POWER SYSTEM OPERATION AND CONTROL

SYLLABUS

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PART - A

UNIT - 1

CONTROL CENTER OPERATION OF POWER SYSTEMS:
Power system control and operating states, control center, digital computer configuration, automatic generation control, area control error, operation without central computers, expression for tie-line flow and frequency deviation, parallel operation of generators, area lumped dynamic model.

8 Hours

UNIT - 2 & 3

AUTOMATIC VOLTAGE REGULATOR:
Basic generator control loops, Cross-coupling between control loops, Exciter types, Exciter modeling, Generator modeling, Static performance of AVR loop.

AUTOMATIC LOAD FREQUENCY CONTROL:
Automatic Load frequency control of single area systems, Speed governing system, Hydraulic valve actuator, Turbine generator response, Static performance of speed governor, Closing of ALFC loop, Concept of control area, Static response of primary ALFC loop, Integral control, ALFC of multi-control area systems (POOL operation), The Two-Area system, Modeling the Tie-Line, Block Diagram representation of Two-Area system, Static response of Two-Area system and Tie-Line Bias control.

12 Hours

UNIT - 4

CONTROL OF VOLTAGE AND REACTIVE POWER:
Introduction, generation and absorption of reactive power, relation between voltage, power and reactive power at a node,
single machine infinite bus systems, methods of voltage control, sub synchronous resonance, voltage stability, voltage collapse.

6 Hours

PART - B

UNIT -5
OPTIMAL SYSTEM OPERATION AND UNIT COMMITMENT: Introduction, Optimal operation of generators on a bus bar, Statement of the Unit Commitment problem, need and importance of unit commitment, Constraint in Unit Commitment, Unit Commitment solution methods-Priority lists method, Forward Dynamic Programming method (excluding problem), Spinning reserve.

6 Hours

UNIT -6

6 Hours

UNIT 7
SYSTEM MONITORING AND CONTROL: Introduction, Energy management system, the basis of power system state estimation (PSSE), mathematical description of PSSE process, minimization technique for PSSE, Least Square estimation, Error and detection in PSSE, System security and emergency control.

6 Hours

UNIT-8
POWER SYSTEM RELIABILITY: Introduction, Modes of failures of a system, Generating system and its performance, derivation of reliability index, reliability measure for N-unit system, cumulative probability outages-Recursive Relation, Loss of load probability, Frequency and duration of a state.

8 Hours
TEXT BOOKS:


REFERENCE:

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PART - A

UNIT - 1

CONTROL CENTER OPERATION OF POWER SYSTEMS:

Power system control and operating states, control center, digital computer configuration, automatic generation control, area control error, operation without central computers, expression for tie-line flow and frequency deviation, parallel operation of generators, area lumped dynamic model.  

General

Electrical Technology was founded on the remarkable discovery by Faraday that a changing magnetic flux creates an electric field. Out of that discovery, grew the largest and most complex engineering achievement of man: the electric power system. Indeed, life without electricity is now unimaginable. Electric power systems form the basic infrastructure of a country. Even as we read this, electrical energy is being produced at rates in excess of hundreds of giga-watts (1 GW = 1,000,000,000 W). Giant rotors spinning at speeds up to 3000 rotations per minute bring us the energy stored in the potential energy of water, or in fossil fuels. Yet we notice electricity only when the lights go out!

While the basic features of the electrical power system have remained practically unchanged in the past century, but there are some significant milestones in the evolution of electrical power systems.

Topics to be studied

(i) Introduction to SCADA
(ii) Control Centre
(iii) Digital Computer Configuration
(iv) Automatic Generation Control
(v) Area Control Error
(vi) Operation Without Central Computers
(vii) Expression for Tie Line Flow
(viii) Parallel Operation of Generators
(ix) Area Lumped Dynamic Model.
Introduction

Electrical energy is an essential ingredient for the industrial and all round development of any country. It is generated centrally in bulk and transmitted economically over long distances.

Electrical energy is conserved at every step in the process of Generation, Transmission, Distribution and utilization of electrical energy. The electrical utility industry is probably the largest and most complex industry in the world and hence very complex and challenging problems to be handled by power engineering particularly, in designing future power system to deliver increasing amounts of electrical energy. This calls for perfect understanding, analysis and decision making of the system. This power system operation and its control play a very important task in the world of Electrical Power Engineering.

Power Quality

Power quality is characterized by –

= Stable AC voltages at near nominal values and at near rated frequency subject to acceptable minor variations, free from annoying voltage flicker, voltage sags and frequency fluctuations.

= Near sinusoidal current and voltage wave forms free from higher order harmonics

All electrical equipments are rated to operate at near rated voltage and rated frequency.

Effects of Poor Power Quality

= Maloperation of control devices, relays etc.

= Extra losses in capacitors, transformers and rotating machines

= Fast ageing of equipments

= Loss of production due to service interruptions

= Electro-magnetic interference due to transients

= Power fluctuation not tolerated by power electronic parts
Major causes of Poor Power Quality

- Nonlinear Loads
- Adjustable speed drives
- Traction Drives
- Start of large motor loads
- Arc furnaces
- Intermittent load transients
- Lightning
- Switching Operations
- Fault Occurrences

Steps to address Power Quality issues

• Detailed field measurements

• Monitor electrical parameters at various places to assess the operating conditions in terms of power quality.

• Detailed studies using a computer model. The accuracy of computer model is first built to the degree where the observed simulation values matches with those of the field measurement values. This provides us with a reliable computer model using which we workout remedial measures.

• For the purpose of the analysis we may use load flow studies, dynamic simulations, EMTP simulations, harmonic analysis depending on the objectives of the studies.

• We also evaluate the effectiveness of harmonic filters through the computer model built, paying due attention to any reactive power compensation that these filters may provide at fundamental frequency for normal system operating conditions.

• The equipment ratings will also be addressed to account for harmonic current flows and consequent overheating.
**Power Quality Solutions:**

Poor power quality in the form of harmonic distortion or low power factor increases stress on a facility’s electrical system. Over time this increased electrical stress will shorten the life expectancy of electrical equipment. In addition to system degradation, poor power quality can cause nuisance tripping and unplanned shutdowns within electrical system.

In an increasingly automated electrical world, it is important for a facility to evaluate power quality. Harmonic distortion, low power factor, and the presence of other transients can cause severe damage to electrical system equipment. PSE provides system analysis and evaluation of power quality issues and makes recommendations for system design solutions.

**Structure of Power Systems**

Generating Stations, transmission lines and the distribution systems are the main components of an electric power system. Generating stations and distribution systems are connected through transmission lines, which also connect one power system (grid, area) to another. A distribution system connects all the loads in a particular area to the transmission lines.

For economical technical reasons, individual power systems are organized in the form of electrically connected areas or regional grids.

As power systems increased in size, so did the number of lines, substations, transformers, switchgear etc. Their operation and interactions became more complex and hence it is necessary to monitor this information simultaneously for the total system at a focal point called as Energy Control Centre. The fundamental design feature is increase in system reliability and economic feasibility.
Major Concerns of Power System Design and Operation

- Quality: Continuous at desired frequency and voltage level
- Reliability: Minimum failure rate of components and systems
- Security: Robustness - normal state even after disturbances
- Stability: Maintain synchronism under disturbances
- Economy: Minimize Capital, running and maintenance Costs

1.2 Need for Power System Management

- Demand for Power Increasing every day
  - No of transmission line, Sub-stations, Transformers, switchgear etc.,
- Operation and Interaction is more and more complex
- Essential to monitor simultaneously for the total system at a focal point – ENERGY LOAD CENTRE

Components of power system operation and control

- Information gathering and processing
- Decision and control
- System integration

Energy Load Centre

The function of energy load centre is to control the function of coordinating the response in both normal and emergency conditions. Digital Computers are very effectively used for the purpose. Their function is to process the data, detect abnormalities, alarm the human operator by lights, buzzers, screens etc., depending on the severity of the problem.
Control Centre of a Power System

- Human Machine Interface – equipped with
- CRT presentations
- Keyboards – change parameters
- Special function keyboards- alter transformer taps, switch line capacitors etc.,
- Light-Pen cursor – open or close circuit breakers
- Alarm lights, alarms, dedicated telephone communications with generating stations and transmission substations, neighboring power utilities

Control Features – Control Centre

- System Commands – Mode of control
- Units – base / peak load
- AGC – Automatic Generation Control
- Data Entry
- Alarms – To find source of alarm and necessary action
- Plant/Substation selection
- Special Functions - To send/retrieve data etc.,
- Readout control – Output to CRT/printers etc.,
- CPU control – Selection for the computer

Functions of Control Centre

- Short, Medium and Long-term Load Forecasting
- System Planning
- Unit Commitment and maintenance Scheduling
- Security Monitoring
- State Estimation
- Economic Dispatch
- Load Frequency Control
SCADA – Supervisory Control and Data Acquisition:

One of key processes of SCADA is the ability to monitor an entire system in real time. This is facilitated by data acquisitions including meter reading, checking statuses of sensors, etc that are communicated at regular intervals depending on the system.

