>( V_{o(dc)} )</th>
<th>Normalized dc output voltage ( V_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.636619 ( V_m )</td>
<td>1</td>
</tr>
<tr>
<td>30(^\circ)</td>
<td>0.593574 ( V_m )</td>
<td>0.9330</td>
</tr>
<tr>
<td>60(^\circ)</td>
<td>0.477464 ( V_m )</td>
<td>0.75</td>
</tr>
<tr>
<td>90(^\circ)</td>
<td>0.3183098 ( V_m )</td>
<td>0.5</td>
</tr>
<tr>
<td>120(^\circ)</td>
<td>0.191549 ( V_m )</td>
<td>0.25</td>
</tr>
<tr>
<td>150(^\circ)</td>
<td>0.04264 ( V_m )</td>
<td>0.06698</td>
</tr>
<tr>
<td>180(^\circ)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Fig.:** Control characteristic of a single phase full wave controlled rectifier with R load or RL load with FWD
CONTINUOUS LOAD CURRENT OPERATION (WITHOUT FWD)

For large values of load inductance the load current flows continuously without decreasing and falling to zero and there is always a load current flowing at any point of time. This type of operation is referred to as continuous current operation.

Generally the load current is continuous for large load inductance and for low trigger angles.

The load current is discontinuous for low values of load inductance and for large values of trigger angles.

The waveforms for continuous current operation are as shown.

Fig.: Load voltage and load current waveform of a single phase full wave controlled rectifier with RL load & without FWD for continuous load current operation

In the case of continuous current operation the thyristor $T_1$ which is triggered at a delay angle of $\alpha$, conducts from $\omega t = \alpha$ to $(\pi + \alpha)$. Output voltage follows the input supply voltage across the upper half of the transformer secondary winding $v_o = v_{ao} = V_m \sin \omega t$.

The next thyristor $T_2$ is triggered at $\omega t = (\pi + \alpha)$, during the negative half cycle input supply. As soon as $T_2$ is triggered at $\omega t = (\pi + \alpha)$, the thyristor $T_1$ will be reverse biased and $T_1$ turns off due to natural commutation (ac line commutation). The load current flows through the thyristor $T_2$ from $\omega t = (\pi + \alpha)$ to $(2\pi + \alpha)$. Output voltage across the load follows the input supply voltage across the lower half of the transformer secondary winding $v_o = v_{bo} = -V_m \sin \omega t$.

Each thyristor conducts for $\pi$ radians $(180^\circ)$ in the case of continuous current operation.
TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH LARGE LOAD INDUCTANCE ASSUMING CONTINUOUS LOAD CURRENT OPERATION.

\[
V_{O_{dc}}(s) = V_{dc} = \frac{1}{\pi} \int_{-\alpha}^{\alpha} v_o.d(\omega t)
\]

\[
V_{O_{dc}} = V_{dc} = \frac{1}{\pi} \int_{-\alpha}^{\alpha} V_m \sin \omega t \cdot d(\omega t)
\]

\[
V_{O_{dc}} = V_{dc} = \frac{V_m}{\pi} \left[ -\cos \omega t \right]_{-\alpha}^{\alpha}
\]

\[
V_{O_{dc}} = V_{dc} = \frac{V_m}{\pi} \left[ \cos \alpha - \cos (\pi + \alpha) \right] ; \quad \cos (\pi + \alpha) = -\cos \alpha
\]

\[
V_{O_{dc}} = V_{dc} = \frac{V_m}{\pi} \left[ \cos \alpha + \cos \alpha \right]
\]

\[
\therefore \quad V_{O_{dc}} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha
\]

The above equation can be plotted to obtain the control characteristic of a single phase full wave controlled rectifier with RL load assuming continuous load current operation.

Normalizing the dc output voltage with respect to its maximum value, the normalized dc output voltage is given by

\[
V_{dcn} = V_{dc} = \frac{2V_m}{\pi} \left( \cos \alpha \right)
\]

Therefore \( V_{dcn} = V_n = \cos \alpha \)
We notice from the control characteristic that by varying the trigger angle $\alpha$ we can vary the output dc voltage across the load. Thus it is possible to control the dc output voltage by changing the trigger angle $\alpha$. For trigger angle $\alpha$ in the range of 0 to 90 degrees ($0 \leq \alpha \leq 90^\circ$), $V_{dc}$ is positive and the circuit operates as a controlled rectifier to convert ac supply voltage into dc output power which is fed to the load.

For trigger angle $\alpha > 90^\circ$, $\cos \alpha$ becomes negative and as a result the average dc output voltage $V_{dc}$ becomes negative, but the load current flows in the same positive direction. Hence the output power becomes negative. This means that the power flows from the load circuit to the input ac source. This is referred to as line commutated inverter operation. During the inverter mode operation for $\alpha > 90^\circ$ the load energy can be fed back from the load circuit to the input ac source.
TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE

The rms value of the output voltage is calculated by using the equation

\[ V_{O(RMS)} = \left[ \frac{2}{2\pi} \int_{a}^{(\pi+\alpha)} v_{O}^2 d(o t) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = \left[ \frac{1}{\pi} \int_{a}^{(\pi+\alpha)} V_{m}^2 \sin^2 o t d(o t) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = \left[ \frac{V_{m}^2}{\pi} \int_{a}^{(\pi+\alpha)} \sin^2 o t d(o t) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = \left[ \frac{V_{m}^2}{\pi} \int_{a}^{(\pi+\alpha)} \frac{1 - \cos 2o t}{2} d(o t) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = V_{m} \left[ \frac{1}{2\pi} \left\{ \int_{a}^{(\pi+\alpha)} d(o t) - \int_{a}^{(\pi+\alpha)} \cos 2o t d(o t) \right\} \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = V_{m} \left[ \frac{1}{2\pi} \left( \frac{(\pi+\alpha) - (\sin 2(\pi+\alpha) - \sin 2\alpha)}{2} \right) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = V_{m} \left[ \frac{1}{2\pi} \left( \pi - \left( \frac{\sin 2\pi \times \cos 2\alpha + \cos 2\pi \times \sin 2\alpha - \sin 2\alpha}{2} \right) \right) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = V_{m} \left[ \frac{1}{2\pi} \left( \pi - \left( \frac{0 + \sin 2\alpha - \sin 2\alpha}{2} \right) \right) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = V_{m} \left[ \frac{1}{2\pi} \left( \pi \right) \right]^{\frac{1}{2}} = \frac{V_{m}}{\sqrt{2}} \]

Therefore

\[ V_{O(RMS)} = \frac{V_{m}}{\sqrt{2}} ; \text{The rms output voltage is same as the input rms supply voltage.} \]
SINGLE PHASE SEMICONVERTER

**Errata:** Consider diode $D_2$ as $D_1$ in the figure and diode $D_1$ as $D_2$.

Single phase semi-converter circuit is a full wave half controlled bridge converter which uses two thyristors and two diodes connected in the form of a full wave bridge configuration.

The two thyristors are controlled power switches which are turned on one after the other by applying suitable gating signals (gate trigger pulses). The two diodes are uncontrolled power switches which turn-on and conduct one after the other as and when they are forward biased.

The circuit diagram of a single phase semi-converter (half controlled bridge converter) is shown in the above figure with highly inductive load and a dc source in the load circuit. When the load inductance is large the load current flows continuously and we can consider the continuous load current operation assuming constant load current, with negligible current ripple (i.e., constant and ripple free load current operation).

The ac supply to the semiconverter is normally fed through a mains supply transformer having suitable turns ratio. The transformer is suitably designed to supply the required ac supply voltage (secondary output voltage) to the converter.

During the positive half cycle of input ac supply voltage, when the transformer secondary output line ‘A’ is positive with respect to the line ‘B’ the thyristor $T_1$ and the diode $D_1$ are both forward biased. The thyristor $T_1$ is triggered at $\omega t = \alpha$; $(0 \leq \alpha \leq \pi)$ by applying an appropriate gate trigger signal to the gate of $T_1$. The current in the circuit flows through the secondary line ‘A’, through $T_1$, through the load in the downward direction, through diode $D_1$ back to the secondary line ‘B’.

$T_1$ and $D_1$ conduct together from $\omega t = \alpha$ to $\pi$ and the load is connected to the input ac supply. The output load voltage follows the input supply voltage (the secondary output voltage of the transformer) during the period $\omega t = \alpha$ to $\pi$.

At $\omega t = \pi$, the input supply voltage decreases to zero and becomes negative during the period $\omega t = \pi$ to $(\pi + \alpha)$. The free wheeling diode $D_m$ across the load becomes forward biased and conducts during the period $\omega t = \pi$ to $(\pi + \alpha)$. 
Fig.: Waveforms of single phase semi-converter for RLE load and constant load current for $\alpha > 90^\circ$
The load current is transferred from \( T_1 \) and \( D_1 \) to the FWD \( D_m \). \( T_1 \) and \( D_1 \) are turned off. The load current continues to flow through the FWD \( D_m \). The load current free wheels (flows continuously) through the FWD during the free wheeling time period \( \pi \) to \( (\pi + \alpha) \).

During the negative half cycle of input supply voltage the secondary line ‘A’ becomes negative with respect to line ‘B’. The thyristor \( T_2 \) and the diode \( D_2 \) are both forward biased. \( T_2 \) is triggered at \( \omega t = (\pi + \alpha) \), during the negative half cycle. The FWD is reverse biased and turns-off as soon as \( T_2 \) is triggered. The load current continues to flow through \( T_2 \) and \( D_2 \) during the period \( \omega t = (\pi + \alpha) \) to \( 2\pi \).

**TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A SINGLE PHASE SEMI-CONVERTER**

The average output voltage can be found from

\[
V_{dc} = \frac{2}{2\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t)
\]

\[
V_{dc} = \frac{2V_m}{2\pi} \left[ -\cos(\omega t) \right]_{\alpha}^{\pi}
\]

\[
V_{dc} = \frac{V_m}{\pi} \left[ -\cos \pi + \cos \alpha \right] \quad \text{if} \quad \cos \pi = -1
\]

Therefore

\[
V_{dc} = \frac{V_m}{\pi} [1 + \cos \alpha]
\]

\( V_{dc} \) can be varied from \( \frac{2V_m}{\pi} \) to 0 by varying \( \alpha \) from 0 to \( \pi \).

The maximum average output voltage is

\[
V_{dc(max)} = V_{dm} = \frac{2V_m}{\pi}
\]

Normalizing the average output voltage with respect to its maximum value

\[
V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1 + \cos \alpha)
\]

The output control characteristic can be plotted by using the expression for \( V_{dc} \).
TO DERIVE AN EXPRESSION FOR THE RMS OUTPUT VOLTAGE OF A
SINGLE PHASE SEMI-CONVERTER

The rms output voltage is found from

\[ V_{O(RMS)} = \left[ \frac{2}{2\pi} \int_{0}^{\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = \left[ \frac{V_m^2}{2\pi} \int_{0}^{\pi} (1 - \cos 2\omega t) \cdot d(\omega t) \right]^{\frac{1}{2}} \]

\[ V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[ \frac{1}{\pi} \left( \pi - \alpha + \sin 2\alpha \right) \right]^{\frac{1}{2}} \]

SINGLE PHASE FULL CONVERTER (FULLY CONTROLLED BRIDGE CONVERTER)

The circuit diagram of a single phase fully controlled bridge converter is shown in the figure with a highly inductive load and a dc source in the load circuit so that the load current is continuous and ripple free (constant load current operation).

The fully controlled bridge converter consists of four thyristors \( T_1, T_2, T_3 \) and \( T_4 \) connected in the form of full wave bridge configuration as shown in the figure. Each thyristor is controlled and turned on by its gating signal and naturally turns off when a reverse voltage appears across it. During the positive half cycle when the upper line of the transformer secondary winding is at a positive potential with respect to the lower end the thyristors \( T_1 \) and \( T_2 \) are forward biased during the time interval \( o\alpha = 0 \) to \( \pi \). The thyristors \( T_1 \) and \( T_2 \) are triggered simultaneously \( o\alpha = \alpha \); \( 0 \leq \alpha \leq \pi \), the load is connected to the input supply through the conducting thyristors \( T_1 \) and \( T_2 \). The output voltage across the load follows the input supply voltage and hence output voltage \( V_o = V_m \sin o\alpha \). Due to the inductive load \( T_1 \) and \( T_2 \) will continue to conduct beyond \( o\alpha = \pi \), even though the input voltage becomes negative. \( T_1 \) and \( T_2 \) conduct together

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During the time period $\alpha$ to $(\pi + \alpha)$, the thyristors $T_3$ and $T_4$ are forward biased. $T_3$ and $T_4$ are triggered at $\omega t = (\pi + \alpha)$. As soon as the thyristors $T_3$ and $T_4$ are triggered a reverse voltage appears across the thyristors $T_1$ and $T_2$, and they naturally turn-off and the load current is transferred from $T_1$ and $T_2$ to the thyristors $T_3$ and $T_4$. The output voltage across the load follows the supply voltage and $v_o = -V_m \sin \omega t$ during the time period $\omega t = (\pi + \alpha)$ to $(2\pi + \alpha)$. In the next positive half cycle when $T_1$ and $T_2$ are triggered, $T_3$ and $T_4$ are reverse biased and they turn-off. The figure shows the waveforms of the input supply voltage, the output load voltage, the constant load current with negligible ripple and the input supply current.
During the time period \( t_0 = \omega t = \alpha \) to \( \omega \alpha \pi = \omega \pi \), the input supply voltage \( v_s \) and the input supply current \( i_s \) are both positive and the power flows from the supply to the load. The converter operates in the rectification mode during \( \omega t = \alpha \) to \( \omega \pi \).

During the time period \( \omega t = \pi \) to \( \pi + \alpha \), the input supply voltage \( v_s \) is negative and the input supply current \( i_s \) is positive and there will be reverse power flow from the load circuit to the input supply. The converter operates in the inversion mode during the time period \( \omega t = \pi \) to \( \pi + \alpha \) and the load energy is fed back to the input source.

The single phase full converter is extensively used in industrial applications up to about 15kW of output power. Depending on the value of trigger angle \( \alpha \), the average output voltage may be either positive or negative and two quadrant operation is possible.

**TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE**

The average (dc) output voltage can be determined by using the expression

\[
V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_0^{\pi} v_{Od}(\omega t) d\omega t ;
\]

The output voltage waveform consists of two output pulses during the input supply time period between \( 0 \) & \( 2\pi \) radians. In the continuous load current operation of a single phase full converter (assuming constant load current) each thyristor conduct for \( \pi \) radians (180°) after it is triggered. When thyristors \( T_1 \) and \( T_2 \) are triggered at \( \omega t = \alpha \), \( T_1 \) and \( T_2 \) conduct from \( \alpha \) to \( \pi + \alpha \) and the output voltage follows the input supply voltage. Therefore output voltage \( v_o = V_o \sin \omega t \) for \( \omega t = \alpha \) to \( \pi + \alpha \).

Hence the average or dc output voltage can be calculated as

\[
V_{O(dc)} = V_{dc} = \frac{2}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t d\omega t ;
\]

\[
V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t d\omega t ;
\]

\[
V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} \sin \omega t d\omega t ;
\]

\[
V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[ -\cos \omega t \right]_{\alpha}^{\pi+\alpha} ;
\]

\[
V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[ -\cos (\pi + \alpha) + \cos \alpha \right] ; \cos (\pi + \alpha) = -\cos \alpha
\]

Therefore

\[
V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha
\]
The dc output voltage $V_{dc}$ can be varied from a maximum value of $\frac{2V_m}{\pi}$ for $\alpha = 0^\circ$ to a minimum value of $\frac{-2V_m}{\pi}$ for $\alpha = \pi$ radians $= 180^\circ$.

The maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^\circ$ and is obtained as

$$V_{dc(max)} = V_{dm} = \frac{2V_m}{\pi} \times \cos(0) = \frac{2V_m}{\pi}$$

Therefore $V_{dc(max)} = V_{dm} = \frac{2V_m}{\pi}$

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(max)}} = \frac{V_{dc}}{V_{dm}} = \frac{2V_m \cos \alpha}{\frac{2V_m}{\pi}} = \cos \alpha$$

Therefore $V_{dcn} = V_n = \cos \alpha$ for a single phase full converter assuming continuous and constant load current operation.

**CONTROL CHARACTERISTIC OF SINGLE PHASE FULL CONVERTER**

The dc output control characteristic can be obtained by plotting the average or dc output voltage $V_{dc}$ versus the trigger angle $\alpha$.

For a single phase full converter the average dc output voltage is given by the equation $V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha$

<table>
<thead>
<tr>
<th>Trigger angle $\alpha$ in degrees</th>
<th>$V_{O(dc)}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$V_{dm} = \left(\frac{2V_m}{\pi}\right)$</td>
<td>Maximum dc output voltage $V_{dc(max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>$0.866 V_{dm}$</td>
<td></td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>$0.5 V_{dm}$</td>
<td></td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>$0 V_{dm}$</td>
<td></td>
</tr>
<tr>
<td>$120^\circ$</td>
<td>$-0.5 V_{dm}$</td>
<td></td>
</tr>
<tr>
<td>$150^\circ$</td>
<td>$-0.866 V_{dm}$</td>
<td></td>
</tr>
<tr>
<td>$180^\circ$</td>
<td>$-V_{dm} = \left(-\frac{2V_m}{\pi}\right)$</td>
<td></td>
</tr>
</tbody>
</table>
We notice from the control characteristic that by varying the trigger angle $\alpha$ we can vary the output dc voltage across the load. Thus it is possible to control the dc output voltage by changing the trigger angle $\alpha$. For trigger angle $\alpha$ in the range of 0 to 90 degrees (i.e., $0 \leq \alpha \leq 90^\circ$), $V_{dc}$ is positive and the average dc load current $I_{dc}$ is also positive. The average or dc output power $P_{dc}$ is positive, hence the circuit operates as a controlled rectifier to convert ac supply voltage into dc output power which is fed to the load.

For trigger angle $\alpha > 90^\circ$, $\cos \alpha$ becomes negative and as a result the average dc output voltage $V_{dc}$ becomes negative, but the load current flows in the same positive direction i.e., $I_{dc}$ is positive. Hence the output power becomes negative. This means that the power flows from the load circuit to the input ac source. This is referred to as line commutated inverter operation. During the inverter mode operation for $\alpha > 90^\circ$ the load energy can be fed back from the load circuit to the input ac source.

**TWO QUADRANT OPERATION OF A SINGLE PHASE FULL CONVERTER**
The above figure shows the two regions of single phase full converter operation in the $V_{dc}$ versus $I_{dc}$ plane. In the first quadrant when the trigger angle $\alpha$ is less than $90^0$, $V_{dc}$ and $I_{dc}$ are both positive and the converter operates as a controlled rectifier and converts the ac input power into dc output power. The power flows from the input source to the load circuit. This is the normal controlled rectifier operation where $P_{dc}$ is positive.

When the trigger angle is increased above $90^0$, $V_{dc}$ becomes negative but $I_{dc}$ is positive and the average output power (dc output power) $P_{dc}$ becomes negative and the power flows from the load circuit to the input source. The operation occurs in the fourth quadrant where $V_{dc}$ is negative and $I_{dc}$ is positive. The converter operates as a line commutated inverter.

**TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE**

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v_O^2(d(\omega t))}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi} \int_0^{\pi/\omega} v_O^2(d(\omega t))}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi} \int_0^{\pi/\omega} V_m^2 \sin^2(\alpha)(\omega t)(d(\omega t))}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi} \int_0^{\pi/\omega} \sin^2(\alpha)(\omega t)(d(\omega t))}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi} \int_0^{\pi/\omega} (1-\cos 2(\omega t))(d(\omega t))}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \int_0^{\pi/\omega} d(\omega t) - \int_0^{\pi/\omega} \cos 2(\omega t)(d(\omega t))}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[ \frac{(\sin 2(\omega t))}{(2\omega t)} \right]_0^{\pi/\omega}}$$
\[ V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[ \pi + \alpha - \frac{\sin 2(\pi + \alpha) - \sin 2\alpha}{2} \right]} \]

\[ V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[ \pi - \frac{\sin (2\pi + 2\alpha) - \sin 2\alpha}{2} \right]} ; \quad \sin (2\pi + 2\alpha) = \sin 2\alpha \]

\[ V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[ \pi - \frac{\sin 2\alpha - \sin 2\alpha}{2} \right]} \]

\[ V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[ \pi - 0 \right]} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} \]

Therefore \[ V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S \]

Hence the rms output voltage is same as the rms input supply voltage.

The rms thyristor current can be calculated as

Each thyristor conducts for \( \pi \) radians or \( 180^\circ \) in a single phase full converter operating at continuous and constant load current.