A well planned and implemented SCADA system not only helps utilities deliver power reliably and safely to their customers but it also helps to lower the costs and achieve higher customer satisfaction and retention.

SCADA – Why do we need it?

- If we did not have SCADA, we would have very inefficient use of human resources and this would cost us (Rs,Rs,Rs)
- In today’s restructured environment SCADA is critical in handling the volume of data needed in a timely fashion
- Service restoration would involve travel time and would be significantly higher
- It is essential to maintain reliability

SCADA - Architecture

- Basic elements are sensors which measure the desired quantities
- Current Transformers CTs – measure currents and Potential Transformers PTs- measure voltages.
- Today there is a whole new breed of Intelligent electronic devices (IEDs)
- This data is fed to a remote terminal unit (RTU)
- The master computer or unit resides at the control center EMS
SCADA - Process

- Master unit scan RTUs for reports, if reports exist, RTU sends back the data and the master computer places it in memory
- In some new substation architectures there could be significant local processing of data which could then be sent to the control center.
- The data is then displayed on CRTs and printed

SCADA - Logging

- The SCADA provides a complete log of the system
- The log could be provided for the entire system or part of the system
- Type of information provided –
  - Time of event
    - Circuit breaker status
  - Current measurements, voltage measurements, calculated flows, energy, etc.
  - Line and equipment ratings

SCADA as a System

There are many parts of a working SCADA system. A SCADA system usually includes signal hardware (input and output), controllers, networks, user interface (HMI), communications equipment and software. All together, the term SCADA refers to the entire central system. The central system usually monitors data from various sensors that are either in close proximity or off site (sometimes miles away).

For the most part, the brains of a SCADA system are performed by the Remote Terminal Units (sometimes referred to as the RTU). The Remote Terminal Units consists
of a programmable logic converter. The RTU are usually set to specific requirements, however, most RTU allow human intervention, for instance, in a factory setting, the RTU might control the setting of a conveyer belt, and the speed can be changed or overridden at any time by human intervention. In addition, any changes or errors are usually automatically logged for and/or displayed. Most often, a SCADA system will monitor and make slight changes to function optimally; SCADA systems are considered closed loop systems and run with relatively little human intervention.

SCADA can be seen as a system with many data elements called points. Usually each point is a monitor or sensor. Usually points can be either hard or soft. A hard data point can be an actual monitor; a soft point can be seen as an application or software calculation. Data elements from hard and soft points are usually always recorded and logged to create a time stamp or history.

User Interface – Human Machine Interface (HMI)

A SCADA system includes a user interface, usually called Human Machine Interface (HMI). The HMI of a SCADA system is where data is processed and presented to be viewed and monitored by a human operator. This interface usually includes controls where the individual can interface with the SCADA system.

HMI's are an easy way to standardize the facilitation of monitoring multiple RTU's or PLC's (programmable logic controllers). Usually RTU's or PLC's will run a pre programmed process, but monitoring each of them individually can be difficult, usually because they are spread out over the system. Because RTU's and PLC's historically had no standardized method to display or present data to an operator, the SCADA system communicates with PLC's throughout the system network and processes information that is easily disseminated by the HMI. HMI's can also be linked to a database, which can use data gathered from PLC's or RTU's to provide graphs on trends, logistic info, schematics for a specific sensor or
Machine or even make troubleshooting guides accessible. In the last decade, practically all SCADA systems include an integrated HMI and PLC device making it extremely easy to run and monitor a SCADA system.

Today’s SCADA systems, in response to changing business needs, have added new functionalities and are aiding strategic advancements towards interactive, self-healing smart grids of the future. A modern SCADA system is also a strategic investment which is a must-have for utilities of all sizes facing the challenges of the competitive market and increased levels of real time data exchange that comes with it (Independent Market Operator, Regional Transmission Operator, Major C&I establishments etc). A well planned and implemented SCADA system not only helps utilities deliver power reliably and safely to their customers but it also helps to lower the costs and achieve higher customer satisfaction and retention. Modern SCADA systems are already contributing and playing a key role at many utilities towards achieving:

- New levels in electric grid reliability – increased revenue.
- Proactive problem detection and resolution – higher reliability.
- Meeting the mandated power quality requirements – increased customer satisfaction.
- Real time strategic decision making – cost reductions and increased revenue.

Critical Functions of SCADA
Following functions are carried out every 2 seconds:

- Switchgear Position, Transformer taps, Capacitor banks
- Tie line flows and interchange schedules
- Generator loads, voltage etc.,
- Verification on links between computer and remote equipment

Modern SCADA systems are already contributing and playing a key role at many utilities towards achieving:

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- Meeting the mandated power quality requirements – increased customer satisfaction.
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1.4 Digital Computer Configuration

Major functions
- Data acquisition control
- Energy Management
- System Security

For best/secured operation 100% redundancy is used – Dual Digital Computers

i) on-line computer – monitors and controls the system
ii) Backup computer – load forecasting or hydro thermal allocations

The digital computers are usually employed in a fixed-cycle operating mode with priority interrupts wherein the computer periodically performs a list of operation. The most critical functions have the fastest scan cycle. Typically the following categories are scanned every 2 seconds:

- All status points such as switchgear position (open or closed), substation loads and voltages, transformer tap positions, and capacitor banks etc.,
- Tie line flows and interchange schedules
- Generator loads, voltage, operating limits and boiler capacity
- Telemetry verification to detect failures and errors in the bilateral communication links between the digital computer and the remote equipment.

1.5 Important Areas of Concern in power System

- Automatic Generation Control (AGC)
On-line Computer Control that maintains overall system frequency and net tie-line load exchange through interconnection
  - Economic Load Dispatch
    On-line computer control to supply load demand using all interconnected system’s power in the most economical manner

AGC is the name given to a control system having three major objectives:
  a. To hold system frequency at or very close to a specified nominal value (50 or 60Hz)
  b. To maintain the correct value of interchange power between control areas
  c. To maintain each unit’s generation at the most economic value.

To implement an AGC system, the following information is required:
  - Unit megawatt output of each committed unit
  - Megawatt flow over each tie line to neighboring systems
  - System frequency

Usually, neighboring power companies are interconnected by one or more transmission lines called Tie Lines. The objective is to buy or sell power with neighboring systems whose operating costs make such transactions profitable. Also, even if no power is being transmitted over ties to neighboring system, if one system has a sudden loss of a generating unit, the units throught all the interconnection will experience a frequency change and can help in restoring frequency.

**Advantages of interconnected system**
- Reduces Reserve Capacity – thus reduces installed capacity
- Capital Cost/kW is less for larger Unit
  - in India single unit can support >500MW because of interconnection
- Effective Use of Generators
- Optimization of Generation – installed capacity is reduced
- Reliability
Disadvantages of interconnected system

- Fault get Propagated – calls for fast switchgear
- CB rating increases
- Proper management required – EMS and it must be automated – Economic load dispatch
  - Base load and Peak Load

National Regional Electricity Boards

- Northern Regional Electricity Board
- Western Regional Electricity Board
- Southern Regional Electricity Board
- Eastern Regional Electricity Board
- North-east Regional Electricity Board

Goal – To have National Grid to improve efficiency of the whole National Power Grid

Control Area Concept

All generators are tightly coupled together to form – Coherent Group
  - all generators respond to changes in load or speed changer setting

Control Area – frequency is assumed to be constant throughout in static and dynamic conditions

For the purpose of analysis, a control area can be reduced to a single speed governor, turbo generator and load system.
Interconnected Power System

Functions
- Exchange or sale of power
- Disturbed areas taking other area’s help
- Long distance sale and transfer of power

1.6 Area Control Error – ACE
To maintain a net interchange of power with its area neighbors, an AGC uses real power flow measurements of all tie lines emanating from the area and subtracts the scheduled interchange to calculate an error value. The net power interchange, together with a gain, $B$ (MW/0.1Hz), called the frequency bias, as a multiplier on the frequency deviation is called the Area Control Error (ACE) given by,

$$
\sum_{k=1}^{k} P_k - P_s + B(f_{\text{act}} - f_0) \text{ MW}
$$

$P_k = $ Power in Tie Line - +ve – out of the area
$P_s$ – Scheduled Power Interchange
$f_0$ – Base frequency, $f_{\text{act}}$ – Actual frequency
+ve ACE indicates flow out of the area.
ACE – Input to AGC
The real power summation of ACE loses information as to the flow of individual tie lines but is concerned with area net generation. The tie lines transfer power through the area from one neighbor to the next, called ‘Wheeling Power’. The wheeling power cancels algebraically in the ACE. Thus one area purchases or sells blocks of power (MWh) with non-neighbor utilities.

**Power Sale from A to C**

- A – selling a power ‘p’ to C, then ACE for A = p
- Power export starts until its AGC forces ACE to become zero
- Area C introduces ‘-p’ into its ACE
- Power flows in to area C until its ACE becomes zero
- Areas B & C must be aware of the power exchange as they are also interconnected

The minimum requirements of AGC on controlling the interchange of power and frequency have been established by NERC – North American Electric Reliability Council, which is comprised of representatives of the major operating power pools. This committee specifies the following criteria as minimum performance expected by AGC.

**A. Normal System Conditions**
- ACE = 0 at least once in 10 min period
- Deviation of ACE from zero must be within allowable limits

**B. Disturbances Conditions**
- ACE must return to zero within 10 min
- Corrective action from AGC must be within a minimum disturbance
Daily Load Cycle

The allowable limit, $L_d$ of the average deviation on power systems (averaged over 10 minutes) is:

$$L_d = 0.025\Delta L + 5.0 \text{ MW}$$

$$\Delta L = \frac{\Delta P}{\Delta t} \text{ MW/hr}$$

The value of $\Delta L$ is determined annually and is taken from the daily load cycle. A power system is said to be in a disturbance condition if the ACE signal exceeds $3L_d$.

1.7 Operation without Central Computers or AGC

Power Systems are capable of functioning even \textit{without} Central Computer and/or AGC

- Due to a result of Turbine Generator speed controls in the generating station and natural load regulation
- Thus generators within an area are forced to share load and cause interconnected areas to share load
The area connected load is defined by the curve LL as shown. As there is increase in load the rotating machinery in the area is forced to increase the speed.

**Basic Equations**

\[ G_A = G_0 + 10\beta_1 (f_{act} - f_0) \quad \text{MW} \]
\[ L_A = L_0 + 10\beta_2 (f_{act} - f_0) \quad \text{MW} \]

- \( G_A \) = Total Generation, \( G_0 \) = Base generation
- \( L_A \) = Total Load, \( L_0 \) = Base load, \( f_{act} \) = System frequency, \( f_0 \) = Base frequency
- \( \beta_1 \) = Cotangent of generation-frequency characteristic, MW/0.1 Hz < 0
- \( \beta_2 \) = Cotangent of load-frequency characteristic, MW/0.1 Hz > 0

1.7.2 Isolated Operation in A – response to load change
For Steady State Frequency – Total generation = Total effective load This is defined by the intersection of GG and LL curves as shown – Io.

Combined characteristic of GG and LL is CC. The composite generation load frequency characteristics is given by,

\[ G_A = G_0 + 10\beta_1 (f_{act} - f_0), \quad L_A = L_0 + 10\beta_2 (f_{act} - f_0) \]

\[ G_A - L_A = G_0 + 10\beta_1 (f_{act} - f_0) - L_0 - 10\beta_2 (f_{act} - f_0) \]

Increase in load in ‘A’ moves the load frequency curve to position L’L’. The new system frequency will now be defined by the intersection labeled as I₁ at 49.9Hz. Then it is desired to return the system frequency to 50.0Hz i.e., point I₂.

Setting AGC in ‘A’- shifting of GG to G’G’ takes place to meet frequency demand of 50.0Hz – I₂.

Resulting combined characteristic is C’C’ In terms of increments,

\[ A = G_A - G_0 + L_0 - L_A = 10\beta_1 (f_{act} - f_0) - 10\beta_2 (f_{act} - f_0) \]

\[ A = G_A - G_0 + L_0 - L_A = 10B_A X_A \Delta f \quad \text{MW} \]

\[ B_A = \text{Natural regulation characteristic} - \% \text{gen} \text{ for} 0.1\text{Hz} \]

\[ X_A = \text{Generating Capacity of} \quad A, \text{MW} \]

\[ \text{Frequency deviation} = \Delta f = A / 10B_A X_A \quad \text{Hz} \]

Considering Tie line flow, Frequency deviation

\[ \Delta f = (\Delta_A + \Delta T_L) / (10B_A X_A) \quad \text{Hz} \]

\[ \Delta A + \Delta T_L \quad \text{Net Megawatt change} \]

\[ \Delta T_L = \Delta G_A - \Delta L_A \]

1.7.3 Effect of Tie Line Flow - Interconnected operation
Let two areas A and B are interconnected through a Tie Line. Thus both Generation and Load frequency are equal to 50.0 Hz. There is no initial Tie Line Power Flow.

- Disturbance occur at B causing frequency to drop to 49.9Hz
- Area generation does not match with effective load in A
- Difference between I1 and I2 – difference between generation and load – net excess power in the area – flows out of A towards B
- Contributory effects in A are decrease in load power ΔL and increase in generation ΔG
- Tie Line Flow from A to B = ΔT_L = (ΔG_A - ΔL_A) MW
- If area A has AGC, tie line flows increases – ΔT_L’ and ΔT_L’’ representing increased amounts of bias due to AGC.

Frequency change due to disturbance B for a tie line power flow from A to B is Δf_B -

\[ \Delta T_L / (10B_B X_B) \text{ Hz} \]

\[ \Delta T_L = (10B_A X_A)_{AB} / (10B_A X_A + 10B_B X_B) \text{ MW} \]

Net power change in B is

\[ = \Delta f_A - \Delta T_L \]

\[ = (10B_B X_B)_{AB} / (10B_A X_A + 10B_B X_B) \]

\[ \Delta f_A = (10B_A X_A + 10B_B X_B) \Delta f \]

Hence, \[ \Delta f_A / (10B_A X_A + 10B_B X_B) \]
Example

Two areas A and B are interconnected. Generating capacity of A is 36,000Mw with regulating characteristic of 1.5%/0.1Hz. B has 4000MW with 1%/0.1Hz. Find each area’s share of +400MW disturbance (load increase) occurring in B and resulting tie line flow.

Sol:

\[ \Delta f = \frac{\Delta f_{AB}}{10 B_A X_A + 10 B_B X_B} = \frac{400}{-10(0.015)(36,000) - 10(0.01)(4000)} = -0.06896 \text{ Hz} \]

Tie Line flow = \( \Delta T_L = \frac{10 B_A X_A}{10 B_A X_A + 10 B_B X_B} = \frac{5400*400}{4800} = 372.4 \text{MW} \)

Smaller system need only 27.6 MW

Frequency regulation is much better

1.8 Parallel Operation of Generators

Tie line flows and frequency droop described for interconnected power areas are composite characteristics based on parallel operation of generators. Each area could maintain its speed \( w = 2f \), then load common to both areas, by superposition have the terminal voltage,

\[ V_{load} = V_1 \sin w_1 t + V_2 \sin w_2 t \]

Where, 1&2 represents areas and ‘t’ time in secs. Generator speed versus load characteristics is a function of the type of the governor used on the prime mover-
type 0 – for a speed droop system and type 1 – for constant speed system.