Therefore rms value of the thyristor current is calculated as

\[ I_{i_{(RMS)}} = I_{i_{(RMS)}} \times \frac{\pi}{2\pi} = I_{i_{(RMS)}} \times \frac{1}{2} \]

\[ I_{i_{(RMS)}} = I_{i_{(RMS)}} \times \frac{1}{\sqrt{2}} \]

The average thyristor current can be calculated as

\[ I_{i_{(Avg)}} = I_{i_{(dc)}} \times \frac{\pi}{2\pi} = I_{i_{(dc)}} \times \frac{1}{2} \]

\[ I_{i_{(Avg)}} = \frac{I_{i_{(dc)}}}{2} \]
SINGLE PHASE DUAL CONVERTER

Converter 1

\[ T_1 \quad T_3 \quad \frac{I_r}{2} \quad i_o \quad T_2 \quad T_4 \quad \frac{I_r}{2} \quad i_r \]

Converter 2

\[ T_1' \quad T_3' \quad \frac{I_r}{2} \quad i_o \quad T_2' \quad T_4' \quad \frac{I_r}{2} \quad i_r \]

(a) Circuit

\[ V_m \]

\[ v = V_m \sin \omega t \]

\[ \omega t \]

\[ \alpha \quad \pi \quad \pi + \alpha \quad 2\pi \]

\[ v_{oi} \]

\[ -V_m \sin \omega t \]

Converter 1 output

\[ \alpha_1 \quad \pi \quad \pi + \alpha \quad 2\pi \]

\[ v_{o2} \]

\[ -V_m \sin \omega t \]

Converter 2 output

\[ \pi - \alpha_1 \quad \pi \quad \pi + \alpha \quad 2\pi \]

\[ v_i(t) = v_{oi} + v_{o2} \]

Voltage generating circulating current

\[ \omega t \]

\[ \pi - \alpha_1 \quad 2\pi - \alpha_1 \]

(b) Waveforms
We have seen in the case of a single phase full converter with inductive loads the converter can operate in two different quadrants in the $V_{dc}$ versus $I_{dc}$ operating diagram. If two single phase full converters are connected in parallel and in opposite direction (connected in back to back) across a common load four quadrant operation is possible. Such a converter is called as a dual converter which is shown in the figure.

The dual converter system will provide four quadrant operation and is normally used in high power industrial variable speed drives. The converter number 1 provides a positive dc output voltage and a positive dc load current, when operated in the rectification mode.

The converter number 2 provides a negative dc output voltage and a negative dc load current when operated in the rectification mode. We can thus have bi-directional load current and bi-directional dc output voltage. The magnitude of output dc load voltage and the dc load current can be controlled by varying the trigger angles $\alpha_1$ & $\alpha_2$ of the converters 1 and 2 respectively.

There are two modes of operations possible for a dual converter system.
- Non circulating current mode of operation (circulating current free mode of operation).
- Circulating current mode of operation.

**NON CIRCULATING CURRENT MODE OF OPERATION (CIRCULATING CURRENT FREE MODE OF OPERATION)**

In this mode of operation only one converter is switched on at a time while the second converter is switched off. When the converter 1 is switched on and the gate trigger signals are released to the gates of thyristors in converter 1, we get an average output voltage across the load, which can be varied by adjusting the trigger angle $\alpha_1$ of the converter 1. If $\alpha_1$ is less than 90°, the converter 1 operates as a controlled rectifier and converts the input ac power into dc output power to feed the load. $V_{dc}$ and $I_{dc}$ are both positive and the operation occurs in the first quadrant. The average output power $P_{dc} = V_{dc} \times I_{dc}$ is positive. The power flows from the input ac supply to the load. When $\alpha_1$ is increased above 90° converter 1 operates as a line commutated inverter and $V_{dc}$ becomes negative while $I_{dc}$ is positive and the output power $P_{dc}$ becomes negative. The power is fed back from the load circuit to the input ac source through the converter 1. The load current falls to zero when the load energy is utilized completely.

The second converter 2 is switched on after a small delay of about 10 to 20 mill seconds to allow all the thyristors of converter 1 to turn off completely. The gate signals...
are released to the thyristor gates of converter 2 and the trigger angle $\alpha_2$ is adjusted such that $0 \leq \alpha_2 \leq 90^\circ$ so that converter 2 operates as a controlled rectifier. The dc output voltage $V_{dc}$ and $I_{dc}$ are both negative and the load current flows in the reverse direction. The magnitude of $V_{dc}$ and $I_{dc}$ are controlled by the trigger angle $\alpha_2$. The operation occurs in the third quadrant where $V_{dc}$ and $I_{dc}$ are both negative and output power $P_{dc}$ is positive and the converter 2 operates as a controlled rectifier and converts the ac supply power into dc output power which is fed to the load.

When we want to reverse the load current flow so that $I_{dc}$ is positive we have to operate converter 2 in the inverter mode by increasing the trigger angle $\alpha_2$ above $90^\circ$. When $\alpha_2$ is made greater than $90^\circ$, the converter 2 operates as a line commutated inverter and the load power (load energy) is fed back to ac mains. The current falls to zero when all the load energy is utilized and the converter 1 can be switched on after a short delay of 10 to 20 milli seconds to ensure that the converter 2 thyristors are completely turned off.

The advantage of non circulating current mode of operation is that there is no circulating current flowing between the two converters as only one converter operates and conducts at a time while the other converter is switched off. Hence there is no need of the series current limiting inductors between the outputs of the two converters. The current rating of thyristors is low in this mode.

But the disadvantage is that the load current tends to become discontinuous and the transfer characteristic becomes non linear. The control circuit becomes complex and the output response is sluggish as the load current reversal takes some time due to the time delay between the switching off of one converter and the switching on of the other converter. Hence the output dynamic response is poor. Whenever a fast and frequent reversal of the load current is required, the dual converter is operated in the circulating current mode.

**CIRCULATING CURRENT MODE OF OPERATION**

In this mode of operation both the converters 1 and 2 are switched on and operated simultaneously and both the converters are in a state of conduction. If converter 1 is operated as a controlled rectifier by adjusting the trigger angle $\alpha_1$ between 0 to $90^\circ$ the second converter 2 is operated as a line commutated inverter by increasing its trigger angle $\alpha_2$ above $90^\circ$. The trigger angles $\alpha_1$ and $\alpha_2$ are adjusted such that they produce the same average dc output voltage across the load terminals.

The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi} \cos \alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi} \cos \alpha_2$$
In the dual converter operation one converter is operated as a controlled rectifier with $\alpha_1 < 90^\circ$ and the second converter is operated as a line commutated inverter in the inversion mode with $\alpha_2 > 90^\circ$.

$$V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi} \cos \alpha_1 = -\frac{2V_m}{\pi} \cos \alpha_2 = \frac{2V_m}{\pi}(-\cos \alpha_2)$$

Therefore $\cos \alpha_1 = -\cos \alpha_2$ or $\cos \alpha_2 = -\cos \alpha_1 = \cos(\pi - \alpha_1)$

Therefore $\alpha_2 = (\pi - \alpha_1)$ or $(\alpha_1 + \alpha_2) = \pi$ radians

Which gives $\alpha_2 = (\pi - \alpha_1)$

When the trigger angle $\alpha_1$ of converter 1 is set to some value the trigger angle $\alpha_2$ of the second converter is adjusted such that $\alpha_2 = (180^\circ - \alpha_1)$. Hence for circulating current mode of operation where both converters are conducting at the same time $(\alpha_1 + \alpha_2) = 180^\circ$ so that they produce the same dc output voltage across the load.

When $\alpha_1 < 90^\circ$ (say $\alpha_1 = 30^\circ$) the converter 1 operates as a controlled rectifier and converts the ac supply into dc output power and the average load current $I_{dc}$ is positive. At the same time the converter 2 is switched on and operated as a line commutated inverter, by adjusting the trigger angle $\alpha_2$ such that $\alpha_2 = (180^\circ - \alpha_1)$, which is equal to $150^\circ$, when $\alpha_1 = 30^\circ$. The converter 2 will operate in the inversion mode and feeds the load energy back to the ac supply. When we want to reverse the load current flow we have to switch the roles of the two converters.

When converter 2 is operated as a controlled rectifier by adjusting the trigger angle $\alpha_2$ such that $\alpha_2 < 90^\circ$, the first converter 1 is operated as a line commutated inverter by adjusting the trigger angle $\alpha_1$ such that $\alpha_1 > 90^\circ$. The trigger angle $\alpha_1$ is adjusted such that $\alpha_1 = (180^\circ - \alpha_2)$ for a set value of $\alpha_2$.

In the circulating current mode a current builds up between the two converters even when the load current falls to zero. In order to limit the circulating current flowing between the two converters, we have to include current limiting reactors in series between the output terminals of the two converters.

The advantage of the circulating current mode of operation is that we can have faster reversal of load current as the two converters are in a state of conduction simultaneously. This greatly improves the dynamic response of the output giving a faster dynamic response. The output voltage and the load current can be linearly varied by adjusting the trigger angles $\alpha_1$ & $\alpha_2$ to obtain a smooth and linear output control. The control circuit becomes relatively simple. The transfer characteristic between the output voltage and the trigger angle is linear and hence the output response is very fast. The load current is free to flow in either direction at any time. The reversal of the load current can be done in a faster and smoother way.
The disadvantage of the circulating current mode of operation is that a current flows continuously in the dual converter circuit even at times when the load current is zero. Hence we should connect current limiting inductors (reactors) in order to limit the peak circulating current within specified value. The circulating current flowing through the series inductors gives rise to increased power losses, due to dc voltage drop across the series inductors which decreases the efficiency. Also the power factor of operation is low. The current limiting series inductors are heavier and bulkier which increases the cost and weight of the dual converter system.

The current flowing through the converter thyristors is much greater than the dc load current. Hence the thyristors should be rated for a peak thyristor current of \( I_{T(\text{max})} = I_{dc(\text{max})} + i_{r(\text{max})} \), where \( I_{dc(\text{max})} \) is the maximum dc load current and \( i_{r(\text{max})} \) is the maximum value of the circulating current.

**TO CALCULATE THE CIRCULATING CURRENT**

The waveforms of dual converter circuit are shown in Fig. 1.

\[ v = V_m \sin \omega t \]

\[ v_{o1} = V_m \sin \omega t \]

\[ v_{o2} = V_m \sin \omega t \]

\[ v_t(t) = v_{o1} + v_{o2} \]

\[ \pi - \alpha_1 \]

\[ 2\pi - \alpha_1 \]
As the instantaneous output voltages of the two converters are out of phase, there will be an instantaneous voltage difference and this will result in circulating current between the two converters. In order to limit the circulating current, current limiting reactors are connected in series between the outputs of the two converters. This circulating current will not flow through the load and is normally limited by the current reactor \( L_r \).

If \( v_{o1} \) and \( v_{o2} \) are the instantaneous output voltages of the converters 1 and 2, respectively, the circulating current can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor \( L_r \)), starting from \( \omega t = (2\pi - \alpha_1) \). As the two average output voltages during the interval \( \omega t = (\pi + \alpha_1) \) to \( (2\pi - \alpha_1) \) are equal and opposite their contribution to the instantaneous circulating current \( i_r \) is zero.

\[
i_r = \frac{1}{\omega L_r} \left[ \int_{(2\pi-\alpha_1)}^{\omega t} v_r d(\omega t) \right]; \quad v_r = (v_{o1} - v_{o2})
\]

As the output voltage \( v_{o2} \) is negative
\[
v_r = (v_{o1} + v_{o2})
\]

Therefore
\[
i_r = \frac{1}{\omega L_r} \left[ \int_{(2\pi-\alpha_1)}^{\omega t} (v_{o1} + v_{o2}) d(\omega t) \right];
\]
\[
v_{o1} = -V_m \sin \omega t \text{ for } (2\pi - \alpha_1) \text{ to } \omega t
\]

\[
i_r = \frac{V_m}{\omega L_r} \left[ -\int_{(2\pi-\alpha_1)}^{\omega t} \sin \omega t d(\omega t) - \int_{(2\pi-\alpha_1)}^{\omega t} \sin \omega t d(\omega t) \right]
\]

\[
i_r = \frac{V_m}{\omega L_r} \left[ \cos \omega t \right]_{(2\pi-\alpha_1)}^{\omega t} + \left[ \cos \omega t \right]_{(2\pi-\alpha_1)}^{\omega t}
\]

\[
i_r = \frac{V_m}{\omega L_r} \left[ (\cos \omega t) - \cos(2\pi - \alpha_1) + (\cos \omega t) - \cos(2\pi - \alpha_1) \right]
\]

\[
i_r = \frac{V_m}{\omega L_r} \left[ 2\cos \omega t - 2\cos(2\pi - \alpha_1) \right]
\]

\[
i_r = \frac{2V_m}{\omega L_r} (\cos \omega t - \cos \alpha_1)
\]

The instantaneous value of the circulating current depends on the delay angle.
For trigger angle (delay angle) $\alpha = 0$, magnitude becomes minimum when $\omega t = n\pi$, $n = 0, 2, 4, \ldots$ and magnitude becomes maximum when $\omega t = n\pi$, $n = 1, 3, 5, \ldots$.

If the peak load current is $I_p$, one of the converters that controls the power flow may carry a peak current of $I_p + \frac{4V_m}{\omega L_r}$,

Where $I_p = I_{L(max)} = \frac{V_m}{R_L}$, & $i_r(max) = \frac{4V_m}{\omega L_r}$

Problems

1. What will be the average power in the load for the circuit shown, when $\alpha = \frac{\pi}{4}$.

Assume SCR to be ideal. Supply voltage is $330 \sin 314t$. Also calculate the RMS power and the rectification efficiency.

The circuit is that of a single phase half wave controlled rectifier with a resistive load

$V_{dc} = \frac{V_m}{2\pi} (1 + \cos \alpha) \quad ; \quad \alpha = \frac{\pi}{4} \text{ radians}$

$V_{dc} = \frac{330}{2\pi} \left(1 + \cos \left(\frac{\pi}{4}\right)\right)$

$V_{dc} = 89.66 \text{ Volts}$

Average Power $= \frac{V_{dc}^2}{R} = \frac{89.66^2}{100} = 80.38 \text{ Watts}$

$I_{dc} = \frac{V_{dc}}{R} = \frac{89.66}{100} = 0.8966 \text{ Amps}$

$V_{RMS} = \frac{V_m}{2} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$
1. \[ V_{\text{RMS}} = 157.32 \, V \]

RMS Power (AC power)

\[ \frac{V_{\text{RMS}}^2}{R} = \frac{157.32^2}{100} = 247.50 \, \text{Watts} \]

Rectification Efficiency

\[ \text{Rectification Efficiency} = \frac{\text{Average power}}{\text{RMS power}} \]

\[ = \frac{80.38}{247.47} = 0.3248 \]

2. In the circuit shown find out the average voltage across the load assuming that the conduction drop across the SCR is 1 volt. Take \( \alpha = 45^\circ \).

\[ V_{\text{AK}} = V_{m} \sin \gamma = 1 \, \text{Volt (given)} \]

The wave form of the load voltage is shown below (not to scale).
Therefore \( \gamma = \sin^{-1}\left( \frac{V_{AK}}{V_m} \right) = \sin^{-1}\left( \frac{1}{330} \right) = 0.17^0 (0.003 \text{ radians}) \)

\[ \beta = (180^0 - \gamma) \quad ; \quad \text{By symmetry of the curve.} \]

\( \beta = 179.83^0 \quad ; \quad 3.138 \text{ radians}. \)

\[ V_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\beta} (V_m \sin \omega t - V_{AK}) d(\omega t) \]

\[ V_{dc} = \frac{1}{2\pi} \left[ \int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) - V_{AK} \int_{\alpha}^{\beta} d(\omega t) \right] \]

\[ V_{dc} = \frac{1}{2\pi} \left[ V_m (-\cos \omega t) \bigg|_{\alpha}^{\beta} - V_{AK} (\omega t) \bigg|_{\alpha}^{\beta} \right] \]

\[ V_{dc} = \frac{1}{2\pi} \left[ V_m (\cos \alpha - \cos \beta) - V_{AK} (\beta - \alpha) \right] \]

\[ V_{dc} = \frac{1}{2\pi} \left[ 330 (\cos 45^0 - \cos 179.83^0) - 1(3.138 - 0.003) \right] \]

\[ V_{dc} = 89.15 \text{ Volts} \]

**Note:** \( \beta \) and \( \alpha \) values should be in radians

3. In the figure find out the battery charging current when \( \alpha = \frac{\pi}{4} \). Assume ideal SCR.

![Diagram](attachment:image.png)

**Solution**

It is obvious that the SCR cannot conduct when the instantaneous value of the supply voltage is less than 24 V, the battery voltage. The load voltage waveform is as shown (voltage across ion).
\[ V_\theta = V_m \sin \gamma \]

\[ 24 = 200 \sqrt{2} \sin \gamma \]

\[ \gamma = \sin^{-1}\left( \frac{24}{200 \times \sqrt{2}} \right) = 4.8675^0 = 0.085 \text{ radians} \]

\[ \beta = \pi - \gamma = 3.056 \text{ radians} \]

Average value of voltage across 10Ω

\[ \frac{1}{2\pi} \left[ \int_{\alpha}^{\beta} (V_m \sin \omega t - V_m) \, d(\omega t) \right] \]

(The integral gives the shaded area)

\[ \frac{1}{2\pi} \left[ \int_{\alpha}^{\beta} (200 \times \sqrt{2} \sin \omega t - 24) \, d(\omega t) \right] \]

\[ \frac{1}{2\pi} \left[ 200 \sqrt{2} \left( \cos \frac{\pi}{4} - \cos 3.056 \right) - 24 \left( 3.056 - \frac{\pi}{4} \right) \right] \]

\[ = 68 \text{ Vots} \]

Therefore charging current

\[ \frac{\text{Average voltage across } R}{R} \]

\[ = \frac{68}{10} = 6.8 \text{ Amps} \]

Note: If value of \( \gamma \) is more than \( \alpha \), then the SCR will trigger only at \( \omega t = \gamma \), (assuming that the gate signal persists till then), when it becomes forward biased.
Therefore \[ V_{dc} = \frac{1}{2\pi} \int_{\gamma}^{\pi} (V_m \sin \omega t - V_B) d(\omega t) \]

4. In a single phase full wave rectifier supply is 200 V AC. The load resistance is \(10\Omega\), \(\alpha = 60^\circ\). Find the average voltage across the load and the power consumed in the load.

**Solution**

In a single phase full wave rectifier

\[ V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha) \]

\[ V_{dc} = \frac{200 \times \sqrt{2}}{\pi} (1 + \cos 60^\circ) \]

\[ V_{dc} = 135 \text{ Volts} \]

**Average Power**

\[ \frac{V_{dc}^2}{R} = \frac{135^2}{10} = 1.823 \text{ kW} \]

5. In the circuit shown find the charging current if the trigger angle \(\alpha = 90^\circ\).

**Solution**

With the usual notation

\[ V_B = V_m \sin \gamma \]

\[ 10 = 200\sqrt{2} \sin \gamma \]

Therefore \[ \gamma = \sin^{-1}\left(\frac{10}{200 \times \sqrt{2}}\right) = 0.035 \text{ radians} \]
Average voltage across $10\Omega = \frac{2}{2\pi} \left[ \int_{\alpha}^{\beta} (V_m \sin \omega t - V_B) d(\omega t) \right]$

\[ = \frac{1}{\pi} \left[ -V_m \cos \omega t - V_B (\omega t) \right]_{\alpha}^{\beta} \]

\[ = \frac{1}{\pi} \left[ V_m (\cos \alpha - \cos \beta) - V_B (\beta - \alpha) \right] \]

\[ = \frac{1}{\pi} \left[ 200 \times \sqrt{2} \left( \cos \frac{\pi}{2} - \cos 3.106 \right) - 10 \left( 3.106 - \frac{\pi}{2} \right) \right] \]

\[ = 85 \text{ V} \]

Note that the values of $\alpha$ & $\beta$ are in radians.

Charging current = \frac{dc \text{ voltage across resistance}}{resistance} = \frac{85}{10} = 8.5 \text{ Amps}

6. A single phase full wave controlled rectifier is used to supply a resistive load of $10 \Omega$ from a 230 V, 50 Hz, supply and firing angle of $90^0$. What is its mean load voltage? If a large inductance is added in series with the load resistance, what will be the new output load voltage?

Solution

For a single phase full wave controlled rectifier with resistive load,

\[ V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha) \]

\[ V_{dc} = \frac{230 \times \sqrt{2}}{\pi} \left( 1 + \cos \frac{\pi}{2} \right) \]

\[ V_{dc} = 103.5 \text{ Volts} \]

When a large inductance is added in series with the load, the output voltage waveform will be as shown below, for trigger angle $\alpha = 90^0$. 

\[ V_{dc} = 103.5 \text{ Volts} \]

Note that the values of $\alpha$ & $\beta$ are in radians.

Charging current = \frac{dc \text{ voltage across resistance}}{resistance} = \frac{85}{10} = 8.5 \text{ Amps}
\[ V_{dc} = \frac{2V_m}{\pi} \cos \alpha \]

Since \( \alpha = \frac{\pi}{2} \); \( \cos \alpha = \cos \left( \frac{\pi}{2} \right) = 0 \)

Therefore \( V_{dc} = 0 \) and this is evident from the waveform also.