Parallel operation of generator with infinite bus
The generator characteristic is such that it is loaded to 50% of its capacity when paralleled to the bus.

Therefore, Unit speed regulation = \( R = \frac{\Delta f(\text{pu})}{\Delta P(\text{pu})} \)

If it is desired to increase the load on the generator, the prime mover torque is increased, which results in a shift of the speed-droop curve as shown below. The real power flow is given by, \( P = \frac{V_1V_2 \sin(\theta_1 - \theta_2)}{X} \), where \( X \) = synchronous reactance
2. Two generators paralleled have different governor-speed-droop characteristics. Because they are in parallel, power exchange between them forces them to synchronize at a common frequency. Since the two units are of equal capacity having equal regulation are initially operating at 1.0 base speed as shown above.

If unit is operated at point $A_1$ satisfies 25% of the total load and unit 2 at point $A_2$ supplies 75%. If the total load is increased to 150%, the frequency decreases to $f_1$.

Since the droop curves are linear, unit 1 will increase its load to 50% of rating and unit 2 to be overloaded.

**Parallel operation of two units with different capacity and regulation**

The case when two units of different frequency and regulation characteristics are operated in parallel is as shown below. The regulation characteristics are

\[ R_1 = \frac{\Delta f (pu)}{\Delta P_1 (pu)}, \quad R_2 = \frac{\Delta f (pu)}{\Delta P_2 (pu)} \]

Initial Loads - $P_1$ and $P_2$, change in load

\[ \Delta L = \Delta P_1 + \Delta P_2 = \]

Equivalent System Regulation = $\frac{\Delta f}{\Delta L} = \frac{P_1 \text{ rate}}{R_1} + \frac{P_2 \text{ rate}}{R_2}$
1.9 Area Lumped Dynamic Model

The model discussed so far is one macroscopic behavior because there is no effort made to indicate instantaneous power flow within the system due to a tie line disturbance, magnitudes of the internal line flows, the time history of generator phase angles and so on. The power system macro model may be described by means of a block diagram as shown in the block diagram.
H_A = Effective Inertia of rotating machinery loads 
B_2 = Load frequency characteristics, MW/0.1Hz 
P_{rate} = Rated power output of Gen ‘i’

\[ \Delta P_i = \text{Power Increment for gen ‘i’} \]

\[ \frac{1}{R_i} = \text{Droop characteristic of gen ‘i’, Hz/MW} \]

**Analysis – Isolated Power Area without Tie Lines**

Steady State value of Frequency deviation \( \Delta f \) for a load change \( \Delta L = \frac{A}{S} \)

Hence,

\[ \Delta f_A = \frac{1}{10\beta_1 - 10\beta_2} \]

Combining droop characteristics of M gen,
UNIT -2 & 3

AUTOMATIC VOLTAGE REGULATOR:  Basic generator control loops, Cross coupling between control loops, Exciter types, Exciter modeling, Generator modeling, Static performance of AVR loop.

AUTOMATIC LOAD FREQUENCY CONTROL:  Automatic Load frequency control of single area systems, Speed governing system, Hydraulic valve actuator, Turbine generator response, Static performance of speed governor, Closing of ALFC loop, Concept of control area, Static response of primary ALFC loop, Integral control, ALFC of multi-control area systems (POOL operation). The Two-Area system, Modeling the Tie-Line, Block Diagram representation of Two-Area system, Static response of Two-Area system and Tie-Line Bias control.

12 Hours

2.1 Introduction

The main objective of power system operation and control is to maintain continuous supply of power with an acceptable quality, to all the consumers in the system. The system will be in equilibrium, when there is a balance between the power demand and the power generated. As the power in AC form has real and reactive components: the real power balance; as well as the reactive power balance is to be achieved.

There are two basic control mechanisms used to achieve reactive power balance (acceptable voltage profile) and real power balance (acceptable frequency values). The former is called the automatic voltage regulator (AVR) and the latter is called the automatic load frequency control (ALFC) or automatic generation control (AGC).

2.2 Generator Voltage Control System

The voltage of the generator is proportional to the speed and excitation (flux) of the generator. The speed being constant, the excitation is used to control the voltage. Therefore, the voltage control system is also called as excitation control system or automatic voltage regulator (AVR).
For the alternators, the excitation is provided by a device (another machine or a static device) called exciter. For a large alternator the exciter may be required to supply a field current of as large as 6500A at 500V and hence the exciter is a fairly large machine. Depending on the way the dc supply is given to the field winding of the alternator (which is on the rotor), the exciters are classified as: i) DC Exciters; ii) AC Exciters; and iii) Static Exciters. Accordingly, several standard block diagrams are developed by the IEEE working group to represent the excitation system. A schematic of an excitation control system is shown in Fig2.1.

![A schematic of excitation (voltage) control system](image)

*Fig2.1: A schematic of Excitation (Voltage) Control System.*

A simplified block diagram of the generator voltage control system is shown in Fig2.2. The generator terminal voltage $V_t$ is compared with a voltage reference $V_{ref}$ to obtain a voltage error signal $\Delta V$. This signal is applied to the voltage regulator shown as a block with transfer function $K_A/(1+T_A s)$. The output of the regulator is then applied to exciter shown with a block of transfer function $K_E/(1+T_E s)$. The output of the exciter $E_{fd}$ is then applied to the field winding which adjusts the generator terminal voltage. The generator field can be represented by a block with a transfer function $K_F/(1+sT_F)$. The total transfer

The stabilizing compensator shown in the diagram is used to improve the dynamic response of the exciter. The input to this block is the exciter voltage and the output is a stabilizing feedback signal to reduce the excessive overshoot.
The performance of the AVR loop is measured by its ability to regulate the terminal voltage of the generator within prescribed static accuracy limit with an acceptable speed of response. Suppose the static accuracy limit is denoted by $A_c$ in percentage with reference to the nominal value. The error voltage is to be less than $\frac{A_c}{100} \Delta |V|_{ref}$.

From the block diagram, for a steady state error voltage $\Delta e$;

**Example 1:** Find the open loop gain of an AVR loop if the static accuracy required is 3%.

**Solution:** Given $A_c = 3\%$. $K > \left\{ \frac{100}{1} - 1 \right\} = K > \left\{ \frac{100}{1} - 1 \right\} = 32.33$. Thus, if the open loop gain of the AVR loop is greater than 32.33, then the terminal voltage will be within 3% of the base voltage.
2.3 Automatic Load Frequency Control

The ALFC is to control the frequency deviation by maintaining the real power balance in the system. The main functions of the ALFC are to i) to maintain the steady frequency; ii) control the tie-line flows; and iii) distribute the load among the participating generating units. The control (input) signals are the tie-line deviation $\Delta P_{\text{tie}}$ (measured from the tie-line flows), and the frequency deviation $\Delta f$ (obtained by measuring the angle deviation $\Delta \delta$). These error signals $\Delta f$ and $\Delta P_{\text{tie}}$ are amplified, mixed and transformed to a real power signal, which then controls the valve position. Depending on the valve position, the turbine (prime mover) changes its output power to establish the real power balance. The complete control schematic is shown in Fig2.3

![Fig2.3: The Schematic representation of ALFC system](image)

For the analysis, the models for each of the blocks in Fig2 are required. The generator and the electrical load constitute the power system. The valve and the hydraulic amplifier represent the speed governing system. Using the swing equation, the generator can be

![Fig2.4: The block diagram representation of the Generator](image)

The load on the system is composite consisting of a frequency independent component and a frequency dependent component. The load can be written as $\Delta P_e = \Delta P_0 + \Delta P_f$ where, $\Delta P_e$ is the change in the load; $\Delta P_0$ is the frequency independent load component; $\Delta P_f$ is the frequency dependent load component. $\Delta P_f = \Delta \omega$ where, D is called frequency
characteristic of the load (also called as damping constant) expressed in percent change in load for 1% change in frequency. If $D=1.5\%$, then a 1% change in frequency causes 1.5% change in load. The combined generator and the load (constituting the power system) can then be represented as shown in Fig2.5

![Fig2.5: The block diagram representation of the Generator and load](image)

The turbine can be modeled as a first order lag as shown in the Fig2.6

![Fig2.6: The turbine model.](image)

$G_t(s)$ is the TF of the turbine; $\Delta P_v(s)$ is the change in valve output (due to action). $\Delta P_m(s)$ is the change in the turbine output

The governor can similarly modeled as shown in Fig2.7. The output of the governor is by

where $\Delta P_{ref}$ is the reference set power, and $\Delta \omega/R$ is the power given $R$ by governor speed characteristic. The hydraulic amplifier transforms this signal $\Delta P_g$ into valve/gate position corresponding to a power $\Delta P_v$. Thus $\Delta P_v(s) = \left( \frac{K_g}{(1+sT_g)} \right) \Delta P_g(s)$.

![Fig2.7: The block diagram representation of the Governor](image)
All the individual blocks can now be connected to represent the complete ALFC loop as shown in Fig2.8

![Fig2.8: The block diagram representation of the ALFC](image)

**2.4 Steady State Performance of the ALFC Loop**

In the steady state, the ALFC is in ‘open’ state, and the output is obtained by substituting $s \rightarrow 0$ in the TF.

With $s \rightarrow 0$, $G_g(s)$ and $G_t(s)$ become unity, then, (note that $\Delta P_m = \Delta P_T = \Delta P_G = \Delta P_e = \Delta P_D$; That is turbine output = generator/electrical output = load demand)

$$\Delta P_m = \Delta P_{ref} - \frac{1}{R} \Delta \omega$$

or

$$P_m = \Delta P_{ref} - \left(\frac{1}{R}\right) \Delta f$$

When the generator is connected to infinite bus ($f = 0$, and $\Delta V = 0$), then

$$\Delta P_m = \Delta P_{ref}.$$  

If the network is finite, for a fixed speed changer setting ($\Delta P_{ref} = 0$), then

$$\Delta P_m = -\left(\frac{1}{R}\right) \Delta f$$

or

$$\Delta f = -\frac{1}{R} \Delta P_m.$$  

If the frequency dependent load is present, then

$$\Delta P_m \Delta f = \Delta P_{ref} - \left(\frac{1}{R} + D\right) \Delta f$$

or

$$\Delta = -\frac{P_m}{D \times 1/R}$$

If there are more than one generator present in the system, then

$$\Delta P_{m, eq} = \Delta P_{ref, eq} - (D + 1/R_{eq}) \Delta f$$

where,

$$\Delta P_{m, eq} = \Delta P_{m1} + \Delta P_{m2} + \Delta P_{m3} + \ldots.$$  

$$\Delta P_{ref, eq} = V_{Pref1} + \Delta P_{ref2} + V_{Pref3} + \ldots.$$  

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots.$$  

The quantity $\beta = (D + 1/R_{eq})$ is called the area frequency (bias) characteristic (response) or simply the stiffness of the system.
2.5 Concept of AGC (Supplementary ALFC Loop)

The ALFC loop shown in Fig2.8, is called the primary ALFC loop. It achieves the primary goal of real power balance by adjusting the turbine output $\Delta P_m$ to match the change in load demand $\Delta P_D$. All the participating generating units contribute to the change in generation. But a change in load results in a steady state frequency deviation $\Delta f$. The restoration of the frequency to the nominal value requires an additional control loop called the supplementary loop. This objective is met by using integral controller which makes the frequency deviation zero. The ALFC with the supplementary loop is generally called the AGC. The block diagram of an AGC is shown in Fig2.9. The main objectives of AGC are i) to regulate the frequency (using both primary and supplementary controls); ii) and to maintain the scheduled tie-line flows. A secondary objective of the AGC is to distribute the required change in generation among the connected generating units economically (to obtain least operating costs).

![Fig2.9: The block diagram representation of the AGC](image-url)
2.6 AGC in a Single Area System

In a single area system, there is no tie-line schedule to be maintained. Thus the function of the AGC is only to bring the frequency to the nominal value. This will be achieved using the supplementary loop (as shown in Fig.2.9) which uses the integral controller to change the reference power setting so as to change the speed set point. The integral controller gain $K_I$ needs to be adjusted for satisfactory response (in terms of overshoot, settling time) of the system. Although each generator will be having a separate speed governor, all the generators in the control area are replaced by a single equivalent generator, and the ALFC for the area corresponds to this equivalent generator.

2.7 AGC in a Multi Area System

In an interconnected (multi area) system, there will be one ALFC loop for each control area (located at the ECC of that area). They are combined as shown in Fig2.10 for the interconnected system operation. For a total change in load of $\Delta P_D$, the steady state

![Diagram of AGC for a multi-area operation](image)
2.8 Expression for tie-line flow in a two-area interconnected system

Consider a change in load $\Delta P_{D1}$ in area 1. The steady state frequency deviation $\Delta f$ is the same for both the areas. That is $\Delta f = \Delta f_1 = \Delta f_2$. Thus, for area 1, we have

$$\text{where, } \beta_1 \text{ and } \beta_2 \text{ are the composite frequency response characteristic of Area 1 and Area 2 respectively.}$$

An increase of load in area 1 by $\Delta P_{D1}$ results in a frequency reduction in both areas and a tie-line flow of $\Delta P_{12}$. A positive $\Delta P_{12}$ is indicative of flow from Area 1 to Area 2 while a negative $\Delta P_{12}$ means flow from Area 2 to Area 1. Similarly, for a change in Area Frequency bias tie line control

The tie line deviation reflects the contribution of regulation characteristic of one area to another. The basic objective of supplementary control is to restore balance between each area load generation. This objective is met when the control action maintains

- Frequency at the scheduled value
- Net interchange power (tie line flow) with neighboring areas at the scheduled values

The supplementary control should ideally correct only for changes in that area. In other words, if there is a change in Area 1 load, there should be supplementary control only in Area 1 and not in Area 2. For this purpose the area control error (ACE) is used (Fig2.9). The ACE of the two areas are given by

For area 1: $\text{ACE}_1 = \Delta P_{12} + \beta_1 \Delta f$

For area 2: $\text{ACE}_2 = \Delta P_{21} + \beta_2 \Delta f$

2.9 Economic Allocation of Generation

An important secondary function of the AGC is to allocate generation so that each generating unit is loaded economically. That is, each generating unit is to generate that amount to meet the present demand in such a way that the operating cost is the minimum. This function is called Economic Load Dispatch (ELD).
Systems with more than two areas

The method described for the frequency bias control for two area system is applicable to multi-area system also.

Examples:

Ex 1. A 500 MVA, 50 Hz, generating unit has a regulation constant R of 0.05 p.u. on its own rating. If the frequency of the system increases by 0.01 Hz in the steady state, what is the decrease in the turbine output? Assume fixed reference power setting.

Solution: In p.u. $\Delta f = 0.01/50 = 0.0002$ p.u.

With $\Delta p_{\text{ref}} = 0$, $\Delta p_m = -1/R(\Delta f) = - 0.004$ p.u. Hence, $\Delta p_m = -0.004 S_{\text{base}} = -2$ MW.

Ex. 2. An interconnected 60 Hz power system consists of one area with three generating units rated 500, 750, and 1000 MVA respectively. The regulation constant of each unit is R= 0.05 per unit on its own rating. Each unit is initially operating at one half of its rating, when the system load suddenly increases by 200MW. Determine (i) the area frequency response characteristic on a 1000 MVA system base, (ii) the steady state frequency deviation of the area, and (iii) the increase in turbine power output.

Regulation constants on common system base are ($R_{\text{pu new}} = R_{\text{pu old}} (S_{\text{base new}}/S_{\text{base old}})$: $R_1 = 0.1$; $R_2 = 0.0667$; and $R_3 = 0.05$.

Hence $\beta = (1/R_1 + 1/R_2 + 1/R_3) = 45$ per unit.