7. The figure shows a battery charging circuit using SCRs. The input voltage to the circuit is 230 V RMS. Find the charging current for a firing angle of 45°. If any one of the SCR is open circuited, what is the charging current?

Solution

With the usual notations

\[ V_S = V_m \sin \omega t \]

\[ V_S = \sqrt{2} \times 230 \sin \omega t \]

\( V_m \sin \gamma = V_B \), the battery voltage

\[ \sqrt{2} \times 230 \sin \gamma = 100 \]
Therefore \[ \gamma = \sin^{-1}\left(\frac{100}{\sqrt{2} \times 230}\right) \]

\[ \gamma = 17.9^\circ \text{ or } 0.312 \text{ radians} \]

\[ \beta = (\pi - \gamma) = (\pi - 0.312) \]

\[ \beta = 2.829 \text{ radians} \]

Average value of voltage across load resistance

\[ = \frac{2}{2\pi} \left[ \frac{\beta}{\alpha} \int_a^\beta (V_m \sin \omega t - V_b) d(\omega t) \right] \]

\[ = \frac{1}{\pi} \left[ -V_m \cos \omega t - V_b (\omega t) \right]_{a}^{\beta} \]

\[ = \frac{1}{\pi} \left[ V_m (\cos \alpha - \cos \beta) - V_b (\beta - \alpha) \right] \]

\[ = \frac{1}{\pi} \left[ 230 \times \sqrt{2} \left( \cos \frac{\pi}{4} - \cos 2.829 \right) - 100 \left( 2.829 - \frac{\pi}{4} \right) \right] \]

\[ = \frac{1}{\pi} \left[ 230 \times \sqrt{2} (0.707 + 0.9517) - 204.36 \right] \]

\[ = 106.68 \text{ Volts} \]

Charging current \[ = \frac{\text{Voltage across resistance}}{R} \]

\[ = \frac{106.68}{10} = 10.668 \text{ Amps} \]

If one of the SCRs is open circuited, the circuit behaves like a half wave rectifier. The average voltage across the resistance and the charging current will be half of that of a full wave rectifier.

Therefore Charging Current \[ = \frac{10.668}{2} = 5.334 \text{ Amps} \]
THREE PHASE CONTROLLED RECTIFIERS

INTRODUCTION TO 3-PHASE CONTROLLED RECTIFIERS

Single phase half controlled bridge converters & fully controlled bridge converters are used extensively in industrial applications up to about 15kW of output power. The single phase controlled rectifiers provide a maximum dc output of \( V_{dc(\text{max})} = \frac{2V_m}{\pi} \).

The output ripple frequency is equal to the twice the ac supply frequency. The single phase full wave controlled rectifiers provide two output pulses during every input supply cycle and hence are referred to as two pulse converters.

Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load.

Features of 3-phase controlled rectifiers are
- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.

Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives.

3-PHASE HALF WAVE CONVERTER

Three single phase half-wave converters are connected together to form a three phase half-wave converter as shown in the figure.
THEE PHASE SUPPLY VOLTAGE EQUATIONS

We define three line neutral voltages (3 phase voltages) as follows

\[ v_{an} = V_m \sin(\omega t) \]

\[ v_{bn} = V_m \sin \left( \omega t - \frac{2\pi}{3} \right) \]

\[ v_{cn} = V_m \sin \left( \omega t - 120^\circ \right) \]

\[ v_{bn} = V_m \sin \left( \omega t + \frac{2\pi}{3} \right) \]

\[ v_{cn} = V_m \sin \left( \omega t + 120^\circ \right) \]

\[ v_{bn} = V_m \sin \left( \omega t - 240^\circ \right) \]

(c) For inductive load

Vector diagram of 3-phase supply voltages
The 3-phase half wave converter combines three single phase half wave controlled rectifiers in one single circuit feeding a common load. The thyristor $T_1$ in series with one of the supply phase windings $'a-n'$ acts as one half wave controlled rectifier. The second thyristor $T_2$ in series with the supply phase winding $'b-n'$ acts as the second half wave controlled rectifier. The third thyristor $T_3$ in series with the supply phase winding $'c-n'$ acts as the third half wave controlled rectifier.

The 3-phase input supply is applied through the star connected supply transformer as shown in the figure. The common neutral point of the supply is connected to one end of the load while the other end of the load connected to the common cathode point.

When the thyristor $T_1$ is triggered at $\omega t = \left( \frac{\pi}{6} + \alpha \right) = (30^0 + \alpha)$, the phase voltage $v_{an}$ appears across the load when $T_1$ conducts. The load current flows through the supply phase winding $'a-n'$ and through thyristor $T_1$ as long as $T_1$ conducts.

When thyristor $T_2$ is triggered at $\omega t = \left( \frac{5\pi}{6} + \alpha \right) = (150^0 + \alpha)$, $T_1$ becomes reverse biased and turns-off. The load current flows through the thyristor $T_2$ and through the supply phase winding $'b-n'$. When $T_2$ conducts the phase voltage $v_{bn}$ appears across the load until the thyristor $T_3$ is triggered.

When the thyristor $T_1$ is triggered at $\omega t = \left( \frac{3\pi}{2} + \alpha \right) = (270^0 + \alpha)$, $T_2$ is reversed biased and hence $T_2$ turns-off. The phase voltage $v_{cn}$ appears across the load when $T_3$ conducts.

When $T_1$ is triggered again at the beginning of the next input cycle the thyristor $T_3$ turns off as it is reverse biased naturally as soon as $T_1$ is triggered. The figure shows the 3-phase input supply voltages, the output voltage which appears across the load, and the load current assuming a constant and ripple free load current for a highly inductive load and the current through the thyristor $T_1$.

For a purely resistive load where the load inductance `$L = 0$' and the trigger angle $\alpha > \left( \frac{\pi}{6} \right)$, the load current appears as discontinuous load current and each thyristor is naturally commutated when the polarity of the corresponding phase supply voltage reverses. The frequency of output ripple frequency for a 3-phase half wave converter is $3f_s$, where $f_s$ is the input supply frequency.

The 3-phase half wave converter is not normally used in practical converter systems because of the disadvantage that the supply current waveforms contain dc components (i.e., the supply current waveforms have an average or dc value).
TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

The reference phase voltage is $v_{RN} = v_{an} = V_m \sin \omega t$. The trigger angle $\alpha$ is measured from the cross over points of the 3-phase supply voltage waveforms. When the phase supply voltage $v_{an}$ begins its positive half cycle at $\omega t = 0$, the first cross over point appears at $\omega t = \left( \frac{\pi}{6} \right)$ radians $= 30^\circ$.

The trigger angle $\alpha$ for the thyristor $T_1$ is measured from the cross over point at $\omega t = 30^\circ$. The thyristor $T_1$ is forward biased during the period $\omega t = 30^\circ$ to $150^\circ$, when the phase supply voltage $v_{an}$ has a higher amplitude than the other phase supply voltages. Hence $T_1$ can be triggered between $30^\circ$ to $150^\circ$. When the thyristor $T_1$ is triggered at a trigger angle $\alpha$, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi} \left[ \int_{\frac{\pi}{6} - \alpha}^{\frac{5\pi}{6} - \alpha} v_{o} \cdot d(\omega t) \right]$$

Output voltage $v_o = v_{an} = V_m \sin \omega t$ for $\omega t = (30^\circ + \alpha)$ to $(150^\circ + \alpha)$

$$V_{dc} = \frac{3}{2\pi} \left[ \int_{\frac{\pi}{6} - \alpha}^{\frac{5\pi}{6} - \alpha} V_m \sin \omega t \cdot d(\omega t) \right]$$

As the output load voltage waveform has three output pulses during the input cycle of $2\pi$ radians

$$V_{dc} = \frac{3V_m}{2\pi} \left[ \int_{\frac{\pi}{6} - \alpha}^{\frac{5\pi}{6} - \alpha} \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[ \int_{\frac{\pi}{6} - \alpha}^{\frac{5\pi}{6} - \alpha} (-\cos \omega t) \cdot d(\omega t) \right]$$
\[ V_{dc} = \frac{3V_m}{2\pi} \left[ -\cos\left(\frac{5\pi}{6}\right)\cos(\alpha) + \sin\left(\frac{5\pi}{6}\right)\sin(\alpha) + \cos\left(\frac{\pi}{6}\right)\cos(\alpha) - \sin\left(\frac{\pi}{6}\right)\sin(\alpha) \right] \]

Note from the trigonometric relationship
\[
\cos(A + B) = (\cos A \cdot \cos B - \sin A \cdot \sin B)
\]

\[
V_{dc} = \frac{3V_m}{2\pi} \left[ -\cos\left(150^\circ\right)\cos(\alpha) + \sin\left(150^\circ\right)\sin(\alpha) + \cos\left(30^\circ\right)\cos(\alpha) - \sin\left(30^\circ\right)\sin(\alpha) \right]
\]

\[
V_{dc} = \frac{3V_m}{2\pi} \left[ -\cos\left(180^\circ - 30^\circ\right)\cos(\alpha) + \sin\left(180^\circ - 30^\circ\right)\sin(\alpha) + \cos\left(30^\circ\right)\cos(\alpha) - \sin\left(30^\circ\right)\sin(\alpha) \right]
\]

Note: \(\cos\left(180^\circ - 30^\circ\right) = -\cos\left(30^\circ\right)\)

\[
\sin\left(180^\circ - 30^\circ\right) = \sin\left(30^\circ\right)
\]

Therefore
\[
V_{dc} = \frac{3V_m}{2\pi} \left[ +\cos\left(30^\circ\right)\cos(\alpha) + \sin\left(30^\circ\right)\sin(\alpha) + \cos\left(30^\circ\right)\cos(\alpha) - \sin\left(30^\circ\right)\sin(\alpha) \right]
\]

\[
V_{dc} = \frac{3V_m}{2\pi} \left[ 2\cos\left(30^\circ\right)\cos(\alpha) \right]
\]

\[
V_{dc} = \frac{3V_m}{2\pi} \left[ 2 \times \frac{\sqrt{3}}{2} \cos(\alpha) \right]
\]

\[
V_{dc} = \frac{3V_m}{2\pi} \left[ \sqrt{3} \cos(\alpha) \right] = \frac{3\sqrt{3}V_m}{2\pi} \cos(\alpha)
\]
\[ V_{dc} = \frac{3V_{Lm}}{2\pi} \cos(\alpha) \]

Where

\[ V_{Lm} = \sqrt{3}V_m = \text{Max. line to line supply voltage for a 3-phase star connected transformer.} \]

The maximum average or dc output voltage is obtained at a delay angle \( \alpha = 0 \) and is given by

\[ V_{dc(\text{max})} = V_{dm} = \frac{3\sqrt{3}V_m}{2\pi} \]

Where

\( V_m \) is the peak phase voltage.

And the normalized average output voltage is

\[ V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha \]

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

The rms value of output voltage is found by using the equation

\[
V_{O(\text{RMS})} = \left[ \frac{3}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6} + \alpha} V_m^2 \sin^2 \omega t \, d(\omega t) \right]^{\frac{1}{2}}
\]

and we obtain

\[
V_{O(\text{RMS})} = \sqrt{3}V_m \left[ \frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}
\]
3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH RL LOAD

\[ \alpha = 30^\circ \]

\[ \alpha = 60^\circ \]

\[ \alpha = 90^\circ \]
3 PHASE HALF WAVE RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH R LOAD

\[
\begin{align*}
V_\text{s} & \quad 0  \\
30' & \quad 60' & \quad 90' & \quad 120' & \quad 150' & \quad 180' & \quad 210' & \quad 240' & \quad 270' & \quad 300' & \quad 330' & \quad 360' & \quad 390' & \quad 420' \\
\alpha = 0 \\
\alpha = 15^0 \\
\alpha = 30^0 \\
\alpha = 60^0
\end{align*}
\]
TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A 3 PHASE HALF WAVE CONVERTER WITH RESISTIVE LOAD OR RL LOAD WITH FWD.

In the case of a three-phase half wave controlled rectifier with resistive load, the thyristor $T_1$ is triggered at $\omega t = (30^0 + \alpha)$ and $T_1$ conducts up to $\omega t = 180^0 = \pi$ radians. When the phase supply voltage $v_{an}$ decreases to zero at $\omega t = \pi$, the load current falls to zero and the thyristor $T_1$ turns off. Thus $T_1$ conducts from $\omega t = (30^0 + \alpha)$ to $180^0$.

Hence the average dc output voltage for a 3-pulse converter (3-phase half wave controlled rectifier) is calculated by using the equation

$$V_{dc} = \frac{3}{2\pi} \left[ \int_{\alpha + 30^0}^{180^0} v_{\omega} d(\omega t) \right]$$

$$v_{\omega} = v_{an} = V_m \sin \omega t; \text{ for } \omega t = (\alpha + 30^0) \text{ to } 180^0$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[ \int_{\alpha + 30^0}^{180^0} \sin \omega t d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[ \int_{\alpha + 30^0}^{180^0} \sin \omega t d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[ -\cos \omega t \right]_{\alpha + 30^0}^{180^0}$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[ -\cos 180^0 + \cos (\alpha + 30^0) \right]$$

Since $\cos 180^0 = -1$,

We get

$$V_{dc} = \frac{3V_m}{2\pi} \left[ 1 + \cos (\alpha + 30^0) \right]$$
THREE PHASE SEMICONVERTERS

3-phase semi-converters are three phase half controlled bridge controlled rectifiers which employ three thyristors and three diodes connected in the form of a bridge configuration. Three thyristors are controlled switches which are turned on at appropriate times by applying appropriate gating signals. The three diodes conduct when they are forward biased by the corresponding phase supply voltages.

3-phase semi-converters are used in industrial power applications up to about 120kW output power level, where single quadrant operation is required. The power factor of 3-phase semi-converter decreases as the trigger angle $\alpha$ increases. The power factor of a 3-phase semi-converter is better than three phase half wave converter.

The figure shows a 3-phase semi-converter with a highly inductive load and the load current is assumed to be a constant and continuous load current with negligible ripple.

Thyristor $T_1$ is forward biased when the phase supply voltage $v_{an}$ is positive and greater than the other phase voltages $v_{bn}$ and $v_{cn}$. The diode $D_1$ is forward biased when the phase supply voltage $v_{cn}$ is more negative than the other phase supply voltages.

Thyristor $T_2$ is forward biased when the phase supply voltage $v_{bn}$ is positive and greater than the other phase voltages. Diode $D_2$ is forward biased when the phase supply voltage $v_{bn}$ is more negative than the other phase supply voltages.

Thyristor $T_3$ is forward biased when the phase supply voltage $v_{cn}$ is positive and greater than the other phase voltages. Diode $D_3$ is forward biased when the phase supply voltage $v_{bn}$ is more negative than the other phase supply voltages.

The figure shows the waveforms for the three phase input supply voltages, the output voltage, the thyristor and diode current waveforms, the current through the free wheeling diode $D_m$ and the supply current $i_a$. The frequency of the output supply waveform is $3f_s$, where $f_s$ is the input ac supply frequency. The trigger angle $\alpha$ can be varied from $0^\circ$ to $180^\circ$.

During the time period $\left(\frac{\pi}{6}\right) \leq \omega t \leq \left(\frac{7\pi}{6}\right)$ i.e., for $30^\circ \leq \omega t \leq 210^\circ$, thyristor $T_1$ is forward biased. If $T_1$ is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$, $T_1$ and $D_1$ conduct together and the
line to line supply voltage $v_{ac}$ appears across the load. At $\omega t = \left(\frac{7\pi}{6}\right)$, $v_{ac}$ starts to become negative and the free wheeling diode $D_m$ turns on and conducts. The load current continues to flow through the free wheeling diode $D_m$ and thyristor $T_1$ and diode $D_1$ are turned off.

If the free wheeling diode $D_m$ is not connected across the load, then $T_1$ would continue to conduct until the thyristor $T_2$ is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right)$ and the free wheeling action is accomplished through $T_1$ and $D_2$, when $D_2$ turns on as soon as $v_{an}$ becomes more negative at $\omega t = \left(\frac{7\pi}{6}\right)$. If the trigger angle $\alpha \leq \left(\frac{\pi}{3}\right)$ each thyristor conducts for $\frac{2\pi}{3}$ radians ($120^\circ$) and the free wheeling diode $D_m$ does not conduct. The waveforms for a 3-phase semi-converter with $\alpha \leq \left(\frac{\pi}{3}\right)$ is shown in figure
for $\alpha = 30^\circ$
We define three line neutral voltages (3 phase voltages) as follows:

\[ v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m = \text{Max. Phase Voltage} \]

\[ v_{YN} = v_{bn} = V_m \sin \left( \omega t - \frac{2\pi}{3} \right) \]

\[ v_{YN} = v_{cn} = V_m \sin \left( \omega t - 120^\circ \right) \]

\[ v_{BN} = v_{cn} = V_m \sin \left( \omega t + \frac{2\pi}{3} \right) \]

\[ v_{BN} = v_{cn} = V_m \sin \left( \omega t + 120^\circ \right) \]

\[ v_{BN} = v_{cn} = V_m \sin \left( \omega t - 240^\circ \right) \]

The corresponding line-to-line voltages are

\[ v_{RB} = v_{ac} = (v_{an} - v_{cn}) = \sqrt{3}V_m \sin \left( \omega t - \frac{\pi}{6} \right) \]

\[ v_{YN} = v_{hn} = (v_{hn} - v_{an}) = \sqrt{3}V_m \sin \left( \omega t - \frac{5\pi}{6} \right) \]

\[ v_{BY} = v_{cb} = (v_{cn} - v_{hb}) = \sqrt{3}V_m \sin \left( \omega t + \frac{\pi}{2} \right) \]

\[ v_{RY} = v_{ab} = (v_{an} - v_{hb}) = \sqrt{3}V_m \sin \left( \omega t + \frac{\pi}{6} \right) \]

Where \( V_m \) is the peak phase voltage of a star (Y) connected source.

**TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE SEMICONVERTER FOR \( \alpha > \left( \frac{\pi}{3} \right) \) AND DISCONTINUOUS OUTPUT VOLTAGE**

For \( \alpha \geq \frac{\pi}{3} \) and discontinuous output voltage: the average output voltage is found from

\[ V_{dc} = \frac{3}{2\pi} \int_{\frac{\alpha}{3}}^{\frac{\pi}{6}} v_{ac} \sin(\omega t) \]
\[ V_{dc} = \frac{3}{2\pi} \int_{\pi/6 + \alpha}^{\pi/6} \sqrt{3} V_m \sin \left( \omega t - \frac{\pi}{6} \right) d(\omega t) \]

\[ V_{dc} = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos \alpha) \]

\[ V_{dc} = \frac{3V_m}{2\pi} (1 + \cos \alpha) \]

The maximum average output voltage that occurs at a delay angle of \( \alpha = 0 \) is

\[ V_{dm} = \frac{3\sqrt{3}V_m}{\pi} \]

The normalized average output voltage is

\[ V_n = \frac{V_{dc}}{V_{dm}} = 0.5 (1 + \cos \alpha) \]

The rms output voltage is found from

\[ V_{O(RMS)} = \left[ \frac{3}{2\pi} \int_{\pi/6 + \alpha}^{\pi/6} 3V_m \sin \left( \omega t - \frac{\pi}{6} \right) d(\omega t) \right]^{1/2} \]

\[ V_{O(RMS)} = \sqrt{3}V_m \left[ \frac{3}{4\pi} \left( \pi - \alpha + \frac{1}{2} \sin 2\alpha \right) \right]^{1/2} \]

For \( \alpha \leq \frac{\pi}{3} \) and continuous output voltage

Output voltage \( V_O = V_{ab} = \sqrt{3}V_m \sin \left( \omega t + \frac{\pi}{6} \right) \); for \( \omega t = \left( \frac{\pi}{6} + \alpha \right) \) to \( \left( \frac{\pi}{2} \right) \)

Output voltage \( V_O = V_{ac} = \sqrt{3}V_m \sin \left( \omega t - \frac{\pi}{6} \right) \); for \( \omega t = \left( \frac{\pi}{2} \right) \) to \( \left( \frac{5\pi}{6} + \alpha \right) \)

The average or dc output voltage is calculated by using the equation

\[ V_{dc} = \frac{3}{2\pi} \left[ \int_{\pi/6 + \alpha}^{\pi/6} v_{ab} d(\omega t) + \int_{\pi/2}^{5\pi/6 + \alpha} v_{ac} d(\omega t) \right] \]
The RMS value of the output voltage is calculated by using the equation

\[ V_{O\text{ (RMS)}} = \sqrt{3} \sqrt{V_m} \left[ \frac{3}{4\pi} \left( \frac{2\pi}{3} + \sqrt{3} \cos^2 \alpha \right) \right]^{\frac{1}{2}} \]

THREE PHASE FULL CONVERTER

Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration. All the six thyristors are controlled switches which are turned on at appropriate times by applying suitable gate trigger signals.