Neglecting losses and frequency dependence of the load, the steady state frequency deviation is $\Delta f = (-1/\beta) \Delta p_m = -4.444 \times 10^{-3}$ per unit = $(-4.444 \times 10^{-3})60 = -0.2667$ Hz.

$\Delta p_{m1} = (-1/R_1)( \Delta f) = 0.04444$ per unit = 44.44 MW

$\Delta p_{m2} = (-1/R_2)( \Delta f) = 0.06666$ per unit = 66.66 MW

$\Delta p_{m3} = (-1/R_3)( \Delta f) = 0.08888$ per unit = 88.88 MW
UNIT -4


6 HOURS

Control of Voltage and Reactive Power

Reactive power is an odd topic in AC (Alternating Current) power systems, and it's usually explained with vector mathematics or phase-shift sine wave graphs. However, a non-math verbal explanation is possible.

Note that Reactive power only becomes important when an "electrical load" or a home appliance contains coils or capacitors. If the electrical load behaves purely as a resistor, (such as a heater or incandescent bulb for example,) then the device consumes "real power" only. Reactive power and "power factor" can be ignored, and it can be analysed using an AC version of Ohm's law.

Reactive power is simply this: when a coil or capacitor is connected to an AC power supply, the coil or capacitor stores electrical energy during one-fourth of an AC cycle. But then during the next quarter-cycle, the coil or capacitor dumps all the stored energy back into the distant AC power supply. **Ideal coils and capacitors consume no electrical energy, yet they create a significant electric current.**

This is very different from a resistor which genuinely consumes electrical energy, and where the electrical energy flows continuously in one direction; moving from source to load.

In other words, if your electrical appliance contains inductance or capacitance, then electrical energy will periodically return to the power plant, and it will flow back and forth across the power lines. This leads to an extra current in the power lines, a current which heats the power lines, but which isn't used to provide energy to the appliance. The coil or capacitor causes electrical energy to begin "sloshing" back and forth between the appliance and the distant AC generator. Electric companies must install heavier wires to tolerate the excess current, and they will charge extra for this "unused" energy.
This undesired "energy sloshing" effect can be eliminated. If an electrical load contains both a coil and capacitor, and if their resonant frequency is adjusted to exactly 60Hz, then the coil and capacitor like magic will begin to behave like a pure resistor. The "energy sloshing" still occurs, but now it's all happening between the coil and capacitor, and not in the AC power lines. So, if your appliance contains a large coil induction motor, you can make the motor behave as a pure resistor, and reduce the current in the power lines by connecting the right value of capacitance across the motor coil.

Why is reactive power so confusing? Well, the math is daunting if not entirely obscure. And the concept of "imaginary power" puts many people off. But this is not the only problem. Unfortunately most of us are taught in grade school that an electric current is a flow of energy, and that energy flows back and forth in AC power lines. This is completely wrong. In fact the energy flows constantly forward, going from source to load. It's only the charges of the metal wires which flow back and forth.

Imagine that we connect a battery to a light bulb. Electric charges already present inside the wires will begin to flow in the circle, and then electrical energy moves almost instantly to the light bulb. The charge flow is circular like a belt, but the energy flow is one-way. Now imagine that we suddenly reverse the connections to the battery. The voltage and current will reverse... but the energy still flows in the same direction as before. It still goes from battery to bulb. If we keep reversing the battery connections over and over, we'd have an AC system. So, in an AC system, only the voltage and current are "alternating," while the electrical energy flows one-way, going from source to load. Where AC resistive loads are concerned, electrical energy does not "alternate."

To understand energy flow in AC systems, it's critically important that we understand the difference between charge flow (current, amperes) and energy flow (power, watts.)

What is imaginary power? Simple: it's the unused power which flows backwards and forwards in the power lines, going back and forth between the load's coil or capacitor and the distant AC generator. If your appliance was a pure capacitor or inductor, then it would consume no electrical energy at all, but instead all the flowing energy would take the form of "sloshing energy," and we'd call it "imaginary power." Of course it's not actually imaginary. Instead it's reflected by the load.
What is real power? Even more simple: it's the energy flow which goes continuously from the AC generator and into the appliance, without any of it returning back to the distant generator.

Finally, what is "reactive" power? It's just the combination of the above two ideas: it is the continuous-forward-moving or "real" energy flow, combined with the sloshing or "imaginary" energy flow.

Power in A.C. Networks
Active Power
Reactive Power
Apparent Power
Power Factor (p.f.) Power Factor Correction
Instantaneous power, \( p(t) = v(t)i(t) \) Power, \( p(t) \) value

*positive* – power transmit/dissipate by load *negative* – power return from the load

Since \( p(t) \) is power transmits by load, then it is the average power, \( P \) at load

Sometimes \( P \) is also known as *active power*, *real power* or *true power* measured in unit of Watts.

\[
P = VI = I R = V /R \text{ (Watt)}
\]
REACTIVE POWER

\[ Z = jX_L \text{ (inductive)} \]

Instantaneous power \[ p(t) = v(t)i(t) = VI \sin 2\omega t \]

Average power is zero

The product of \( VI \) is called reactive power \( (Q_L) \) with unit Volt-Amp

Reactive (VAR)

Reactive power (inductive)

\[ Q_L = VI = P X_L = V^2 / X_L \]

\[ Z_C = -jX \text{ (capacitive)} \]
Reactive power (capacitive) \[ Q_C = VI = I X_C = V / X_C \quad \text{(VAR)} \]

Note:
To distinguish between inductive reactive power \( Q_L \) and capacitive reactive power \( Q_C \), we use two different signs (+ or –) depending on our reference \( i \) or \( v \), for example \( j Q_L \) and \( – j Q_C \) or otherwise.

Note that Reactive power only becomes important when an "electrical load" or a home appliance contains coils or capacitors. If the electrical load behaves purely as a resistor, (such as a heater or incandescent bulb for example,) then the device consumes "real power" only. Reactive power and "power factor" can be ignored.

![Diagram showing electrical circuits with reactive power](image)

Reactive power is simply this: when a coil or capacitor is connected to an AC power supply, the coil or capacitor stores electrical energy during one-fourth of an AC cycle. But then during the next quarter-cycle, the coil or capacitor dumps all the stored energy back into the distant AC power supply. Ideal coils and capacitors consume no electrical energy, yet they create a significant electric current. This is very different from a resistor which genuinely consumes electrical energy, and where the electrical energy flows continuously in one direction; moving from source to load.
ACTIVE/REACTIVE POWER – Example

\[ I = 100 \ \text{V}/25 \ \Omega = 4 \ \text{A}, \quad P = VI = (100 \ \text{V})(4 \ \text{A}) = 400 \ \text{W}, \quad = 0 \ \text{VAR} \]

\[ I = 100 \ \text{V}/20 \ \Omega = 5 \ \text{A}, \quad P = 0, \quad Q_L = VI = (100 \ \text{V})(5 \ \text{A}) = 500 \ \text{VAR} \text{ (inductive)} \]

\[ I = 100 \ \text{V}/40 \ \Omega = 2.5 \ \text{A}, \quad P = 0, \quad Q_C = VI = (100 \ \text{V})(2.5) = \quad \text{VAR} \text{ (capacitive)} = -250 \ \text{VAR} \]

Note: use the magnitude of I and V

Determine the total \( P_T \) and \( Q_T \) for the circuit. Sketch the series equivalent circuit.
APPARENT POWER

For load consisting of series resistance and reactance, \( Z = R + jX = Z/\Delta\theta \), the power produced is called *Apparent Power* or *Complex Power*), \( S \) or \( P_S \) with unit Volt-Amp (VA)

\[
S = V \times I
\]

\[
\begin{align*}
S &= P + jQ \\
\theta &= \text{positive, inductive load} \\
\theta &= \text{negative, capacitive load} \\
S &= VI \text{ (VA)} \\
P &= VI \cos \theta = I \times R = V_R/\Delta R \text{ (W)} \\
&= S \cos \theta \text{ (W)} \\
Q &= VI \sin \theta = I \times X = V_X/\Delta X \text{ (VAR)} = S \sin \theta \\
S &= \sqrt{(P + Q)} = V \times I \\
\end{align*}
\]

Power Triangle
POWER TRIANGLE – Example

Sketch the power triangle.