The three phase full converter is extensively used in industrial power applications upto about 120kW output power level, where two quadrant operation is required. The figure shows a three phase full converter with highly inductive load. This circuit is also known as three phase full wave bridge or as a six pulse converter.

The thyristors are triggered at an interval of \( \frac{\pi}{3} \) radians (i.e. at an interval of 60°). The frequency of output ripple voltage is \( 6f_s \) and the filtering requirement is less than that of three phase semi and half wave converters.
At $\omega t = \left(\frac{\pi}{6} + \alpha\right)$, thyristor $T_1$ is already conducting when the thyristor $T_1$ is turned on by applying the gating signal to the gate of $T_1$. During the time period $\omega t = \left(\frac{\pi}{6} + \alpha\right)$ to $\left(\frac{\pi}{2} + \alpha\right)$, thyristors $T_1$ and $T_6$ conduct together and the line to line supply voltage $v_{ab}$ appears across the load.

At $\omega t = \left(\frac{\pi}{2} + \alpha\right)$, the thyristor $T_2$ is triggered and $T_6$ is reverse biased immediately and $T_6$ turns off due to natural commutation. During the time period $\omega t = \left(\frac{\pi}{2} + \alpha\right)$ to $\left(\frac{5\pi}{6} + \alpha\right)$, thyristor $T_1$ and $T_2$ conduct together and the line to line supply voltage $v_{ac}$ appears across the load.

The thyristors are numbered in the circuit diagram corresponding to the order in which they are triggered. The trigger sequence (firing sequence) of the thyristors is 12, 23, 34, 45, 56, 61, 12, 23, and so on. The figure shows the waveforms of three phase input supply voltages, output voltage, the thyristor current through $T_1$ and $T_4$, the supply current through the line ‘a’.

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t ; \quad V_m = \text{Max. Phase Voltage}$$

$$v_{JN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3}\right) = V_m \sin \left(\omega t - 120^\circ\right)$$

$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3}\right) = V_m \sin \left(\omega t + 120^\circ\right) = V_m \sin \left(\omega t - 240^\circ\right)$$

Where $V_m$ is the peak phase voltage of a star (Y) connected source.

The corresponding line-to-line voltages are

$$v_{ab} = v_{an} - v_{bn} = \sqrt{3}V_m \sin \left(\omega t + \frac{\pi}{6}\right)$$

$$v_{bc} = v_{bn} - v_{cn} = \sqrt{3}V_m \sin \left(\omega t - \frac{\pi}{2}\right)$$

$$v_{ca} = v_{cn} - v_{an} = \sqrt{3}V_m \sin \left(\omega t + \frac{\pi}{2}\right)$$
Gating (Control) Signals of 3-phase full converter
TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE FULL CONVERTER WITH HIGHLY INDUCTIVE LOAD ASSUMING CONTINUOUS AND CONSTANT LOAD CURRENT

The output load voltage consists of 6 voltage pulses over a period of $2\pi$ radians, hence the average output voltage is calculated as

\[ V_{O\text{ (dc)}} = V_{dc} = \frac{6}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} v_o \, d\omega t \; ; \]

\[ v_o = v_{ab} = \sqrt{3}V_m \sin \left( \omega t + \frac{\pi}{6} \right) \]

\[ V_{dc} = \frac{2}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} \sqrt{3}V_m \sin \left( \omega t + \frac{\pi}{6} \right) \, d\omega t \]

\[ V_{dc} = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha = \frac{3V_{ml}}{\pi} \cos \alpha \]

Where $V_{ml} = \sqrt{3}V_m$ = Max. line-to-line supply voltage

The maximum average dc output voltage is obtained for a delay angle $\alpha = 0$, \[ V_{dc\text{ (max)}} = \frac{3\sqrt{3}V_m}{\pi} = \frac{3V_{ml}}{\pi} \]

The normalized average dc output voltage is \[ V_{dcn} = V_n = \frac{V_{dc}}{V_{dc\text{ (max)}}} = \cos \alpha \]

The rms value of the output voltage is found from \[ V_{O\text{ (rms)}} = \left[ \frac{6}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} v_o^2 \, d(\omega t) \right]^{\frac{1}{2}} \]
\[ V_{O(mv)} = \left[ \frac{6}{2\pi} \int_{-\pi/6}^{\pi/6} v_{im}^2 d(\omega t) \right]^{1/2} \]

\[ V_{O(mv)} = \left[ \frac{3}{2\pi} \int_{-\pi/6}^{\pi/6} 3v_m^2 \sin^2 \left( \omega t + \frac{\pi}{6} \right) d(\omega t) \right]^{1/2} \]

\[ V_{O(mv)} = \sqrt{3}v_m \left( \frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right)^{1/2} \]
THREE PHASE DUAL CONVERTERS

In many variable speed drives, the four quadrant operation is generally required and three phase dual converters are extensively used in applications up to the 2000 kW level. Figure shows three phase dual converters where two three phase full converters are connected back to back across a common load. We have seen that due to the instantaneous voltage differences between the output voltages of converters, a circulating current flows through the converters. The circulating current is normally limited by circulating reactor, $L_c$. The two converters are controlled in such a way that if $\alpha_1$ is the delay angle of converter 1, the delay angle of converter 2 is $\alpha_2 = (\pi - \alpha_1)$.

The operation of a three phase dual converter is similar that of a single phase dual converter system. The main difference being that a three phase dual converter gives much higher dc output voltage and higher dc output power than a single phase dual converter system. But the drawback is that the three phase dual converter is more expensive and the design of control circuit is more complex.

The figure below shows the waveforms for the input supply voltages, output voltages of converter 1 and converter 2, and the voltage across current limiting reactor (inductor) $L_r$. The operation of each converter is identical to that of a three phase full converter.

During the interval $\left(\frac{\pi}{6} + \alpha_1\right)$ to $\left(\frac{\pi}{2} + \alpha_1\right)$, the line to line voltage $v_{ab}$ appears across the output of converter 1 and $v_{bc}$ appears across the output of converter 2.

We define three line neutral voltages (3 phase voltages) as follows:

$$v_{RN} = v_{an} = V_m \sin \omega t$$
$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3}\right) = V_m \sin \left(\omega t - 120^\circ\right)$$
$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3}\right) = V_m \sin \left(\omega t + 120^\circ\right) = V_m \sin \left(\omega t - 240^\circ\right)$$
The corresponding line-to-line supply voltages are:

\[ v_{K}\bar{Y} = v_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin \left( \omega t + \frac{\pi}{6} \right) \]

\[ v_{Y\bar{M}} = v_{bc} = (v_{ba} - v_{cn}) = \sqrt{3}V_m \sin \left( \omega t - \frac{\pi}{2} \right) \]

\[ v_{B\bar{R}} = v_{ca} = (v_{cn} - v_{an}) = \sqrt{3}V_m \sin \left( \omega t + \frac{\pi}{2} \right) \]
TO OBTAIN AN EXPRESSION FOR THE CIRCULATING CURRENT

If \( v_{o1} \) and \( v_{o2} \) are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval \( \left( \frac{\pi}{6} + \alpha_1 \right) \leq \omega t \leq \left( \frac{\pi}{2} + \alpha_1 \right) \) is

\[
v_r = (v_{o1} + v_{o2}) = (v_{ab} - v_{bc})
\]

\[
v_r = \sqrt{3}V_m \left[ \sin \left( \omega t + \frac{\pi}{6} \right) - \sin \left( \omega t - \frac{\pi}{2} \right) \right]
\]

\[
v_r = 3V_m \cos \left( \omega t - \frac{\pi}{6} \right)
\]

The circulating current can be calculated by using the equation

\[
i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\pi_1} v_r \, d(\omega t)
\]

\[
i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\pi_1} 3V_m \cos \left( \omega t - \frac{\pi}{6} \right) \, d(\omega t)
\]

\[
i_r(t) = \frac{3V_m}{\omega L_r} \left[ \sin \left( \omega t - \frac{\pi}{6} \right) - \sin \alpha_1 \right]
\]

\[
i_{r(max)} = \frac{3V_m}{\omega L_r} \text{ maximum value of the circulating current.}
\]

There are two different modes of operation of a three phase dual converter system.

- Circulating current free (non-circulating) mode of operation
- Circulating current mode of operation

CIRCULATING CURRENT FREE (NON-CIRCULATING) MODE OF OPERATION

In this mode of operation only one converter is switched on at a time when the converter number 1 is switched on and the gate signals are applied to the thyristors the average output voltage and the average load current are controlled by adjusting the trigger angle \( \alpha_1 \) and the gating signals of converter 1 thyristors.

The load current giving a positive average load current when the converter 1 is switched on. For \( \alpha_1 < 90^\circ \) the converter 1 operates in the rectification mode \( V_{dc} \) is positive, \( I_{dc} \) is positive and hence the average load power \( P_{dc} \) is positive.
The converter 1 converts the input ac supply and feeds a dc power to the load. Power flows from the ac supply to the load during the rectification mode. When the trigger angle $\alpha_1$ is increased above $90^\circ$, $V_{dc}$ becomes negative where as $I_{dc}$ is positive because the thyristors of converter 1 conduct in only one direction and reversal of load current through thyristors of converter 1 is not possible.

For $\alpha_1 > 90^\circ$ converter 1 operates in the inversion mode & the load energy is supplied back to the ac supply. The thyristors are switched-off when the load current decreases to zero & after a short delay time of about 10 to 20 milliseconds, the converter 2 can be switched on by releasing the gate control signals to the thyristors of converter 2.

We obtain a reverse or negative load current when the converter 2 is switched ON. The average or dc output voltage and the average load current are controlled by adjusting the trigger angle $\alpha_2$ of the gate trigger pulses supplied to the thyristors of converter 2. When $\alpha_2$ is less than $90^\circ$, converter 2 operates in the rectification mode and converts the input ac supply in to dc output power which is fed to the load.

When $\alpha_2$ is less than $90^\circ$ for converter 2, $V_{dc}$ is negative & $I_{dc}$ is negative, converter 2 operates as a controlled rectifier & power flows from the ac source to the load circuit. When $\alpha_2$ is increased above $90^\circ$, the converter 2 operates in the inversion mode with $V_{dc}$ positive and $I_{dc}$ negative and hence $P_{dc}$ is negative, which means that power flows from the load circuit to the input ac supply.

The power flow from the load circuit to the input ac source is possible if the load circuit has a dc source of appropriate polarity.

When the load current falls to zero the thyristors of converter 2 turn-off and the converter 2 can be turned off.

**CIRCULATING CURRENT MODE OF OPERATION**

Both the converters are switched on at the same time in the mode of operation. One converter operates in the rectification mode while the other operates in the inversion mode. Trigger angles $\alpha_1$ & $\alpha_2$ are adjusted such that $(\alpha_1 + \alpha_2) = 180^\circ$

When $\alpha_1 < 90^\circ$, converter 1 operates as a controlled rectifier. When $\alpha_2$ is made greater than $90^\circ$, converter 2 operates in the inversion mode. $V_{dc}$, $I_{dc}$, $P_{dc}$ are positive.

When $\alpha_2 < 90^\circ$, converter 2 operates as a controlled rectifier. When $\alpha_1$ is made greater than $90^\circ$, converter 1 operates as an Inverter. $V_{dc}$ and $I_{dc}$ are negative while $P_{dc}$ is positive.
Problems

1. A 3 phase fully controlled bridge rectifier is operating from a 400 V, 50 Hz supply. The thyristors are fired at $\alpha = \frac{\pi}{4}$. There is a FWD across the load. Find the average output voltage for $\alpha = 45^\circ$ and $\alpha = 75^\circ$.

Solution

For $\alpha = 45^\circ$, $V_{dc} = \frac{3V_m}{\pi} \cos \alpha$

$$V_{dc} = \frac{3\sqrt{2} \times 400}{\pi} \cos 45^\circ = 382 \text{ Volts}$$

For $\alpha = 75^\circ$, $V_{dc} = \frac{6V_m}{2\pi} \left[1 + \cos \left(60^\circ + \alpha\right)\right]$

$$V_{dc} = \frac{6\sqrt{2} \times 400}{2\pi} \left[1 + \cos \left(60^\circ + 75^\circ\right)\right]$$

$$V_{dc} = 158.4 \text{ Volts}$$

2. A 6 pulse converter connected to 415 V ac supply is controlling a 440 V dc motor. Find the angle at which the converter must be triggered so that the voltage drop in the circuit is 10% of the motor rated voltage.

Solution

![Diagram of 3 phase Full Wave Rectifier](image)

$R_a$ - Armature resistance of motor.

$L_a$ - Armature Inductance.

If the voltage across the armature has to be the rated voltage i.e., 440 V, then the output voltage of the rectifier should be $440 + \text{drop in the motor}$

That is $440 + 0.1 \times 440 = 484 \text{ Volts}$.
Therefore

\[ V_o = \frac{3V_m}{\pi} \cos \frac{48\pi}{m} = 484 \]

That is

\[ \frac{3 \times \sqrt{2} \times 415 \times \cos \alpha}{\pi} = 484 \]

Therefore \[ \alpha = 30.27^0 \]

3. A 3 phase half controlled bridge rectifier is feeding a RL load. If input voltage is 400 sin314t and SCR is fired at \( \alpha = \frac{\pi}{4} \). Find average load voltage. If any one supply line is disconnected what is the average load voltage.

Solution

\[ \alpha = \frac{\pi}{4} \text{ radians which is less than } \frac{\pi}{3} \]

Therefore

\[ V_{dc} = \frac{3V_m}{2\pi} [1 + \cos \alpha] \]

\[ V_{dc} = \frac{3 \times 400}{2\pi} [1 + \cos 45^0] \]

\[ V_{dc} = 326.18 \text{ Volts} \]

If any one supply line is disconnected, the circuit behaves like a single phase half controlled rectifier with RL load.

\[ V_{dc} = \frac{V_m}{\pi} [1 + \cos \alpha] \]

\[ V_{dc} = \frac{400}{\pi} [1 + \cos 45^0] \]

\[ V_{dc} = 217.45 \text{ Volts} \]
INTRODUCTION

In practice it becomes necessary to turn off a conducting thyristor. (Often thyristors are used as switches to turn on and off power to the load). The process of turning off a conducting thyristor is called commutation. The principle involved is that either the anode should be made negative with respect to cathode (voltage commutation) or the anode current should be reduced below the holding current value (current commutation).

The reverse voltage must be maintained for a time at least equal to the turn-off time of SCR otherwise a reappplication of a positive voltage will cause the thyristor to conduct even without a gate signal. On similar lines the anode current should be held at a value less than the holding current at least for a time equal to turn-off time otherwise the SCR will start conducting if the current in the circuit increases beyond the holding current level even without a gate signal. Commutation circuits have been developed to hasten the turn-off process of Thyristors. The study of commutation techniques helps in understanding the transient phenomena under switching conditions.

The reverse voltage or the small anode current condition must be maintained for a time at least equal to the TURN OFF time of SCR. Otherwise the SCR may again start conducting. The techniques to turn off a SCR can be broadly classified as

- Natural Commutation
- Forced Commutation.

NATURAL COMMUTATION (CLASS F)

This type of commutation takes place when supply voltage is AC, because a negative voltage will appear across the SCR in the negative half cycle of the supply voltage and the SCR turns off by itself. Hence no special circuits are required to turn off the SCR. That is the reason that this type of commutation is called Natural or Line Commutation. Figure 1.1 shows the circuit where natural commutation takes place and figure 1.2 shows the related waveforms. \( t_c \) is the time offered by the circuit within which the SCR should turn off completely. Thus \( t_c \) should be greater than \( t_q \), the turn off time of the SCR. Otherwise, the SCR will become forward biased before it has turned off completely, and will start conducting even without a gate signal.

![Fig. 1.1: Circuit for Natural Commutation](image-url)
This type of commutation is applied in ac voltage controllers, phase controlled rectifiers and cyclo converters.

**FORCED COMMUTATION**

When supply is DC, natural commutation is not possible because the polarity of the supply remains unchanged. Hence special methods must be used to reduce the SCR current below the holding value or to apply a negative voltage across the SCR for a time interval greater than the turn off time of the SCR. This technique is called FORCED COMMUTATION and is applied in all circuits where the supply voltage is DC - namely, Choppers (fixed DC to variable DC), inverters (DC to AC). Forced commutation techniques are as follows:

- Self Commutation
- Resonant Pulse Commutation
- Complementary Commutation
- Impulse Commutation
- External Pulse Commutation
- Load Side Commutation
• Line Side Commutation

SELF COMMUTATION OR LOAD COMMUTATION OR CLASS A COMMUTATION: (COMMUTATION BY RESONATING THE LOAD)

In this type of commutation the current through the SCR is reduced below the holding current value by resonating the load. i.e., the load circuit is so designed that even though the supply voltage is positive, an oscillating current tends to flow and when the current through the SCR reaches zero, the device turns off. This is done by including an inductance and a capacitor in series with the load and keeping the circuit under-damped. Figure 1.3 shows the circuit.

This type of commutation is used in Series Inverter Circuit.

![Fig. 1.3: Circuit for Self Commutation](image)

**EXPRESSION FOR CURRENT**

At $t = 0$, when the SCR turns ON on the application of gate pulse assume the current in the circuit is zero and the capacitor voltage is $V_c(0)$.

Writing the Laplace Transformation circuit of figure 1.3 the following circuit is obtained when the SCR is conducting.

![Fig.: 1.4.](image)
\[
I(S) = \frac{[V - V_c(0)]}{S} \frac{1}{R + sL + \frac{1}{C_s}}
\]

\[
C_s[V - V_c(0)] = \frac{S}{RC_s + s^2LC + 1}
\]

\[
= \frac{C[V - V_c(0)]}{LC \left[s^2 + s\frac{R}{L} + \frac{1}{LC}\right]}
\]

\[
= \frac{V - V_c(0)}{L} \frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC}}
\]

\[
= \frac{(V - V_c(0))}{L} \frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC} + \left(\frac{R}{2L}\right)^2 - \left(\frac{R}{2L}\right)^2}
\]

\[
= \frac{(V - V_c(0))}{L} \frac{1}{s^2 + s\frac{R}{2L} + \left(\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right)^2}
\]

\[
= \frac{A}{(s + \delta)^2 + \omega^2},
\]

Where

\[
A = \frac{(V - V_c(0))}{L}, \quad \delta = \frac{R}{2L}, \quad \omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}
\]

\(\omega\) is called the natural frequency

\[
I(S) = \frac{A}{\omega} \frac{\omega}{(s + \delta)^2 + \omega^2}
\]
Taking inverse Laplace transforms

\[ i(t) = \frac{A}{\omega} e^{-\delta t} \sin \omega t \]

Therefore expression for current

\[ i(t) = \frac{V - V_c(0)}{\omega L} e^{-\frac{R}{2L}} \sin \omega t \]

Peak value of current \( \frac{(V - V_c(0))}{\omega L} \)

**Expression for voltage across capacitor at the time of turn off**

Applying KVL to figure 1.3

\[ v_c = V - v_r - V_L \]
\[ v_c = V - iR - L \frac{di}{dt} \]

Substituting for \( i \),

\[ v_c = V - R \frac{A}{\omega} e^{-\delta t} \sin \omega t - L \frac{d}{dt} \left( \frac{A}{\omega} e^{-\delta t} \sin \omega t \right) \]
\[ v_c = V - R \frac{A}{\omega} e^{-\delta t} \sin \omega t - L \frac{A}{\omega} \left( e^{-\delta t} \omega \cos \omega t - \delta e^{-\delta t} \sin \omega t \right) \]
\[ v_c = V - \frac{A}{\omega} e^{-\delta t} \left[ R \sin \omega t + \omega L \cos \omega t - L \delta \sin \omega t \right] \]
\[ v_c = V - \frac{A}{\omega} e^{-\delta t} \left[ R \sin \omega t + \omega L \cos \omega t - L \frac{R}{2L} \sin \omega t \right] \]
\[ v_c = V - \frac{A}{\omega} e^{-\delta t} \left[ \frac{R}{2} \sin \omega t + \omega L \cos \omega t \right] \]

Substituting for A,

\[ v_c(t) = V - \frac{(V - V_c(0))}{\omega L} e^{-\delta t} \left[ \frac{R}{2} \sin \omega t + \omega L \cos \omega t \right] \]
\[ v_c(t) = V - \frac{(V - V_c(0))}{\omega} e^{-\delta t} \left[ \frac{R}{2L} \sin \omega t + \omega \cos \omega t \right] \]
SCR turns off when current goes to zero, i.e., \( t = \frac{\pi}{\omega} \).

Therefore at turn off

\[
v_c = V - \frac{(V - V_c(0))}{\omega} e^{\frac{-5\pi}{\omega}} (0 + \omega \cos \pi)
\]

\[
v_c = V + [V - V_c(0)] e^{\frac{-5\pi}{\omega}}
\]

Therefore

\[
v_c = V + [V - V_c(0)] e^{2\pi \omega}
\]

**Note:** For effective commutation the circuit should be under damped.

That is

\[
\left( \frac{R}{2L} \right)^2 < \frac{1}{LC}
\]

- With \( R = 0 \), and the capacitor initially uncharged that is \( V_c(0) = 0 \)

\[
i = \frac{V}{\omega L} \sin \frac{t}{\sqrt{LC}}
\]

But \( \omega = \frac{1}{\sqrt{LC}} \)

Therefore

\[
i = \frac{V}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}} = V \left( \frac{C}{L} \right) \sin \frac{t}{\sqrt{LC}}
\]

and capacitor voltage at turn off is equal to 2V.

- Figure 1.5 shows the waveforms for the above conditions. Once the SCR turns off voltage across it is negative voltage.

- Conduction time of SCR \( \frac{\pi}{\omega} \).
Problem 1.1: Calculate the conduction time of SCR and the peak SCR current that flows in the circuit employing series resonant commutation (self commutation or class A commutation), if the supply voltage is 300 V, $C = 1 \mu F$, $L = 5 \text{ mH}$ and $R_L = 100 \Omega$. Assume that the circuit is initially relaxed.
Solution:

\[ \omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \]

\[ \omega = \sqrt{\frac{1}{5 \times 10^{-3} \times 1 \times 10^{-6}} - \left(\frac{100}{2 \times 5 \times 10^{-3}}\right)^2} \]

\[ \omega = 10,000 \text{ rad/sec} \]

Since the circuit is initially relaxed, initial voltage across capacitor is zero as also the initial current through L and the expression for current \( i \) is

\[ i = \frac{V}{\omega L} e^{-\delta t} \sin \omega t \], where \( \delta = \frac{R}{2L} \).

Therefore peak value of \( i = \frac{V}{\omega L} \)

\[ i = \frac{200}{1000 \times 5 \times 10^{-3}} = 6A \]

Conducting time of SCR \( \frac{\pi}{\omega} = \frac{\pi}{10000} = 0.314 \text{msec} \)

Problem 1.2: Figure 1.7 shows a self commutating circuit. The inductance carries an initial current of 200 A and the initial voltage across the capacitor is \( V \), the supply voltage. Determine the conduction time of the SCR and the capacitor voltage at turn off.

\[ V = 100V \]

\[ 10 \mu H \]

\[ 50 \mu F \]

\[ V_c(0) = V \]

\[ \text{Fig. 1.7.} \]
Solution:
The transformed circuit of figure 1.7 is shown in figure 1.8.

Fig.1.8: Transformed Circuit of Fig. 1.7

The governing equation is

\[
\frac{V}{s} = I(S) sL - I_o L + \frac{V_c(0)}{s} + I(S) \frac{1}{Cs}
\]

Therefore

\[
I(S) = \frac{\frac{V}{s} - \frac{V_c(0)}{s}}{sL + \frac{1}{Cs}} + \frac{I_o LC_s}{s^2 LC + 1}
\]

\[
I(S) = \left[\frac{V - V_c(0)}{s^2 + \frac{1}{LC}}\right] + \frac{I_o LC_s}{s^2 + \frac{1}{LC}}
\]

\[
I(S) = \frac{V - V_c(0)}{s^2 + \omega^2} + \frac{sI_o}{s^2 + \omega^2}
\]

Taking inverse LT

\[
i(t) = \left[\frac{V - V_c(0)}{\omega L}\right] \frac{C}{L} \sin \omega t + I_o \cos \omega t
\]
The capacitor voltage is given by:

\[ v_c(t) = \frac{1}{C} \int_0^t i(t) \, dt + V_c(0) \]

\[ v_c(t) = \frac{1}{C} \int_0^t \left\{ [V - V_c(0)] \frac{C}{\sqrt{\omega L}} \sin \omega t + I_o \cos \omega t \right\} \, dt + V_c(0) \]

\[ v_c(t) = \frac{1}{C} \left[ \frac{(V - V_c(0))}{\omega} \sqrt{\frac{C}{L}} (\cos \omega t)^t_o - \frac{I_o}{\omega} (\sin \omega t)^t_o + V_c(0) \right] \]

\[ v_c(t) = \frac{1}{C} \left[ \frac{(V - V_c(0))}{\omega} \sqrt{\frac{C}{L}} (1 - \cos \omega t) + \frac{I_o}{\omega} (\sin \omega t) + V_c(0) \right] \]

\[ v_c(t) = \frac{I_o}{C} \times \sqrt{LC} \sin \omega t + \frac{1}{C} (V - V_c(0)) \sqrt{LC} \frac{C}{\sqrt{L}} (1 - \cos \omega t) + V_c(0) \]

\[ v_c(t) = I_o \sqrt{\frac{C}{L}} \sin \omega t + V - V \cos \omega t - V_c(0) + V_c(0) \cos \omega t + V_c(0) \]

\[ v_c(t) = I_o \sqrt{\frac{C}{L}} \sin \omega t - (V - V_c(0)) \cos \omega t + V \]

In this problem \( V_c(0) = V \)

Therefore we get,

\[ i(t) = I_o \cos \omega t \] and

\[ v_c(t) = I_o \sqrt{\frac{L}{C}} \sin \omega t + V \]
The waveforms are as shown in figure 1.9.

Turn off occurs at a time $t_0$ so that $\omega t_0 = \frac{\pi}{2}$.

Therefore

$$t_0 = \frac{0.5\pi}{\omega} = 0.5\sqrt{\frac{L}{C}}$$

$$t_0 = 0.5 \times \frac{\pi}{10 \times 10^{-6}} \times 50 \times 10^{-6}$$

$$t_0 = 0.5 \times \pi \times 10^{-6} \times \sqrt{500} = 35.1\mu\text{seconds}$$

and the capacitor voltage at turn off is given by

$$v_c(t_0) = I_0 \sqrt{\frac{L}{C}} \sin \omega t_0 + V$$

$$v_c(t_0) = 200 \sqrt{\frac{10 \times 10^{-6}}{50 \times 10^{-6}}} \sin 90^\circ + 100$$

$$v_c(t_0) = 200 \times 0.447 \times \sin \left( \frac{35.12}{22.36} \right) + 100$$

$$v_c(t_0) = 89.4 + 100 = 189.4 \text{ V}$$
**Problem 1.3:** In the circuit shown in Fig. 1.10, $V = 600$ volts, initial capacitor voltage is zero, $L = 20 \, \mu H$, $C = 50 \, \mu F$ and the current through the inductance at the time of SCR triggering is $I_o = 350$ A. Determine (a) the peak values of capacitor voltage and current (b) the conduction time of $T_1$.

![Fig. 1.10](image)

**Solution:**
(Refer to problem 1.2).

The expression for $i(t)$ is given by

$$i(t) = \left[ V - V_c(0) \right] \sqrt{\frac{C}{L}} \sin \omega t + I_o \cos \omega t$$

It is given that the initial voltage across the capacitor, $V_c(0)$ is zero.

Therefore

$$i(t) = V \sqrt{\frac{C}{L}} \sin \omega t + I_o \cos \omega t$$

$i(t)$ can be written as

$$i(t) = \sqrt{I_o^2 + V^2 \frac{C}{L}} \sin (\omega t + \alpha)$$

where

$$\alpha = \tan^{-1} \left( \frac{L}{V} \frac{I_o}{\sqrt{C}} \right)$$

and

$$\omega = \frac{1}{\sqrt{LC}}$$

The peak capacitor current is

$$\sqrt{I_o^2 + V^2 \frac{C}{L}}$$

Substituting the values, the peak capacitor current
The expression for capacitor voltage is

\[ v_c(t) = I_0 \sqrt{\frac{L}{C}} \sin \omega t - (V - V_c(0)) \cos \omega t + V \]

with \( V_c(0) = 0 \), \( v_c(t) = I_0 \sqrt{\frac{L}{C}} \sin \omega t - V \cos \omega t + V \)

This can be rewritten as

\[ v_c(t) = \sqrt{V^2 + I_0^2 \frac{L}{C}} \sin (\omega t - \beta) + V \]

Where \( \beta = \tan^{-1} \frac{V}{I_0} \sqrt{\frac{L}{C}} \)

The peak value of capacitor voltage is

\[ = \sqrt{V^2 + I_0^2 \frac{L}{C} + V} \]

Substituting the values, the peak value of capacitor voltage

\[ = \sqrt{600^2 + 350^2 \times \frac{20 \times 10^{-6}}{50 \times 10^{-6}}} + 600 \]

\[ = 639.5 + 600 = 1239.5 V \]

To calculate conduction time of \( T_i \)

The waveform of capacitor current is shown in figure 1.11. When the capacitor current becomes zero the SCR turns off.
Therefore conduction time of SCR

\[
\frac{\pi - \alpha}{\omega}
\]

\[
\pi - \tan^{-1}\left(\frac{I_o \sqrt{L/C}}{V}\right)
\]

\[
= \frac{1}{\sqrt{LC}}
\]

Substituting the values

\[
\alpha = \tan^{-1}\left(\frac{I_o \sqrt{L/C}}{V}\right)
\]

\[
\alpha = \tan^{-1}\left(\frac{350 \times 10^{-6}}{600 \times 50 \times 10^{-6}}\right)
\]

\[
\alpha = 20.25^0 \text{ i.e., } 0.3534 \text{ radians}
\]

\[
\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-6} \times 50 \times 10^{-6}}} = 31622.8 \text{ rad/sec}
\]

Therefore conduction time of SCR

\[
\frac{\pi - 0.3534}{31622.8} = 88.17 \mu\text{sec}
\]

**RESONANT PULSE COMMUTATION (CLASS B COMMUTATION)**

The circuit for resonant pulse commutation is shown in figure 1.12.
Fig. 1.12: Circuit for Resonant Pulse Commutation

This is a type of commutation in which a LC series circuit is connected across the SCR. Since the commutation circuit has negligible resistance it is always under-damped i.e., the current in LC circuit tends to oscillate whenever the SCR is on.

Initially the SCR is off and the capacitor is charged to $V$ volts with plate ‘a’ being positive. Referring to figure 1.13 at $t = t_0$ the SCR is turned ON by giving a gate pulse. A current $I_L$ flows through the load and this is assumed to be constant. At the same time SCR short circuits the LC combination which starts oscillating. A current ‘i’ starts flowing in the direction shown in figure. As ‘i’ reaches its maximum value, the capacitor voltage reduces to zero and then the polarity of the capacitor voltage reverses ‘b’ becomes positive). When ‘i’ falls to zero this reverse voltage becomes maximum, and then direction of ‘i’ reverses i.e., through SCR the load current $I_L$ and ‘i’ flow in opposite direction. When the instantaneous value of ‘i’ becomes equal to $I_L$, the SCR current becomes zero and the SCR turns off. Now the capacitor starts charging and its voltage reaches the supply voltage with plate a being positive. The related waveforms are shown in figure 1.13.
Fig. 1.13: Resonant Pulse Commutation – Various Waveforms

**EXPRESSION FOR $t_c$, THE CIRCUIT TURN OFF TIME**

Assume that at the time of turn off of the SCR the capacitor voltage $v_{ab} \approx -V$ and load current $I_L$ is constant. $t_c$ is the time taken for the capacitor voltage to reach 0 volts from $-V$ volts and is derived as follows.

$$V = \frac{1}{C} \int_{t_0}^{t_c} I_L dt$$

$$V = \frac{I_L t_c}{C}$$

$$t_c = \frac{VC}{I_L} \text{ seconds}$$
For proper commutation of the inverter, it is necessary that $t_q$, the turn off time of T. Also, the magnitude of $I_p$, the peak value of $i$ should be greater than the load current $I_L$ and the expression for $i$ is derived as follows:

The LC circuit during the commutation period is shown in figure 1.14.

The transformed circuit is shown in figure 1.15.

\[
I(S) = \frac{\frac{V}{s}Cs}{sL + \frac{1}{Cs}}
\]

\[
I(S) = \frac{\left(\frac{V}{s}\right)Cs}{s^2LC + 1}
\]

\[
I(S) = \frac{VC}{LC\left(s^2 + \frac{1}{LC}\right)}
\]
\[ I(S) = \frac{V}{L} \times \frac{1}{s^2 + \frac{1}{LC}} \]

\[ I(S) = \frac{V}{L} \times \frac{\left( \frac{1}{\sqrt{LC}} \right)}{s^2 + \frac{1}{LC}} \times \frac{1}{\left( \frac{1}{\sqrt{LC}} \right)} \]

\[ I(S) = V \sqrt{\frac{C}{L}} \times \frac{1}{s^2 + \frac{1}{LC}} \]

Taking inverse LT

\[ i(t) = V \sqrt{\frac{C}{L}} \sin \omega t \]

Where \( \omega = \frac{1}{\sqrt{LC}} \)

Or \[ i(t) = \frac{V}{\omega L} \sin \omega t = I_0 \sin \omega t \]

Therefore \[ I_p = V \sqrt{\frac{C}{L}} \text{ amps} \]
EXPRESSION FOR CONDUCTION TIME OF SCR

For figure 1.13 (waveform of i), the conduction time of SCR is given by:

$$t = \frac{\pi}{\omega} + \Delta t$$

$$= \frac{\pi}{\omega} + \frac{\sin^{-1}\left(\frac{I_L}{I_p}\right)}{\omega}$$

ALTERNATE CIRCUIT FOR RESONANT PULSE COMMUTATION

The working of the circuit can be explained as follows. The capacitor C is assumed to be charged to \(V_c(0)\) with polarity as shown, \(T_1\) is conducting and the load current \(I_L\) is a constant. To turn off \(T_1\), \(T_2\) is triggered. L, C, \(T_1\) and \(T_2\) forms a resonant circuit. A resonant current \(i_c(t)\) flows in the direction shown, i.e., in a direction opposite to that of load current \(I_L\).

\[i_c(t) = I_p \sin \omega t\] (refer to the previous circuit description). Where \(I_p = V_c(0)\sqrt{\frac{C}{L}}\)

& the capacitor voltage is given by

\[v_c(t) = \frac{1}{C} \int i_c(t) \, dt\]

\[v_c(t) = \frac{1}{C} \int V_c(0) \sqrt{\frac{C}{L}} \sin \omega t \, dt\]

\[v_c(t) = -V_c(0) \cos \omega t\]

---

**Fig. 1.16: Resonant Pulse Commutation – An Alternate Circuit**
When \( i_c(t) \) becomes equal to \( I_L \) (the load current), the current through \( T_1 \) becomes zero and \( T_1 \) turns off. This happens at time \( t_i \) such that

\[
I_L = I_p \sin \frac{t_i}{\sqrt{L_C}}
\]

\[
I_p = V_c(0) \sqrt{\frac{C}{L}}
\]

\[
t_i = \sqrt{LC} \sin^{-1} \left( \frac{I_L}{V_c(0) \sqrt{C}} \right)
\]

and the corresponding capacitor voltage is

\[
v_c(t_i) = -V_1 = -V_c(0) \cos \omega t_i
\]

Once the thyristor \( T_1 \) turns off, the capacitor starts charging towards the supply voltage through \( T_2 \) and load. As the capacitor charges through the load capacitor current is same as load current \( I_L \), which is constant. When the capacitor voltage reaches \( V \), the supply voltage, the FWD starts conducting and the energy stored in \( L \) charges \( C \) to a still higher voltage. The triggering of \( T_1 \) reverses the polarity of the capacitor voltage and the circuit is ready for another triggering of \( T_1 \). The waveforms are shown in figure 1.17.

**EXPRESSION FOR \( t_c \)**

Assuming a constant load current \( I_L \) which charges the capacitor

\[
t_c = \frac{C V_1}{I_L} \text{ seconds}
\]

Normally \( V_1 = V_c(0) \)

For reliable commutation \( t_c \) should be greater than \( t_q \), the turn off time of SCR \( T_1 \). It is to be noted that \( t_c \) depends upon \( I_L \) and becomes smaller for higher values of load current.
Fig. 1.17: Resonant Pulse Commutation – Alternate Circuit – Various Waveforms

RESONANT PULSE COMMUTATION WITH ACCELERATING DIODE
A diode $D_2$ is connected as shown in the figure 1.17(a) to accelerate the discharging of the capacitor ‘$C$’. When thyristor $T_2$ is fired a resonant current $i_c(t)$ flows through the capacitor and thyristor $T_1$. At time $t = t_1$, the capacitor current $i_c(t)$ equals the load current $I_L$ and hence current through $T_1$ is reduced to zero resulting in turning off of $T_1$. Now the capacitor current $i_c(t)$ continues to flow through the diode $D_2$ until it reduces to load current level $I_L$ at time $t_2$. Thus the presence of $D_2$ has accelerated the discharge of capacitor ‘$C$’. Now the capacitor gets charged through the load and the charging current is constant. Once capacitor is fully charged $T_2$ turns off by itself. But once current of thyristor $T_1$ reduces to zero the reverse voltage appearing across $T_1$ is the forward voltage drop of $D_2$ which is very small. This makes the thyristor recovery process very slow and it becomes necessary to provide longer reverse bias time.

From figure 1.17(b)

$$t_2 = \pi \sqrt{\frac{L}{C}} - t_1$$

$$V_c(t_2) = -V_c(O) \cos \omega t_2$$

Circuit turn-off time \( t_c = t_2 - t_1 \)

**Problem 1.4:** The circuit in figure 1.18 shows a resonant pulse commutation circuit. The initial capacitor voltage $V_c(O) = 200V$, $C = 30\mu F$ and $L = 3\mu H$. Determine the circuit turn off time $t_c$ if the load current $I_L$ is (a) 200 A and (b) 50 A.
Fig. 1.18.

Solution

(a) When $I_L = 200A$

Let $T_2$ be triggered at $t = 0$.

The capacitor current $i_c(t)$ reaches a value $I_L$ at $t = t_1$, when $T_1$ turns off

$$t_1 = \sqrt{\frac{LC}{I_L}} \sin^{-1}\left(\frac{I_L}{V_c(0)}\sqrt{\frac{C}{L}}\right)$$

$$t_1 = \sqrt{\frac{3 \times 10^{-6} \times 30 \times 10^{-6}}{200 \sqrt{\frac{3 \times 10^{-6}}{30 \times 10^{-6}}}}}$$

$$t_1 = 3.05 \mu s$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3 \times 10^{-6} \times 30 \times 10^{-6}}}$$

$$\omega = 0.105 \times 10^6 \text{ rad/sec}$$

At $t = t_1$, the magnitude of capacitor voltage is $V_i = V_c(0) \cos \omega t_1$

That is

$$V_i = 200 \cos 0.105 \times 10^6 \times 3.05 \times 10^{-6}$$

$$V_i = 200 \times 0.9487$$

$$V_i = 189.75 \text{ Volts}$$

and

$$t_c = \frac{CV_i}{I_L}$$
(b) When $I_L = 50A$

$$t_c = \frac{30 \times 10^{-6} \times 200}{200} = 28.46 \mu \text{sec}.$$  

$$t_i = \sqrt{3 \times 10^{-6} \times 30 \times 10^{-6} \sin^{-1} \left( \frac{50}{200} \sqrt{\frac{3 \times 10^{-6}}{30 \times 10^{-6}}} \right)}$$

$$t_i = 0.749 \mu \text{sec}.$$  

$$V_i = 200 \cos 0.105 \times 10^6 \times 0.749 \times 10^{-6}$$

$$V_i = 200 \times 1 = 200 \text{ Volts}.$$  

$$t_c = \frac{CV_i}{I_L}$$

$$t_c = \frac{30 \times 10^{-6} \times 200}{50} = 120 \mu \text{sec}.$$  

It is observed that as load current increases the value of $t_c$ reduces.

**Problem 1.4a :** Repeat the above problem for $I_L = 200A$, if an antiparallel diode $D_2$ is connected across thyristor $T_1$ as shown in figure 1.18a.
Solution

\( I_L = 200A \)

Let \( T_2 \) be triggered at \( t = 0 \).

Capacitor current \( i_C(t) \) reaches the value \( I_L \) at \( t = t_1 \), when \( T_1 \) turns off.

Therefore

\[
t_1 = \sqrt{\frac{L}{LC}} \sin^{-1} \left[ \frac{I_L}{V_C(0)\sqrt{\frac{L}{C}}} \right]
\]

\[
t_1 = \sqrt{\frac{3 \times 10^{-6} \times 30 \times 10^{-6}}{3 \times 10^{-6} \times 30 \times 10^{-6}}} \sin^{-1} \left( \frac{200}{200 \sqrt{\frac{3 \times 10^{-6}}{30 \times 10^{-6}}}} \right)
\]

\[
t_1 = 3.05 \mu \text{sec}.
\]

\[ \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3 \times 10^{-6} \times 30 \times 10^{-6}}} \]

\[ \omega = 0.105 \times 10^6 \text{ radians/sec} \]

At \( t = t_1 \)

\[ V_C(t) = V_1 = V_C(0) \cos \omega t_1 \]

\[ V_C(t) = -200 \cos \left( 0.105 \times 10^6 \times 3.05 \times 10^{-6} \right) \]

\[ V_C(t) = 189.75V \]

\( i_C(t) \) flows through diode \( D_2 \) after \( T_1 \) turns off.