Note that Reactive power only becomes important when an "electrical load" or a home appliance contains coils or capacitors. If the electrical load behaves purely as a resistor, (such as a heater or incandescent bulb for example,) then the device consumes "real power" only. Reactive power and "power factor" can be ignored,

- Reactive power is simply this: when a coil or capacitor is connected to an AC power supply, the coil or capacitor stores electrical energy during one-fourth of an AC cycle. But then during the next quarter-cycle, the coil or capacitor dumps all the stored energy back into the distant AC power supply. Ideal coils and capacitors consume no electrical energy, yet they create a significant electric current. This is very different from a resistor which genuinely consumes electrical energy, and where the electrical energy flows continuously in one direction; moving from source to load.

\[ P_T = 700 + 800 + 80 + 120 = 1700 \text{ W} \]
\[ Q_T = 1300 - 600 - 100 - 1200 = -600 \text{ VAR} = 600 \text{ VAR (cap.)} \]
\[ S_T = P_T + jQ_T = 1700 - j600 = 1803 \angle -19.4^\circ \text{ VA} \]
POWER FACTOR

Power factor, p.f. = \( \cos \theta = \frac{P}{S} = \frac{R}{Z} \)

p.f. depends on the load type:

- Purely resistive load, \( R \), p.f. = 1
- Inductive load, \( RL \), p.f. < 1 (lagging) and
- Capacitive load, \( RC \), p.f. < 1 (leading)
- Most of the loads are inductive (lagging p.f.) and must be corrected until p.f. approximately become unity (p.f. = 1) using capacitor.
POWER FACTOR – Example

Find the complex power for the circuit. Correct the circuit power factor to p.f. = 1 using parallel reactance.

Reactive Power is a Byproduct of Alternating Current (AC) Systems

- Transformers, transmission lines, and motors require reactive power
- Transformers and transmission lines introduce inductance as well as resistance (Both oppose the flow of current)
- Must raise the voltage higher to push the power through the inductance of the lines (Unless capacitance is introduced to offset inductance)
- The farther the transmission of power, the higher the voltage needs to be raised
- Electric motors need reactive power to produce magnetic fields for their operation

Generation and Absorption of Reactive Power

Synchronous Generators - Synchronous machines can be made to generate or absorb reactive power depending upon the excitation (a form of generator control) applied. The ability to supply reactive power is determined by the short circuit ratio.

Synchronous Compensators - Certain smaller generators, once run up to speed and synchronized to the system, can be declutched from their turbine and provide reactive power without producing real power.

Capacitive and Inductive Compensators - These are devices that can be connected to the system to adjust voltage levels. A capacitive compensator produces an electric field
thereby generating reactive power. An inductive compensator produces a magnetic field to absorb reactive power. Compensation devices are available as either capacitive or inductive alone or as a hybrid to provide both generation and absorption of reactive power.

**Overhead Lines, Underground Cables and Transformers.**

- Overhead lines and underground cables, when operating at the normal system voltage, both produce strong electric fields and so generate reactive power.
- When current flows through a line or cable it produces a magnetic field which absorbs reactive power.
- A lightly loaded overhead line is a net generator of reactive power while a heavily loaded line is a net absorber of reactive power. In the case of cables designed for use at 275 or 400kV the reactive power generated by the electric field is always greater than the reactive power absorbed by the magnetic field and so cables are always net generators of reactive power.
- Transformers always absorb reactive power.

**Relation between voltage, Power and Reactive Power at a node**

The phase voltage $V$ at a node is a function of $P$ and $Q$ at that node.

i.e $V = f(P, Q)$

The voltage is also independent of adjacent nodes and assume that these are infinite busses.

The total differential of $V$,

$$dV = \left(\frac{\partial V}{\partial P}\right) \cdot dp + \left(\frac{\partial V}{\partial Q}\right) \cdot dQ$$

and using the relation $\left(\frac{\partial P}{\partial V}\right) \cdot \left(\frac{\partial V}{\partial Q}\right) = 1$ and $\left(\frac{\partial Q}{\partial V}\right) \cdot \left(\frac{\partial V}{\partial P}\right) = 1$

$$dv = \frac{dp}{\left(\frac{\partial P}{\partial V}\right)} + \frac{dQ}{\left(\frac{\partial Q}{\partial V}\right)} \quad \text{---------(1)}$$

From the above equation it is seen that the change in voltage at a node is defined by two quantities, $(\frac{\partial P}{\partial V})$ and $(\frac{\partial Q}{\partial V})$.
Normally \(\frac{\partial Q}{\partial v}\) is the quantity of greater interest and can be experimentally determined using Network Analyser by injecting known quantity of VARs at the node in question and measuring the difference in voltage produced.

**Methods of voltage control**

- By Reactive Power Injection
- By Tap Changing Transformers
- Combined use of Tap Changing Transformers and Reactive Power Injection
- Booster Transformers.

**Reactive Power Injection**

This is the most fundamental method and is used only in places where the transformer alone is not sufficient to control the voltage. Since many years we use capacitors to improve the power factors of industrial loads. The injection of reactive power required for the power factor improvement is determined like this. A load of 1 kw at a lagging power factor of \(\cos_1\) has a KVA of \(\frac{P_1}{\cos \delta_1}\). If this power factor is improved to \(\cos \delta_2\), the new KVA is \(\frac{P_1}{\cos \delta_2}\).

The reactive power required from the capacitors is \((P_1 \tan \delta_1 - P_1 \tan \delta_2)\) KVAR.

Now the question is why the power factor is to be improved. What if the power is transmitted at non unity power factor. We all know very well that the voltage drop depends on reactive power (Q) while the load angle (or) power transmission angle (\(\delta\)) depends on real power (P) At non unity power factors if the power is transmitted then it results in higher line currents which increases the \(I^2 R\) losses and hence reduces the thermal capability. One of the ideal place for the injection of reactive power is at the loads itself.

Generally reactive power injections are of the following types.

- **Static shunt capacitors**
- **Static series capacitors**
- **Synchronous compensators**

**Shunt capacitors and Reactors:**
Shunt capacitors are used for lagging power factor circuits whereas shunt reactors are used for leading power factors that are created by lightly loaded cables. In both the cases the effect is to supply the required amount of reactive power to maintain the voltage. Capacitors are connected either directly to the bus bar or to the tertiary winding of the main transformer and are distributed along the line to minimise the losses and the voltage drops. Now when the voltage drops, the vars produced by shunt capacitor or reactor falls, so when required most, the effectiveness of these capacitors or the reactors also falls. On the other hand, on light loads when the voltage is high, the capacitor output is large and the voltage tends to rise to excessive level, so some of the capacitors or reactors are to be switched out by over voltage relays. For fast control of voltages in power systems, switched capacitors in parallel with semiconductor controlled reactors are generally used to provide var compensation.

Series capacitors:

Here the capacitors are connected in series with the line. The main aim is to reduce the inductive reactance between supply point and the load. The major disadvantage of the method is, whenever a short circuit current flows through the capacitor, protective
devices like spark gaps and non linear resistors are to be incorporated. Phasor diagram for a line with series capacitor is shown in the figure (b).

Relative merits between shunt and series capacitors.

• If the load var requirement is small, series capacitors are of little help.
• If the voltage drop is the limiting factor, series capacitors are effective, also to some extent the voltage fluctuations can be evened.
• If the total line reactance is high, series capacitors are very effective and stability is improved.
• With series capacitors the reduction in line current is small, hence if the thermal considerations limits the current, little advantage is from this, so shunt compensation is to be used.

Synchronous compensators.

A synchronous compensator is a synchronous motor running without a mechanical load and depending on the excitation level, it can either absorb or generate reactive power. when used with a voltage regulator the compensator can automatically run overexcited at times of high loads and under excited at light loads. A typical connection of a compensator is shown in the figure along with the associated voltage – var output characteristics.
A great advantage of the method is the flexible operation for all load conditions.