\( i_C(t) \) current falls back to \( I_L \) at \( t_2 \)

\[
t_2 = \pi \sqrt{LC} - t_1
\]

\[
t_2 = \pi \sqrt{3 \times 10^{-6} \times 30 \times 10^{-6} - 3.05 \times 10^{-6}}
\]

\[
t_2 = 26.75 \mu \text{sec}.
\]

\[ \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3 \times 10^{-6} \times 30 \times 10^{-6}}} \]

\[ \omega = 0.105 \times 10^6 \text{ rad/sec} \]
At \( t = t_2 \)

\[ V_c(t_2) = V_2 = -200 \cos 0.105 \times 10^{-6} \times 26.75 \times 10^{-6} \]

\[ V_c(t_2) = V_2 = 189.02 \text{ V} \]

Therefore \( t_c = t_2 - t_1 = 26.75 \times 10^{-6} - 3.05 \times 10^{-6} \)

\( t_c = 23.7 \mu \text{secs} \)

**Problem 1.5:** For the circuit shown in figure 1.19 calculate the value of \( L \) for proper commutation of SCR. Also find the conduction time of SCR.

![Circuit Diagram](image)

**Solution:**

The load current \( I_L = \frac{V}{R} = \frac{30}{30} = 1 \text{ Amp} \)

For proper SCR commutation \( I_p \), the peak value of resonant current \( i \), should be greater than \( I_L \).

Let \( I_p = 2I_L \), Therefore \( I_p = 2 \text{ Amps} \).

Also \( I_p = \frac{V}{\omega L} = \frac{1}{\sqrt{LC}} \times L = V \sqrt{\frac{C}{L}} \)

Therefore \( 2 = 30 \times \sqrt{\frac{4 \times 10^{-6}}{L}} \)

Therefore \( L = 0.9 \text{ mH} \).

\( \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.9 \times 10^{-3} \times 4 \times 10^{-6}}} = 16666 \text{ rad/sec} \)
Conduction time of SCR = \( \frac{\pi}{\omega} + \frac{\sin^{-1}\left(\frac{1}{2}\right)}{\omega} \)

\[
= \frac{\pi}{16666} + \frac{\sin^{-1}\left(\frac{1}{2}\right)}{16666} \\
= \frac{\pi + 0.523}{16666} \text{ radians} \\
= 0.00022 \text{ seconds} \\
= 0.22 \text{ msec}
\]

**Problem 1.6:** For the circuit shown in figure 1.20 given that the load current to be commutated is 10 A, turn off time required is 40 \( \mu \text{sec} \) and the supply voltage is 100 V. Obtain the proper values of commutating elements.

![Fig. 1.20.](image)

**Solution**

\( I_p \) peak value of \( I = V \sqrt{\frac{C}{L}} \) and this should be greater than \( I_L \). Let \( I_p = 1.5I_L \).

Therefore \( 1.5 \times 10 = 100 \sqrt{\frac{C}{L}} \) \( \ldots (a) \)

Also, assuming that at the time of turn off the capacitor voltage is approximately equal to \( V \) (and referring to waveform of capacitor voltage in figure 1.13) and the load current linearly charges the capacitor

\[ t_c = \frac{CV}{I_L} \text{ seconds} \]

and this \( t_c \) is given to be 40 \( \mu \text{sec} \).

Therefore \( 40 \times 10^{-6} = C \times \frac{100}{10} \)
Therefore \[ C = 4 \mu F \].

Substituting this in equation (a)

\[ 1.5 \times 10 = 100 \sqrt{\frac{4 \times 10^{-6}}{L}} \]

\[ 1.5^2 \times 10^2 = \frac{10^4 \times 4 \times 10^{-6}}{L} \]

Therefore \[ L = 1.777 \times 10^{-4} \text{ } H \]

\[ L = 0.177 \text{ } mH \].

**Problem 1.7:** In a resonant commutation circuit supply voltage is 200 V. Load current is 10 A and the device turn off time is 20\(\mu\)s. The ratio of peak resonant current to load current is 1.5. Determine the value of \(L\) and \(C\) of the commutation circuit.

**Solution**

Given \[ \frac{I_p}{I_L} = 1.5 \]

Therefore \[ I_p = 1.5I_L = 1.5 \times 10 = 15 \text{ } A. \]

That is \[ I_p = V \sqrt{\frac{C}{L}} = 15 \text{ } A \ldots (a) \]

It is given that the device turn off time is 20 \(\mu\)sec. Therefore \(t_c\), the circuit turn off time should be greater than this.

Let \[ t_c = 30 \mu \text{sec} \].

And \[ t_c = \frac{CV}{I_L} \]

Therefore \[ 30 \times 10^{-6} = \frac{200 \times C}{10} \]

Therefore \[ C = 1.5 \mu F \].

Substituting in (a)

\[ 15 = 200 \sqrt{\frac{1.5 \times 10^{-6}}{L}} \]
Therefore

\[ L = 0.2666 \text{ mH} \]

**COMPLEMENTARY COMMUTATION (CLASS C COMMUTATION, PARALLEL CAPACITOR COMMUTATION)**

In complementary commutation the current can be transferred between two loads. Two SCRs are used and firing of one SCR turns off the other. The circuit is shown in figure 1.21.

---

**Fig. 1.21: Complementary Commutation**

---

The working of the circuit can be explained as follows.

Initially both \( T_1 \) and \( T_2 \) are off; Now, \( T_1 \) is fired. Load current \( I_L \) flows through \( R_1 \). At the same time, the capacitor \( C \) gets charged to \( V \) volts through \( R_2 \) and \( T_1 \) (‘b’ becomes positive with respect to ‘a’). When the capacitor gets fully charged, the capacitor current \( i_C \) becomes zero.

To turn off \( T_1 \), \( T_2 \) is fired; the voltage across \( C \) comes across \( T_1 \) and reverse biases it, hence \( T_1 \) turns off. At the same time, the load current flows through \( R_2 \) and \( T_2 \). The capacitor ‘C’ charges towards \( V \) through \( R_1 \) and \( T_2 \) and is finally charged to \( V \) volts with ‘a’ plate positive. When the capacitor is fully charged, the capacitor current becomes zero. To turn off \( T_2 \), \( T_1 \) is triggered, the capacitor voltage (with ‘a’ positive) comes across \( T_2 \) and \( T_2 \) turns off. The related waveforms are shown in figure 1.22.

**EXPRESSION FOR CIRCUIT TURN OFF TIME \( t_c \)**

From the waveforms of the voltages across \( T_1 \) and capacitor, it is obvious that \( t_c \) is the time taken by the capacitor voltage to reach 0 volts from \(- V \) volts, the time constant being \( RC \) and the final voltage reached by the capacitor being \( V \) volts. The equation for capacitor voltage \( v_C(t) \) can be written as
Where $V_f$ is the final voltage, $V_i$ is the initial voltage and $\tau$ is the time constant.

At $t = t_c$, $v_c(t) = 0$,

$$\tau = R_cC, \ V_f = V, \ V_i = -V,$$

Therefore

$$0 = V + (V - V)e^{-\frac{t_c}{RC}}$$

$$0 = V - 2Ve^{-\frac{t_c}{RC}}$$

Therefore

$$V = 2Ve^{-\frac{t_c}{RC}}$$

$$0.5 = e^{-\frac{t_c}{RC}}$$

Taking natural logarithms on both sides

$$\ln 0.5 = -\frac{t_c}{RC}$$

$$t_c = 0.693RC$$

This time should be greater than the turn off time $t_q$ of $T_1$.

Similarly when $T_2$ is commutated

$$t_2 = 0.693R_2C$$

And this time should be greater than $t_q$ of $T_2$.

Usually $R_1 = R_2 = R$
Problem 1.8: In the circuit shown in figure 1.23, the load resistances $R_1 = R_2 = R = 5\Omega$ and the capacitance $C = 7.5 \mu F$, $V = 100$ volts. Determine the circuit turn off time $t_c$.

![Fig. 1.23.](image)

Solution

The circuit turn-off time $t_c = 0.693$ RC seconds

$$t_c = 0.693 \times 5 \times 7.5 \times 10^{-6}$$

$$t_c = 26\mu sec.$$

Problem 1.9: Calculate the values of $R_L$ and $C$ to be used for commutating the main SCR in the circuit shown in figure 1.24. When it is conducting a full load current of 25 A flows. The minimum time for which the SCR has to be reverse biased for proper commutation is $40\mu sec$. Also find $R_1$, given that the auxiliary SCR will undergo natural commutation when its forward current falls below the holding current value of 2 mA.

![Fig. 1.24.](image)

Solution

In this circuit only the main SCR carries the load and the auxiliary SCR is used to turn off the main SCR. Once the main SCR turns off the current through the auxiliary SCR is the sum of the capacitor charging current $i_c$ and the current $i_1$ through $R_1$. $i_c$ reduces to zero after a time $t_c$ and hence the auxiliary SCR turns off automatically after a time $t_c$, $i_1$ should be less than the holding current.
Given $I_L = 25A$

That is $25A = \frac{V}{R_L} = \frac{100}{R_L}$

Therefore $R_L = 4\Omega$

$t_c = 40\mu \text{sec} = 0.693R_LC$

That is $40 \times 10^{-6} = 0.693 \times 4 \times C$

Therefore $C = \frac{40 \times 10^{-6}}{4 \times 0.693}$

$C = 14.43 \mu F$

$i_i = \frac{V}{R_i}$ should be less than the holding current of auxiliary SCR.

Therefore $\frac{100}{R_i}$ should be $< 2\text{mA}$

Therefore $R_i > \frac{100}{2 \times 10^{-4}}$

That is $R_i > 50k\Omega$

**IMPULSE COMMUTATION (CLASS D COMMUTATION)**

The circuit for impulse commutation is as shown in figure 1.25.

![Circuit for Impulse Commutation](image)

Fig. 1.25: Circuit for Impulse Commutation
The working of the circuit can be explained as follows. It is assumed that initially the capacitor $C$ is charged to a voltage $V_C$ with polarity as shown. Let the thyristor $T_1$ be conducting and carry a load current $I_L$. If the thyristor $T_1$ is to be turned off, $T_2$ is fired. The capacitor voltage comes across $T_1$, $T_1$ is reverse biased and it turns off. Now the capacitor starts charging through $T_2$ and the load. The capacitor voltage reaches $V$ with top plate being positive. By this time the capacitor charging current (current through $T_2$) would have reduced to zero and $T_2$ automatically turns off. Now $T_1$ and $T_2$ are both off. Before firing $T_1$ again, the capacitor voltage should be reversed. This is done by turning on $T_3$, $C$ discharges through $T_3$ and $L$ and the capacitor voltage reverses. The waveforms are shown in figure 1.26.

**Fig. 1.26: Impulse Commutation – Waveforms of Capacitor Voltage, Voltage across $T_1$.**
EXPRESSION FOR CIRCUIT TURN OFF TIME (AVAILABLE TURN OFF TIME) $t_c$

$t_c$ depends on the load current $I_L$ and is given by the expression

$$V_C = \frac{1}{C} \int_0^t I_L dt$$

(assuming the load current to be constant)

$$V_C = \frac{I_c t_c}{C}$$

$$t_c = \frac{V_c C}{I_L} \text{ seconds}$$

For proper commutation $t_c$ should be $> t_q$, turn off time of $T_1$.

Note:

- $T_1$ is turned off by applying a negative voltage across its terminals. Hence this is voltage commutation.
- $t_c$ depends on load current. For higher load currents $t_c$ is small. This is a disadvantage of this circuit.
- When $T_2$ is fired, voltage across the load is $V + V_C$; hence the current through load shoots up and then decays as the capacitor starts charging.

AN ALTERNATIVE CIRCUIT FOR IMPULSE COMMUTATION

Is shown in figure 1.27.

Fig. 1.27: Impulse Commutation – An Alternate Circuit
The working of the circuit can be explained as follows:
Initially let the voltage across the capacitor be $V_C(O)$ with the top plate positive. Now $T_1$ is triggered. Load current flows through $T_1$ and load. At the same time, $C$ discharges through $T_1$, $L$ and $D$ (the current is ‘i’) and the voltage across $C$ reverses i.e., the bottom plate becomes positive. The diode $D$ ensures that the bottom plate of the capacitor remains positive.

To turn off $T_1$, $T_2$ is triggered; the voltage across the capacitor comes across $T_1$. $T_1$ is reverse biased and it turns off (voltage commutation). The capacitor now starts charging through $T_2$ and load. When it charges to $V$ volts (with the top plate positive), the current through $T_2$ becomes zero and $T_2$ automatically turns off.

The related waveforms are shown in figure 1.28.
Problem 1.10: An impulse commutated thyristor circuit is shown in figure 1.29. Determine the available turn off time of the circuit if $V = 100$ V, $R = 10$ $\Omega$ and $C = 10$ $\mu$F. Voltage across capacitor before $T_2$ is fired is $V$ volts with polarity as shown.

Fig. 1.29.

Solution

When $T_2$ is triggered the circuit is as shown in figure 1.30.

Fig. 1.30.

Writing the transform circuit, we obtain

Fig. 1.31.
We have to obtain an expression for capacitor voltage. It is done as follows:

\[
I(S) = \frac{1}{s} \left( V + V_c(0) \right) \quad R + \frac{1}{Cs}
\]

\[
I(S) = \frac{C \left( V + V_c(0) \right)}{1 + RCS}
\]

\[
I(S) = \frac{\left(V + V_c(0)\right)}{R \left(s + \frac{1}{RC}\right)}
\]

Voltage across capacitor

\[
V_c(s) = I(s) \frac{1}{Cs} \frac{V_c(0)}{s}
\]

\[
V_c(s) = \frac{1}{RCs} \left(V + \frac{V_c(0)}{s} \frac{V_c(0)}{s + \frac{1}{RC}}\right)
\]

\[
V_c(s) = \frac{V + V_c(0)}{s} \frac{V - V_c(0)}{s + \frac{1}{RC}}
\]

\[
v_c(t) = V \left(1 - e^{-t/RC}\right) - V_c(0) e^{-t/RC}
\]

In the given problem  \(V_c(0) = V\)

Therefore \(v_c(t) = V \left(1 - 2e^{-t/RC}\right)\)

The waveform of \(v_c(t)\) is shown in figure 1.32.
At $t = t_c$, $v_c(t) = 0$

Therefore

$$0 = v \left(1 - 2e^{-t/RC}\right)$$

$$1 = 2e^{-t/RC}$$

$$\frac{1}{2} = e^{-t/RC}$$

Taking natural logarithms

$$\log_e \left(\frac{1}{2}\right) = -\frac{t_c}{RC}$$

$$t_c = RC \ln(2)$$

$$t_c = 10 \times 10^{-6} \times 10 \times 10^{-6} \ln(2)$$

$$t_c = 69.3 \text{ nsec}.$$  

**Problem 1.11:** In the commutation circuit shown in figure 1.33, $C = 20 \mu F$, the input voltage $V$ varies between 180 and 220 V and the load current varies between 50 and 200 A. Determine the minimum and maximum values of available turn off time $t_c$.
Solution

It is given that V varies between 180 and 220 V and \( I_o \) varies between 50 and 200 A.

The expression for available turn off time \( t_c \) is given by

\[
t_c = \frac{CV}{I_o}
\]

\( t_c \) is maximum when V is maximum and \( I_o \) is minimum.

Therefore

\[
t_{c_{\text{max}}} = \frac{CV_{\text{max}}}{I_{o_{\text{min}}}}
\]

\[
t_{c_{\text{max}}} = 20 \times 10^{-6} \times \frac{220}{50} = 88 \mu \text{sec}
\]

and

\[
t_{c_{\text{min}}} = \frac{CV_{\text{min}}}{I_{o_{\text{max}}}}
\]

\[
t_{c_{\text{min}}} = 20 \times 10^{-6} \times \frac{180}{200} = 18 \mu \text{sec}
\]

EXTERNAL PULSE COMMUTATION (CLASS E COMMUTATION)

Fig. 1.34: External Pulse Commutation

In this type of commutation an additional source is required to turn-off the conducting thyristor. Figure 1.34 shows a circuit for external pulse commutation. \( V_s \) is the main voltage source and \( V_{aux} \) is the auxiliary supply. Assume thyristor \( T_1 \) is conducting and load \( R_L \) is connected across supply \( V_s \). When thyristor \( T_3 \) is turned ON at \( t = 0 \), \( V_{aux} \), \( T_3 \), L and C from an oscillatory circuit. Assuming capacitor is initially uncharged, capacitor C is now charged to a voltage \( 2V_{aux} \) with upper plate positive at \( t = \pi \sqrt{LC} \). When current through \( T_3 \) falls to zero, \( T_1 \) gets commutated. To turn-off the
main thyristor $T_1$, thyristor $T_2$ is fired. Then, $T_1$ is subjected to a reverse voltage equal to $V_s - 2V_{aux}$. This results in thyristor $T_1$ being turned-off. Once $T_1$ is off capacitor ‘C’ discharges through the load $R_L$.

**LOAD SIDE COMMUTATION**

In load side commutation the discharging and recharging of capacitor takes place through the load. Hence to test the commutation circuit the load has to be connected. Examples of load side commutation are Resonant Pulse Commutation and Impulse Commutation.

**LINE SIDE COMMUTATION**

In this type of commutation the discharging and recharging of capacitor takes place through the supply.

![Fig. 1.35 Line Side Commutation Circuit](image)

Figure 1.35 shows line side commutation circuit. Thyristor $T_2$ is fired to charge the capacitor ‘C’. When ‘C’ charges to a voltage of 2V, $T_2$ is self commutated. To reverse the voltage of capacitor to -2V, thyristor $T_3$ is fired and $T_3$ commutates by itself. Assuming that $T_1$ is conducting and carries a load current $I_L$ thyristor $T_2$ is fired to turn off $T_1$. The turning ON of $T_2$ will result in forward biasing the diode (FWD) and applying a reverse voltage of 2V across $T_1$. This turns off $T_1$, thus the discharging and recharging of capacitor is done through the supply and the commutation circuit can be tested without load.
INTRODUCTION

A chopper is a static device which is used to obtain a variable dc voltage from a constant dc voltage source. A chopper is also known as dc-to-dc converter. The thyristor converter offers greater efficiency, faster response, lower maintenance, smaller size and smooth control. Choppers are widely used in trolley cars, battery operated vehicles, traction motor control, control of large number of dc motors, etc…. They are also used in regenerative braking of dc motors to return energy back to supply and also as dc voltage regulators.

Choppers are of two types

- Step-down choppers
- Step-up choppers.

In step-down choppers, the output voltage will be less than the input voltage whereas in step-up choppers output voltage will be more than the input voltage.

PRINCIPLE OF STEP-DOWN CHOPPER

Figure 2.1 shows a step-down chopper with resistive load. The thyristor in the circuit acts as a switch. When thyristor is ON, supply voltage appears across the load and when thyristor is OFF, the voltage across the load will be zero. The output voltage and current waveforms are as shown in figure 2.2.
Fig. 2.2: Step-down choppers — output voltage and current waveforms

- $V_{dc}$ = average value of output or load voltage
- $I_{dc}$ = average value of output or load current
- $t_{ON}$ = time interval for which SCR conducts
- $t_{OFF}$ = time interval for which SCR is OFF.
- $T = t_{ON} + t_{OFF}$ = period of switching or chopping period
- $f = \frac{1}{T}$ = frequency of chopper switching or chopping frequency.

Average output voltage

$$V_{dc} = V\left(\frac{t_{ON}}{t_{ON} + t_{OFF}}\right) \quad \ldots(2.1)$$

$$V_{dc} = V\left(\frac{t_{ON}}{T}\right) = V.d \quad \ldots(2.2)$$

but \[\frac{t_{ON}}{T} = d = \text{duty cycle} \quad \ldots(2.3)\]

Average output current,

$$I_{dc} = \frac{V_{dc}}{R} \quad \ldots(2.4)$$

$$I_{dc} = \frac{V}{R}\left(\frac{t_{ON}}{T}\right) = \frac{V}{R}d \quad \ldots(2.5)$$
RMS value of output voltage

\[ V_o = \sqrt{\frac{1}{T} \int_0^{t_{on}} v_o^2 dt} \]

But during \( t_{on} \), \( v_o = V \)

Therefore RMS output voltage

\[ V_o = \sqrt{\frac{1}{T} \int_0^{t_{on}} V^2 dt} \]

\[ V_o = \sqrt{\frac{V^2}{T} t_{on}} = \sqrt{\frac{t_{on}}{T}} V \quad \text{...(2.6)} \]

\[ V_o = \sqrt{d} V \quad \text{...(2.7)} \]

Output power

\[ P_o = V_o I_o \]

But

\[ I_o = \frac{V_o}{R} \]

Therefore output power

\[ P_o = \frac{V_o^2}{R} \]

\[ P_o = \frac{dV^2}{R} \quad \text{...(2.8)} \]

Effective input resistance of chopper

\[ R_i = \frac{V}{I_{dc}} \quad \text{...(2.9)} \]

\[ R_i = \frac{R}{d} \quad \text{...(2.10)} \]

The output voltage can be varied by varying the duty cycle.

METHODS OF CONTROL

The output dc voltage can be varied by the following methods.

- Pulse width modulation control or constant frequency operation.
- Variable frequency control.

PULSE WIDTH MODULATION

In pulse width modulation the pulse width \( t_{on} \) of the output waveform is varied keeping chopping frequency ‘f’ and hence chopping period ‘T’ constant. Therefore output voltage is varied by varying the ON time, \( t_{on} \). Figure 2.3 shows the output voltage waveforms for different ON times.
VARIABLE FREQUENCY CONTROL

In this method of control, chopping frequency \( f \) is varied keeping either \( t_{ON} \) or \( t_{OFF} \) constant. This method is also known as frequency modulation.

Figure 2.4 shows the output voltage waveforms for a constant \( t_{ON} \) and variable chopping period \( T \).

In frequency modulation to obtain full output voltage, range frequency has to be varied over a wide range. This method produces harmonics in the output and for large \( t_{OFF} \) load current may become discontinuous.
STEP-DOWN CHOPPER WITH R-L LOAD

Figure 2.5 shows a step-down chopper with R-L load and free wheeling diode. When chopper is ON, the supply is connected across the load. Current flows from the supply to the load. When chopper is OFF, the load current $i_o$ continues to flow in the same direction through the free-wheeling diode due to the energy stored in the inductor $L$. The load current can be continuous or discontinuous depending on the values of $L$ and duty cycle, $d$. For a continuous current operation the load current is assumed to vary between two limits $I_{\text{min}}$ and $I_{\text{max}}$.

Figure 2.6 shows the output current and output voltage waveforms for a continuous current and discontinuous current operation.

![Fig. 2.5: Step Down Chopper with R-L Load](image)

![Fig. 2.6: Output Voltage and Load Current Waveforms (Continuous Current)](image)
When the current exceeds $I_{\text{max}}$ the chopper is turned off and it is turned on when current reduces to $I_{\text{min}}$.

**EXPRESSIONS FOR LOAD CURRENT $i_o$ FOR CONTINUOUS CURRENT OPERATION WHEN CHOPPER IS ON** ($0 \leq t \leq t_{\text{ON}}$)

Voltage equation for the circuit shown in figure 2.5(a) is

$$V = i_o R + L \frac{di_o}{dt} + E$$  \hspace{0.5cm} ... (2.11)

Taking Laplace Transform

$$\frac{V}{S} = R I_o (S) + L \left[ S I_o (S) - I_o (0^-) \right] + \frac{E}{S}$$  \hspace{0.5cm} ... (2.12)

At $t = 0$, initial current $i_o(0^-) = I_{\text{min}}$

$$I_o (S) = \frac{V - E}{LS \left( S + \frac{R}{L} \right)} + \frac{I_{\text{min}}}{S + \frac{R}{L}}$$  \hspace{0.5cm} ... (2.13)

Taking Inverse Laplace Transform

$$i_o (t) = \frac{V - E}{R} \left[ 1 - e^{-\frac{R}{L}t} \right] + I_{\text{min}} e^{-\frac{R}{L}t}$$  \hspace{0.5cm} ... (2.14)

This expression is valid for $0 \leq t \leq t_{\text{ON}}$. i.e., during the period chopper is ON.

At the instant the chopper is turned off, load current is

$$i_o (t_{\text{ON}}) = I_{\text{max}}$$
When Chopper is OFF \( (0 \leq t \leq t_{OFF}) \)

Voltage equation for the circuit shown in figure 2.5(b) is

\[
0 = Ri_o + L \frac{di_o}{dt} + E
\]  
\[\ldots(2.15)\]

Taking Laplace transform

\[
0 = RI_o(S) + L \left[ SI_o(S) - i_o(0^-) \right] + \frac{E}{S}
\]

Redefining time origin we have at \( t = 0^- \), initial current \( i_o(0^-) = I_{max} \)

Therefore

\[
I_o(S) = \frac{I_{max}}{S + \frac{R}{L}} \frac{E}{LS \left( S + \frac{R}{L} \right)}
\]

Taking Inverse Laplace Transform

\[
i_o(t) = I_{max} e^{-\frac{t}{T}} \left( \frac{R}{E} \right) \frac{E}{R} \left[ 1 - e^{-\frac{t}{T}} \right]
\]  
\[\ldots(2.16)\]

The expression is valid for \( 0 \leq t \leq t_{OFF} \), i.e., during the period chopper is OFF. At the instant the chopper is turned ON or at the end of the off period, the load current is

\[
i_o(t_{OFF}) = I_{min}
\]
TO FIND $I_{\text{max}}$ AND $I_{\text{min}}$

From equation (2.14),

At $t = t_{\text{ON}} = dT$, $i_D(t) = I_{\text{max}}$

Therefore

$$I_{\text{max}} = \frac{V - E}{R} \left[1 - e^{-\frac{dRT}{E}}\right] + I_{\text{min}} e^{-\frac{dRT}{E}} \quad \text{...(2.17)}$$

From equation (2.16),

At $t = t_{\text{OFF}} = T - t_{\text{ON}}$, $i_D(t) = I_{\text{min}}$

$t = t_{\text{OFF}} = (1 - d)T$

Therefore

$$I_{\text{min}} = I_{\text{max}} e^{-\frac{(1 - d)RT}{E}} - \frac{E}{R} \left[1 - e^{-\frac{(1 - d)RT}{E}}\right] \quad \text{...(2.18)}$$

Substituting for $I_{\text{min}}$ in equation (2.17) we get,

$$I_{\text{max}} = \frac{V}{R} \left[\frac{1}{1 - e^{-\frac{dRT}{E}}}\right] - \frac{E}{R} \quad \text{...(2.19)}$$

Substituting for $I_{\text{max}}$ in equation (2.18) we get,

$$I_{\text{min}} = \frac{V}{R} \left[\frac{e^{-\frac{RT}{E}} - 1}{e^{-\frac{RT}{E}} - 1}\right] - \frac{E}{R} \quad \text{...(2.20)}$$

$(I_{\text{max}} - I_{\text{min}})$ is known as the steady state ripple.

Therefore peak-to-peak ripple current

$$\Delta I = I_{\text{max}} - I_{\text{min}}$$

Average output voltage

$$V_{dc} = dV \quad \text{...(2.21)}$$

Average output current

$$I_{dc(\text{approx})} = \frac{I_{\text{max}} + I_{\text{min}}}{2} \quad \text{...(2.22)}$$
Assuming load current varies linearly from $I_{\text{min}}$ to $I_{\text{max}}$, instantaneous load current is given by

$$i_O = I_{\text{min}} + \frac{(\Delta I) t}{dT} \text{ for } 0 \leq t \leq t_{\text{ON}} (dT)$$

$$i_O = I_{\text{min}} + \left( \frac{I_{\text{max}} - I_{\text{min}}}{dT} \right) t \quad \text{(2.23)}$$

RMS value of load current

$$I_{O(\text{RMS})} = \sqrt{\frac{1}{dT} \int_0^{dT} i_O^2 \, dt}$$

$$I_{O(\text{RMS})} = \sqrt{\frac{1}{dT} \int_0^{dT} I_{\text{min}}^2 + \left( \frac{I_{\text{max}} - I_{\text{min}}}{dT} \right)^2 t^2 \, dt}$$

$$I_{O(\text{RMS})} = \sqrt{\frac{1}{dT} \int_0^{dT} I_{\text{min}}^2 + \left( \frac{I_{\text{max}} - I_{\text{min}}}{dT} \right)^2 t^2 + 2I_{\text{min}} \left( I_{\text{max}} - I_{\text{min}} \right) \, dt}$$

RMS value of output current

$$I_{O(\text{RMS})} = \left[ I_{\text{min}}^2 + \left( \frac{I_{\text{max}} - I_{\text{min}}}{3} \right)^2 + I_{\text{min}} \left( I_{\text{max}} - I_{\text{min}} \right) \right]^{1/2} \quad \text{(2.24)}$$

RMS chopper current

$$I_{\text{CH}} = \sqrt{\frac{1}{T} \int_0^T i_{\text{CH}}^2 \, dt}$$

$$I_{\text{CH}} = \sqrt{\frac{1}{T} \int_0^T \left[ I_{\text{min}} + \left( \frac{I_{\text{max}} - I_{\text{min}}}{dT} \right) t \right]^2 \, dt}$$

$$I_{\text{CH}} = \sqrt{a \left[ I_{\text{min}}^2 + \left( \frac{I_{\text{max}} - I_{\text{min}}}{3} \right)^2 + I_{\text{min}} \left( I_{\text{max}} - I_{\text{min}} \right) \right]^{1/2}} \quad \text{(2.25)}$$

Effective input resistance is

$$R_i = \frac{V}{I_S}$$
Where \( I_s \) = Average source current

\[
I_s = \frac{dI_{dc}}{dt}
\]

Therefore \( R_l = \frac{V}{dI_{dc}} \) \text{ ...}(2.26)

**PRINCIPLE OF STEP-UP CHOPPER**

![Fig. 2.13: Step-up Chopper](image)

Figure 2.13 shows a step-up chopper to obtain a load voltage \( V_o \) higher than the input voltage \( V \). The values of \( L \) and \( C \) are chosen depending upon the requirement of output voltage and current. When the chopper is ON, the inductor \( L \) is connected across the supply. The inductor current \( I \) rises and the inductor stores energy during the ON time of the chopper, \( t_{ON} \). When the chopper is off, the inductor current \( I \) is forced to flow through the diode \( D \) and load for a period, \( t_{OFF} \). The current tends to decrease resulting in reversing the polarity of induced EMF in \( L \). Therefore voltage across load is given by

\[
V_{o} = V + L \frac{dI}{dt} \text{ i.e., } V_o > V \text{ ...}(2.27)
\]

If a large capacitor ‘\( C \)’ is connected across the load then the capacitor will provide a continuous output voltage \( V_o \). Diode \( D \) prevents any current flow from capacitor to the source. Step up choppers are used for regenerative braking of dc motors.

**EXPRESSION FOR OUTPUT VOLTAGE**

Assume the average inductor current to be \( I \) during ON and OFF time of Chopper.

**When Chopper is ON**

Voltage across inductor \( L = V \)

Therefore energy stored in inductor = \( V.I.t_{ON} \) \text{ ...}(2.28)
where  \( t_{on} = ON \) period of chopper.

**When Chopper is OFF** (energy is supplied by inductor to load)

Voltage across  \( L = V_o - V \)

Energy supplied by inductor  \( L = (V_o - V) t_{off} \), where  \( t_{off} = OFF \) period of Chopper.

Neglecting losses, energy stored in inductor  \( L = \) energy supplied by inductor  \( L \)

Therefore  \( Vt_{on} = (V_o - V) t_{off} \)

\[
V_o = \frac{V(t_{on} + t_{off})}{t_{off}}
\]

\[
V_o = V \left( \frac{T}{T - t_{on}} \right)
\]

Where  \( T = \) Chopping period or period of switching.

\( T = t_{on} + t_{off} \)

\[
V_o = V \left( \frac{1}{1 - \frac{t_{on}}{T}} \right)
\]

Therefore  \( V_o = V \left( \frac{1}{1 - d} \right) \) \ldots (2.29)

Where  \( d = \frac{t_{on}}{T} = \) duty cyle

For variation of duty cycle ‘d’ in the range of  \( 0 < d < 1 \) the output voltage  \( V_o \) will vary in the range  \( V < V_o < \infty \).

**PERFORMANCE PARAMETERS**

The thyristor requires a certain minimum time to turn  \( ON \) and turn  \( OFF \). Hence duty cycle  \( d \) can be varied only between a minimum and a maximum value, limiting the minimum and maximum value of the output voltage. Ripple in the load current depends inversely on the chopping frequency,  \( f \). Therefore to reduce the load ripple current, frequency should be as high as possible.

**CLASSIFICATION OF CHOPPERS**
Choppers are classified as follows:

- Class A Chopper
- Class B Chopper
- Class C Chopper
- Class D Chopper
- Class E Chopper

**CLASS A CHOPPER**

![Class A Chopper Diagram](image)

**Fig. 2.14: Class A Chopper and \( v_o - i_o \) Characteristic**

Figure 2.14 shows a *Class A Chopper* circuit with inductive load and freewheeling diode. When chopper is ON, supply voltage \( V \) is connected across the load i.e., \( v_o = V \) and current \( i_o \) flows as shown in figure. When chopper is OFF, \( v_o = 0 \) and the load current \( i_o \) continues to flow in the same direction through the free wheeling diode. Therefore the average values of output voltage and current i.e., \( v_o \) and \( i_o \) are always positive. Hence, *Class A Chopper* is a first quadrant chopper (or single quadrant chopper). Figure 2.15 shows output voltage and current waveforms for a continuous load current.
**Class A Chopper** is a step-down chopper in which power always flows from source to load. It is used to control the speed of dc motor. The output current equations obtained in step down chopper with $R-L$ load can be used to study the performance of **Class A Chopper**.

**CLASS B CHOPPER**

Fig. 2.16: **Class B Chopper**

Fig. 2.16 shows a **Class B Chopper** circuit. When chopper is ON, $v_o = 0$ and $E$ drives a current $i_o$ through $L$ and $R$ in a direction opposite to that shown in figure 2.16. During the ON period of the chopper, the inductance $L$ stores energy. When Chopper is OFF, diode $D$ conducts, $v_o = V$ and part of the energy stored in inductor $L$ is returned to the supply. Also the current $i_o$ continues to flow from the load to source. Hence the average output voltage is positive and average output current is negative. Therefore **Class**
**Class B Chopper** operates in second quadrant. In this chopper, power flows from load to source. **Class B Chopper** is used for regenerative braking of dc motor. Figure 2.17 shows the output voltage and current waveforms of a **Class B Chopper**.

The output current equations can be obtained as follows. During the interval diode ‘D’ conducts (chopper is off) voltage equation is given by

\[
V = \frac{Ldi_o}{dt} + Ri_o + E
\]

For the initial condition i.e., \(i_o(t) = I_{min}\) at \(t = 0\).

The solution of the above equation is obtained along similar lines as in step-down chopper with R-L load

Therefore

\[
i_o(t) = \frac{V - E}{R} \left(1 - e^{\frac{R}{L}t}\right) + I_{min} e^{\frac{R}{L}t}, \quad 0 < t < t_{OFF}
\]

At \(t = t_{OFF}\)

\[
i_o(t) = I_{max}
\]

\[
I_{max} = \frac{V - E}{R} \left(1 - e^{\frac{R}{L_{off}}t}\right) + I_{min} e^{\frac{R}{L_{off}}t}
\]

During the interval chopper is ON voltage equation is given by

\[
0 = \frac{Ldi_o}{dt} + Ri_o + E
\]
Redefining the time origin, at $t = t_{ON}$, $i_o(t) = I_{max}$

The solution for the stated initial condition is

$$i_o(t) = I_{max} e^{\frac{R}{L}} \left( \frac{E}{R} \left( 1 - e^{\frac{R}{L}} \right) \right)$$

$0 < t < t_{ON}$

At $t = t_{ON}$

$$i_o(t) = I_{min}$$

Therefore

$$I_{min} = I_{max} e^{\frac{R}{L_{ON}}} \left( \frac{E}{R} \left( 1 - e^{\frac{R}{L_{ON}}} \right) \right)$$

---

**Class C Chopper**

Class C Chopper is a combination of Class A and Class B Choppers. Figure 2.18 shows a Class C two quadrant Chopper circuit. For first quadrant operation, $CH_1$ is ON or $D_2$ conducts and for second quadrant operation, $CH_2$ is ON or $D_1$ conducts. When $CH_1$ is ON, the load current $i_o$ is positive. i.e., $i_o$ flows in the direction as shown in figure 2.18.

The output voltage is equal to $V(v_o = V)$ and the load receives power from the source.
Fig. 2.18: Class C Chopper

When \( CH_1 \) is turned OFF, energy stored in inductance \( L \) forces current to flow through the diode \( D_2 \) and the output voltage \( v_o = 0 \), but \( i_o \) continues to flow in positive direction. When \( CH_2 \) is triggered, the voltage \( E \) forces \( i_o \) to flow in opposite direction through \( L \) and \( CH_2 \). The output voltage \( v_o = 0 \). On turning OFF \( CH_2 \), the energy stored in the inductance drives current through diode \( D_1 \) and the supply; output voltage \( v_o = V \) the input current becomes negative and power flows from load to source.

Thus the average output voltage \( v_o \) is positive but the average output current \( i_o \) can take both positive and negative values. Choppers \( CH_1 \) and \( CH_2 \) should not be turned ON simultaneously as it would result in short circuiting the supply. \textit{Class C Chopper} can be used both for dc motor control and regenerative braking of dc motor. Figure 2.19 shows the output voltage and current waveforms.

![Class C Chopper - Output Voltage and Current Waveforms](image)

Fig. 2.19: Class C Chopper - Output Voltage and Current Waveforms
Figure 2.20: Class D Chopper

Figure 2.20 shows a class D two quadrant chopper circuit. When both $CH_1$ and $CH_2$ are triggered simultaneously, the output voltage $v_o = V$ and output current $i_o$ flows through the load in the direction shown in figure 2.20. When $CH_1$ and $CH_2$ are turned OFF, the load current $i_o$ continues to flow in the same direction through load, $D_1$ and $D_2$, due to the energy stored in the inductor $L$, but output voltage $v_o = -V$. The average load voltage $v_o$ is positive if chopper ON-time ($t_{ON}$) is more than their OFF-time ($t_{OFF}$) and average output voltage becomes negative if $t_{ON} < t_{OFF}$. Hence the direction of load current is always positive but load voltage can be positive or negative. Waveforms are shown in figures 2.21 and 2.22.

Fig. 2.21: Output Voltage and Current Waveforms for $t_{ON} > t_{OFF}$
Fig. 2.22: Output Voltage and Current Waveforms for $t_{ON} < t_{OFF}$

CLASS E CHOPPER

Fig. 2.23: Class E Chopper
Figure 2.23 shows a class E 4 quadrant chopper circuit. When \( CH_1 \) and \( CH_4 \) are triggered, output current \( i_o \) flows in positive direction as shown in figure 2.23 through \( CH_1 \) and \( CH_4 \), with output voltage \( v_o = V \). This gives the first quadrant operation. When both \( CH_1 \) and \( CH_4 \) are OFF, the energy stored in the inductor \( L \) drives \( i_o \) through \( D_3 \) and \( D_2 \) in the same direction, but output voltage \( v_o = -V \). Therefore the chopper operates in the fourth quadrant. For fourth quadrant operation the direction of battery must be reversed. When \( CH_2 \) and \( CH_3 \) are triggered, the load current \( i_o \) flows in opposite direction and output voltage \( v_o = -V \).

Since both \( i_o \) and \( v_o \) are negative, the chopper operates in third quadrant. When both \( CH_2 \) and \( CH_3 \) are OFF, the load current \( i_o \) continues to flow in the same direction through \( D_1 \) and \( D_4 \) and the output voltage \( v_o = V \). Therefore the chopper operates in second quadrant as \( v_o \) is positive but \( i_o \) is negative. Figure 2.23(a) shows the devices which are operative in different quadrants.

**EFFECT OF SOURCE AND LOAD INDUCTANCE**

In choppers, the source inductance should be as small as possible to limit the transient voltage. Usually an input filter is used to overcome the problem of source inductance. Also source inductance may cause commutation problem for the chopper. The load ripple current is inversely proportional to load inductance and chopping frequency. Therefore the peak load current depends on load inductance. To limit the load ripple current, a smoothing inductor is connected in series with the load.

**Problem 2.1** : For the first quadrant chopper shown in figure 2.24, express the following variables as functions of \( V \), \( R \) and duty cycle ‘d’ in case load is resistive.

- Average output voltage and current
- Output current at the instant of commutation
- Average and rms free wheeling diode current.
- RMS value of output voltage
- RMS and average thyristor currents.
Solution

- Average output voltage, \( V_{dc} = \left( \frac{t_{ON}}{T} \right) V = dV \)

Average output current, \( I_{dc} = \frac{V_{dc}}{R} = \frac{dV}{R} \)

- The thyristor is commutated at the instant \( t = t_{ON} \).
Therefore output current at the instant of commutation is \( \frac{V}{R} \), since \( V \) is the output voltage at that instant.

- Free wheeling diode (FWD) will never conduct in a resistive load. Therefore average and RMS free wheeling diode currents are zero.

- RMS value of output voltage
\[
V_{o(RMS)} = \sqrt{\frac{1}{T} \int_0^{t_{ON}} v_o^2 dt}
\]
But \( v_o = V \) during \( t_{ON} \)
\[
V_{o(RMS)} = \sqrt{\frac{1}{T} \int_0^{t_{ON}} V^2 dt}
\]
\[
V_{o(RMS)} = \sqrt{\frac{V^2 \left( \frac{t_{ON}}{T} \right)}{T}}
\]
\[
V_{o(RMS)} = \sqrt{dV}
\]

Where duty cycle, \( d = \frac{t_{ON}}{T} \)
RMS value of thyristor current = RMS value of load current
\[ V_{o(RMS)} = \frac{dV}{R} \]

Average value of thyristor current = Average value of load current
\[ = \frac{dV}{R} \]

Problem 2.2: A Chopper circuit is operating on TRC at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.

Solution
\[ V = 460 \text{ V}, \ V_{dc} = 350 \text{ V}, \ f = 2 \text{ kHz} \]

Chopping period
\[ T = \frac{1}{f} \]
\[ T = \frac{1}{2 \times 10^{-3}} = 0.5 \text{ msec} \]

Output voltage
\[ V_{dc} \left( \frac{t_{ON}}{T} \right) V \]

Conduction period of thyristor
\[ t_{ON} = \frac{T \times V_{dc}}{V} \]
\[ t_{ON} = \frac{0.5 \times 10^{-3} \times 350}{460} \]
\[ t_{ON} = 0.38 \text{ msec} \]

Problem 2.3: Input to the step up chopper is 200 V. The output required is 600 V. If the conducting time of thyristor is 200 \( \mu \text{sec} \). Compute
- Chopping frequency,
- If the pulse width is halved for constant frequency of operation, find the new output voltage.
Solution

\[ V = 200 \text{ V}, \quad t_{ON} = 200 \mu s, \quad V_{dc} = 600V \]

\[ V_{dc} = V \left( \frac{T}{T-t_{ON}} \right) \]

\[ 600 = 200 \left( \frac{T}{T-200 \times 10^{-6}} \right) \]

Solving for T

\[ T = 300 \mu s \]

- Chopping frequency

\[ f = \frac{1}{T} \]

\[ f = \frac{1}{300 \times 10^{-6}} = 3.33 \text{ KHz} \]

- Pulse width is halved

\[ t_{ON} = \frac{200 \times 10^{-6}}{2} = 100 \mu s \]

Frequency is constant

\[ f = 3.33 \text{ KHz} \]

\[ T = \frac{1}{f} = 300 \mu s \]

Therefore output voltage

\[ = V \left( \frac{T}{T-t_{ON}} \right) \]

\[ = 200 \left( \frac{300 \times 10^{-6}}{300-100 \times 10^{-6}} \right) = 300 \text{ Volts} \]

Problem 2.4: A dc chopper has a resistive load of 20Ω and input voltage \( V_s = 220V \). When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.
Solution

\[ V_S = 220V, \quad R = 20\Omega, \quad f = 10 \text{ kHz} \]

\[ d = \frac{t_{ON}}{T} = 0.80 \]

\[ V_{ch} = \text{Voltage drop across chopper} = 1.5 \text{ volts} \]

Average output voltage

\[ V_{dc} = \left( \frac{t_{ON}}{T} \right) (V_S - V_{ch}) \]

\[ V_{dc} = 0.80(220 - 1.5) = 174.8 \text{ Volts} \]

Chopper ON time, \[ t_{ON} = dT \]

Chopping period, \[ T = \frac{1}{f} \]

\[ T = \frac{1}{10 \times 10^3} = 0.1 \times 10^{-3} \text{ secs} = 100 \mu\text{secs} \]

Chopper ON time,

\[ t_{ON} = dT \]

\[ t_{ON} = 0.80 \times 0.1 \times 10^{-3} \]

\[ t_{ON} = 0.08 \times 10^{-3} = 80 \mu\text{secs} \]

Problem 2.5: In a dc chopper, the average load current is 30 Amps, chopping frequency is 250 Hz. Supply voltage is 110 volts. Calculate the ON and OFF periods of the chopper if the load resistance is 2 ohms.

Solution

\[ I_{dc} = 30 \text{ Amps}, \quad f = 250 \text{ Hz}, \quad V = 110 \text{ V}, \quad R = 2\Omega \]

Chopping period, \[ T = \frac{1}{f} = \frac{1}{250} = 4 \times 10^{-3} = 4 \text{ msecs} \]

\[ I_{dc} = \frac{V_{dc}}{R} \quad \text{and} \quad V_{dc} = dV \]

Therefore

\[ I_{dc} = \frac{dV}{R} \]
Problem 2.6: A dc chopper in figure 2.25 has a resistive load of $R = 10\Omega$ and input voltage of $V = 200\ V$. When chopper is ON, its voltage drop is $2\ V$ and the chopping frequency is $1\ kHz$. If the duty cycle is $60\%$, determine

- Average output voltage
- RMS value of output voltage
- Effective input resistance of chopper
- Chopper efficiency.

**Solution**

$V = 200\ V, R = 10\Omega, \ Chopper\ voltage\ drop, V_{ch} = 2V, \ d = 0.60, \ f = 1\ kHz.$

- **Average output voltage**
  
  $V_{dc} = d(V - V_{ch})$
  
  $V_{dc} = 0.60[200 - 2] = 118.8\ Volts$

- **RMS value of output voltage**
  
  $V_{o} = \sqrt{d(V - V_{ch})}$
  
  $V_{o} = \sqrt{0.6(200 - 2)} = 153.37\ Volts$
• Effective input resistance of chopper is
\[ R_i = \frac{V}{I_S} = \frac{V}{I_{dc}} \]
\[ I_{dc} = \frac{V_{dc}}{R} = \frac{118.8}{10} = 11.88 \text{ Amps} \]
\[ R_i = \frac{V}{I_S} = \frac{V}{I_{dc}} = \frac{200}{11.88} = 16.83 \Omega \]

• Output power is
\[ P_o = \frac{1}{T} \int_0^{T} \frac{V_o^2}{R} \, dt \]
\[ P_o = \frac{1}{T} \int_0^{T} \frac{(V - V_{ch})^2}{R} \, dt \]
\[ P_o = \frac{d(V - V_{ch})^2}{R} \]
\[ P_o = \frac{0.6[200-2]^2}{10} = 2352.24 \text{ watts} \]

• Input power,
\[ P_i = \frac{1}{T} \int_0^{T} V_i \, dt \]
\[ P_i = \frac{1}{T} \int_0^{T} V(V - V_{ch}) \, dt \]
\[ P_i = \frac{dV(V - V_{ch})}{R} = \frac{0.6 \times 200[200-2]}{10} = 2376 \text{ watts} \]

• Chopper efficiency,
\[ \eta = \frac{P_o \times 100}{P_i} \]
\[ \eta = \frac{2352.24}{2376} \times 100 = 99\% \]

**Problem 2.7:** A chopper is supplying an inductive load with a free-wheeling diode. The load inductance is 5 H and resistance is 10Ω. The input voltage to the chopper is 200 volts and the chopper is operating at a frequency of 1000 Hz. If the ON/OFF time ratio is 2:3. Calculate
• Maximum and minimum values of load current in one cycle of chopper operation.
• Average load current

Solution:

L = 5 H, R = 10 Ω, f = 1000 Hz, V = 200 V, \( t_{on}:t_{off} = 2:3 \)

Chopping period,

\[
T = \frac{1}{f} = \frac{1}{1000} = 1 \text{ msecs}
\]

\[
\frac{t_{on}}{t_{off}} = \frac{2}{3}
\]

\[
t_{on} = \frac{2}{3} t_{off}
\]

\[
T = t_{on} + t_{off}
\]

\[
T = \frac{2}{3} t_{off} + t_{off}
\]

\[
T = \frac{5}{3} t_{off}
\]

\[
t_{off} = \frac{3}{5} T
\]

\[
T = \frac{3}{5} \times 1 \times 10^{-3} = 0.6 \text{ msec}
\]

\[
t_{on} = T - t_{off}
\]

\[
t_{on} = (1 - 0.6) \times 10^{-3} = 0.4 \text{ msec}
\]

Duty cycle,

\[
d = \frac{t_{on}}{T} = \frac{0.4 \times 10^{-3}}{1 \times 10^{-3}} = 0.4
\]

• Refer equations (2.19) and (2.20) for expressions of \( I_{max} \) and \( I_{min} \).

Maximum value of load current [equation (2.19)] is

\[
I_{max} = \frac{V}{R} \left[ \frac{1 - e^{-\frac{RT}{L}}}{1 - e^{-\frac{RF}{L}}} \right] - \frac{E}{R}
\]

Since there is no voltage source in the load circuit, \( E = 0 \).
Therefore 

\[ I_{\text{max}} = \frac{V}{R} \left[ 1 - e^{-\frac{dRT}{L}} \right] \]

\[ I_{\text{max}} = \frac{200}{10} \left[ 1 - e^{-\frac{0.4 \times 10^{-3}}{5}} \right] \]

\[ I_{\text{max}} = 20 \left[ 1 - e^{-0.8 \times 10^{-3}} \right] \]

\[ I_{\text{max}} = 8.0047 \text{A} \]

Minimum value of load current from equation (2.20) with \( E = 0 \) is

\[ I_{\text{min}} = \frac{V}{R} \left[ e^{\frac{dRT}{L}} - 1 \right] \]

\[ I_{\text{min}} = \frac{200}{10} \left[ e^{\frac{0.4 \times 10^{-3}}{5}} - 1 \right] = 7.995 \text{ A} \]

- Average load current

\[ I_{\text{avg}} = \frac{I_{\text{max}} + I_{\text{min}}}{2} \]

\[ I_{\text{avg}} = \frac{8.0047 + 7.995}{2} \approx 8 \text{ A} \]

**Problem 2.8**: A chopper feeding on RL load is shown in figure 2.26. With \( V = 200 \text{ V}, R = 5 \Omega, L = 5 \text{ mH}, f = 1 \text{ kHz}, d = 0.5 \text{ and } E = 0 \text{ V}. Calculate

- Maximum and minimum values of load current
- Average value of load current
- RMS load current
- Effective input resistance as seen by source
- RMS chopper current.

**Solution**

\[ V = 200 \text{ V}, \quad R = 5 \Omega, \quad L = 5 \text{ mH}, \quad f = 1\text{ kHz}, \quad d = 0.5, \quad E = 0 \]

Chopping period is 

\[ T = \frac{1}{f} = \frac{1}{1 \times 10^3} = 1 \times 10^{-3} \text{ secs} \]
Refer equations (2.19) and (2.20) for expressions of $I_{\text{max}}$ and $I_{\text{min}}$.

Maximum value of load current

$$I_{\text{max}} = \frac{V}{R} \left[ 1 - e^{-\frac{dRT}{L}} \right] - \frac{E}{R}$$

$$I_{\text{max}} = \frac{200}{5} \left[ 1 - e^{-\frac{0.5 \times 5 \times 10^{-3}}{5 \times 10^{-3}}} \right] = 24.9 \text{ A}$$

Minimum value of load current is

$$I_{\text{min}} = \frac{V}{R} \left[ \frac{e^{\frac{dRT}{L}} - 1}{e^{\frac{RT}{L}} - 1} \right] - \frac{E}{R}$$

$$I_{\text{min}} = \frac{200}{5} \left[ \frac{e^{\frac{0.5 \times 5 \times 10^{-3}}{5 \times 10^{-3}}} - 1}{e^{\frac{5 \times 10^{-3}}{5 \times 10^{-3}}} - 1} \right] = 15.1 \text{ A}$$

Average value of load current is

$$I_{dc} = \frac{I_1 + I_2}{2} \text{ for linear variation of currents}$$

Therefore

$$I_{dc} = \frac{24.9 + 15.1}{2} = 20 \text{ A}$$
Refer equations (2.24) and (2.25) for RMS load current and RMS chopper current.

RMS load current from equation (2.24) is

\[
I_{O(RMS)} = \left[ I_{\text{min}}^2 + \frac{(I_{\text{max}}-I_{\text{min}})^2}{3} + I_{\text{min}} \left( I_{\text{max}}-I_{\text{min}} \right) \right]^{\frac{1}{2}}
\]

\[
I_{O(RMS)} = \left[ 15.1^2 + \frac{(24.9-15.1)^2}{3} + 15.1(24.9-15.1) \right]^{\frac{1}{2}}
\]

\[
I_{O(RMS)} = \left[ 228.01 + \frac{96.04}{3} + 147.98 \right]^{\frac{1}{2}} = 20.2 \text{ A}
\]

RMS chopper current from equation is (2.25) is

\[
I_{ch} = \sqrt{d I_{O(RMS)}} = \sqrt{0.5 \times 20.2} = 14.28 \text{ A}
\]

Effective input resistance is

\[
R_e = \frac{V}{I_s}
\]

\[
I_s = \text{Average source current}
\]

\[
I_s = d I_d
\]

\[
I_d = 0.5 \times 20 = 10 \text{ A}
\]

Therefore effective input resistance is

\[
R_e = \frac{V}{I_s} = \frac{200}{10} = 20 \Omega
\]

**Problem 2.9**: A 200 V dc motor fed by a chopper, runs at 1000 rpm with a duty ratio of 0.8. What must be the ON time of the chopper if the motor has to run at 800 rpm. The chopper operates at 100 Hz.

**Solution**

- Speed of motor \( N_1 = 1000 \text{ rpm} \)
- Duty ratio \( d_1 = 0.8 \), \( f = 100 \text{ Hz} \)
- We know that back EMF of motor \( E_b \) is given by

\[
E_b = \frac{\Phi Z N P}{60A}
\]
Where N = speed in rpm
φ = flux/pole in wbs
Z = Number of Armature conductors
P = Number of poles
A = Number of parallel paths

Therefore

\[ E_b \propto \phi N \]
\[ E_b \propto N \text{ if flux } \phi \text{ is constant} \]

\[ V = \frac{V_{dc}}{d_1} \]

\[ V = \frac{200}{0.8} \]
\[ V = 250 \text{ Volts} \]

\[ E_b \propto N_1 \]

\[ 200 \propto 1000 \]  
(2.30)
Now speed changes hence ‘d’ also changes.

Given \( N_2 = 800 \) rpm \( E_{b_2} = ? \)

\[
E_{b_2} \propto N_2
\]

\[
E_{b_2} \propto 800 \quad \ldots(2.31)
\]

Dividing equation (2.30) by equation (2.31) we get

\[
\frac{200}{E_{b_2}} = \frac{1000}{800}
\]

\[
E_{b_2} = \frac{800 \times 200}{1000} = 160 \text{ V}
\]

But \( E_{b_2} = V_{dc_2} = d_2V \)

\[
d_2 = \frac{V_{dc_2}}{V} = \frac{160}{250} = 0.64
\]

Chopping frequency \( f = 100 \text{ Hz} \)

\[
T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ sec}
\]

\( T = 10 \) msecs

\[
\frac{t_{ON}}{T} = d_2
\]

ON time of chopper

\[
t_{ON} = d_2T
\]

\[
t_{ON} = 0.64 \times 10 \times 10^{-3}
\]

\[
t_{ON} = 6.4 \text{ msecs}
\]
IMPULSE COMMUTATED CHOPPER

Impulse commutated choppers are widely used in high power circuits where load fluctuation is not large. This chopper is also known as parallel capacitor turn-off chopper or voltage commutated chopper or classical chopper.

Fig. 2.28 shows an impulse commutated chopper with two thyristors $T_1$ and $T_2$. We shall assume that the load current remains constant at a value $I_L$ during the commutation process.

To start the circuit, capacitor ‘$C$’ is initially charged with polarity (with plate ‘$a$’ positive) as shown in the fig. 2.28 by triggering the thyristor $T_2$. Capacitor ‘$C$’ gets charged through ‘$V_S$’, ‘$C$’, $T_2$ and load. As the charging current decays to zero thyristor $T_2$ will be turned-off. With capacitor charged with plate ‘$a$’ positive the circuit is ready for operation. For convenience the chopper operation is divided into five modes.

**MODE – 1**

Thyristor $T_1$ is fired at $t = 0$. The supply voltage comes across the load. Load current $I_L$ flows through $T_1$ and load. At the same time capacitor discharges through $T_1$, $D_1$, $L_1$, and ‘$C$’ and the capacitor reverses its voltage. This reverse voltage on capacitor is held constant by diode $D_1$. Fig. 2.29 shows the equivalent circuit of Mode 1.
Capacitor Discharge Current

\[ i_c(t) = V \sqrt{\frac{C}{L}} \sin \omega t \]

\[ i_c(t) = I_p \sin \omega t \quad \text{where} \quad I_p = V \sqrt{\frac{C}{L}} \]

Where \( \omega = \frac{1}{\sqrt{LC}} \)

& Capacitor Voltage

\[ V_c(t) = V \cos \omega t \]

**MODE – 2**

Thyristor \( T_2 \) is now fired to commutate thyristor \( T_1 \). When \( T_2 \) is ON capacitor voltage reverse biases \( T_1 \) and turns it off. Now the capacitor discharges through the load from \(-V_S\) to 0 and the discharge time is known as circuit turn-off time.

Circuit turn-off time is given by

\[ t_c = \frac{V_c \times C}{I_L} \]

Where \( I_L \) is load current.

Since \( t_c \) depends on load current, it must be designed for the worst case condition which occur at the maximum value of load current and minimum value of capacitor voltage.

Then the capacitor recharges back to the supply voltage (with plate ‘a’ positive). This time is called the recharging time and is given by

\[ t_d = \frac{V_c \times C}{I_L} \]

The total time required for the capacitor to discharge and recharge is called the commutation time and it is given by

\[ t_r = t_c + t_d \]

At the end of Mode-2 capacitor has recharged to \( V_s \) and the free wheeling diode starts conducting. The equivalent circuit for Mode-2 is shown in fig. 2.30.
MODE – 3

Free wheeling diode $FWD$ starts conducting and the load current decays. The energy stored in source inductance $L_S$ is transferred to capacitor. Instantaneous current is $i(t) = I_L \cos \omega t$ Hence capacitor charges to a voltage higher than supply voltage. $T_2$ naturally turns-off.

The instantaneous capacitor voltage is

$$V_C(t) = V_S + I_L \frac{L_S}{C} \sin \omega_S t$$

Where

$$\omega_S = \frac{1}{\sqrt{L_S C}}$$

Fig. 2.31 shows the equivalent circuit of Mode – 3.

MODE – 4

Since the capacitor has been overcharged i.e. its voltage is above supply voltage it starts discharging in reverse direction. Hence capacitor current becomes negative. The capacitor discharges through $L_S, V_S, FWD, D_1$ and $L$. When this current reduces to zero $D_1$ will stop conducting and the capacitor voltage will be same as the supply voltage fig. 2.32 shows in equivalent circuit of Mode – 4.
MODE – 5
In mode 5 both thyristors are off and the load current flows through the free wheeling diode (FWD). This mode will end once thyristor \( T_1 \) is fired. The equivalent circuit for mode 5 is shown in fig. 2.33.

Fig. 2.33

Fig. 2.34 shows the current and voltage waveforms for a voltage commutated chopper.
Though voltage commutated chopper is a simple circuit it has the following disadvantages.

- A starting circuit is required and the starting circuit should be such that it triggers thyristor $T_2$ first.
- Load voltage jumps to twice the supply voltage when the commutation is initiated.
- The discharging and charging time of commutation capacitor are dependent on the load current and this limits high frequency operation, especially at low load current.
- Chopper cannot be tested without connecting load.
- Thyristor $T_1$ has to carry load current as well as resonant current resulting in increasing its peak current rating.
Jone’s Chopper

Figure 2.35 shows a Jone’s Chopper circuit for an inductive load with free wheeling diode. Jone’s Chopper is an example of class D commutation. Two thyristors are used, T₁ is the main thyristor and T₂ is the auxiliary thyristor. Commutating circuit for T₁ consists of thyristor T₂, capacitor C, diode D and autotransformer (L₁ and L₂).

Initially thyristor T₂ is turned ON and capacitor C is charged to a voltage V with a polarity as shown in figure 2.35. As C charges, the charging current through thyristor T₂ decays exponentially and when current falls below holding current level, thyristor T₂ turns OFF by itself. When thyristor T₁ is triggered, load current flows through thyristor T₁, L₂ and load. The capacitor discharges through thyristor T₁, L₁ and diode D. Due to resonant action of the auto transformer inductance L₂ and capacitance C, the voltage across the capacitor reverses after some time.

It is to be noted that the load current in L₁ induces a voltage in L₂ due to autotransformer action. Due to this voltage in L₂ in the reverse direction, the capacitor charges to a voltage greater than the supply voltage. (The capacitor now tries to discharge in opposite direction but it is blocked by diode D and hence capacitor maintains the reverse voltage across it). When thyristor T₁ is to be commutated, thyristor T₂ is turned ON resulting in connecting capacitor C directly across thyristor T₁. Capacitor voltage reverse biases thyristor T₁ and turns it off. The capacitor again begins to charge through thyristor T₂ and the load for the next cycle of operation.

The various waveforms are shown in figure 2.36.