- Being a rotating machine, its stored energy is useful for riding through transient disturbances, including voltage drops.
Sub Synchronous Resonance

Series capacitors are installed in series with long lines for providing compensation of reactive power and giving higher power transfer ability. Series compensated lines have a tendency to produce series resonance at frequencies lower than power frequencies. This is called **Sub Synchronous Resonance (SSR)**. The sub synchronous resonance currents produce mechanical resonance in Turbo generator shafts, which causes the following in the generator shafts:

(i) Induction generator effect
(ii) torsional torques and (iii) transient torques.

These problems have resulted in damage to rotor shafts of turbine generators. Therefore the sub synchronous resonance is analysed in the design of series compensated lines. Now let us derive a relationship between the normal frequency and the sub synchronous resonance frequency.

Let $f_n$ be the normal frequency (synchronous) let $f_r$ be the sub synchronous frequency of series compensated line. $\frac{1}{2}f_n L$ be the series inductive reactance of EHV line at normal frequency. $\frac{1}{2}f_n C$ be the series capacitive reactance of series compensation at normal frequency.

$$K = \frac{X_c}{X}$$

$L$ be the degree of compensation.

$$X = (XL - Xc) = XL(1 - K)$$
is the equivalent reactance of the compensated line.

Let the SSR occur at a frequency $f_r$. Then $f_r^2 = \left(\frac{1}{2} L \right) \times \left(\frac{1}{2} C \right)$

(OR) $(fr/fn)^2 = Xc/XL = K$ or $fr = fn*sqrt(K)$

Thus SSR occurs at a frequency $f_r$ which is the product of normal frequency and the root of the degree of compensation $K$. The condition of SSR can occur during the faults on the power system, during switching operations and changing system configurations. Solution to SSR problems

1. Use of filters: For eliminating/damping the harmonics. The various filters include: static blocking filters, bypass damping filters, dynamic filters.
2. Bypassing the series capacitor bank under resonance condition
3. Tripping of generator units under conditions of SSR

**Reactive Power and Voltage Collapse**

Voltage collapse is a system instability and it involves many power system components and their variables at once. Indeed, voltage collapse involves an entire power system although it usually has a relatively larger involvement in one particular section of the power system. Voltage collapse occurs in power systems which are usually **Heavily loaded**, faulted and/or have reactive power shortages. Voltage collapses can occur in a transient time scale or in a long term time scale. Voltage collapse in a long term time scale can include effects from the transient time scale; for example, a slow voltage collapse taking several minutes may end in a fast voltage collapse in the transient time scale.

**Changes in power system contributing to voltage collapse**

There are several power system disturbances which contribute to the voltage collapse.

i. Increase in inductive loading

ii. Reactive power limits attained by reactive power compensators and generators.

iii. On Load Tap Changing operation

iv. Load recovery dynamics.

v. Generator outage

vi. Line tripping.

Most of these factors have significant effects on reactive power production, transmission and consumption. Switching of shunt capacitors, blocking of OLTC operation, generation rescheduling, bus voltage control, strategic load shedding and allowing temporary reactive power overloading of generators may be used as some of the effective countermeasures against voltage collapse.

**Voltage Stability**

The voltage stability may be defined as the ability of a power system to maintain steady acceptable voltage at all busses in the system at normal operating conditions and after being subjected to disturbances/perturbations.
Voltage stability is the ability of a system to maintain voltage so that when load admittance is increased, load power will increase, and so that both power and voltage are controllable. Power system is “Voltage Stable “if voltages at respective busses after a disturbance are close to the voltages at normal operating conditions. So voltage instability is that appears when the attempt of load dynamics to restore power consumption is just beyond the capability of the combined transmission and generator system. Though voltage instability may be a local problem, its consequences may have a widespread effect. Voltage collapse is the catastrophic result of a sequence of events leading to a sudden low-voltage profile in a major part of the system, i.e. in a significant part of the system. Voltage Stability can also be called Load Stability. A Power system lacks the capability to transfer an infinite amount of electrical power to the loads. The main factor causing voltage instability is the inability of the power system to meet the demands for reactive power in the heavily stressed system keeping desired voltages. Other factors contributing to voltage instability are the generator reactive power limits. Transfer of reactive power is difficult due to extremely high reactive power losses, which is why the reactive power required for voltage control is generated and consumed at the control area. A classification of power system stability is shown in the table below. The driving forces for instability are named generator– driven and load-driven. It is to be noted that these terms do not exclude the effect of other components to the mechanism. The time scale is divided into short and long-term time scale.

Now let us analyse voltage stability using Q-V curves. Consider a simple system as shown below and its P-V curves.
Fig: Normalised P-V curves for fixed (infinite) source. Now map the normalised P-V curves onto V-Q curves. For constant value of P, note the values of Q and V and then replot to get Q-V curves as shown below. From P-V curves it is observed that the critical voltage is very high for high loadings. V is above 1.0p.u for P = 1.0p.u. The right side represents normal conditions where applying a capacitor bank raises voltage.
Fig : Normalised Q-V curves for fixed (infinite) source.

Fig : Q – V Curves
Figure shows the Q-V diagram of a bus in a particular power system at four different loads: P1, P2, P3, P4. The Q axis shows the amount of additional reactive power that must be injected into the bus to operate at a given voltage. The operating point is the intersection of the power curve with the voltage axis, where no reactive power is required to be injected or absorbed. If the slope of the curve at the intersection point is **positive**, the system is **stable**, because any additional reactive power will raise the voltage and vice-versa. Hence for P1 load, there is a reserve of reactive power that can be used to maintain stability even if the load increases. For load P2 the system is marginally stable. For higher load P3 and P4 the system is not stable (since a certain amount of reactive power must be injected into the bus to cause an intersection with the voltage axis.) Thus the measure of Q reserve gives an indication of the margin between stability and instability. The slope of the Q-V curve represents the stiffness of the test bus. When nearby generators reach their Var limits, the slope of the Q-V curve becomes less steep and the bottom of the curve is approached. V-Q curves are presently the workhorse method of voltage stability analysis at many utilities. Since the method artificially stresses a single bus, conclusions should be confirmed by more realistic methods.
Fig: Reactive Power Margins
PART - B

UNIT -5

OPTIMAL SYSTEM OPERATION AND UNIT COMMITMENT: Introduction , Optimal operation of generators on a bus bar, Statement of the Unit Commitment problem, need and importance of unit commitment, Constraint in Unit Commitment, Unit Commitment solution methods-Priority lists method, Forward Dynamic Programming method( excluding problem), Spinning reserve. 6 Hours

POWER SYSTEM OPTIMIZATION

(x) Introduction
(xi) Problem of economic load scheduling
(xii) Performance curves
(xiii) Constraints in economic operation of power systems
(xiv) Spinning reserve
(xv) Solution to economic load dispatch

Solution to ELD without inequality constraints
Solution to ELD with capacity constraints

Solution to ELD with Transmission losses considered- PENALTY FACTOR METHOD
Solution to ELD with Transmission losses considered- LOSS COEFFICIENTS METHOD (Transmission loss as a function of plant generation- B Coefficients)

(xvi) Examples

INTRODUCTION

Electric Power Systems (EPS): Economic Aspects:

In EPS, the first step is to properly assess the load requirement of a given area where electrical power is to be supplied. This power is to be supplied using the available units such as thermal, hydroelectric, nuclear, etc. Many factors are required to be considered while choosing a type of generation such as: kind of fuel available, fuel cost, availability of suitable sites for major station, nature of load to be supplied, etc.
Variable load: The load is not constant due to the varying demands at the different times of the day. The EPS is expected to supply reliable and quality power. It should ensure the continuity of power supply at all times.

{Qn.: write a note on the choice of the number and size of the generating units at a power station from economic operation point of view}

Single unit Vs. multiple units: the use of a single unit to supply the complete load demand is not practical since, it would not be a reliable one. Alternately, a large number of smaller units can be used to fit the load curve as closely as possible. Again, with a large number of units, the operation and maintenance costs will increase. Further, the capital cost of large number of units of smaller size is more as compared to a small number of units of larger size. Thus, there has to be compromise in the selection of size and number of generating units within a power plant or a station.

Electric Power Systems (EPS): Operational Aspects:

Electric energy is generated at large power stations that are far away from the load centers. Large and long transmission lines (grid lines) wheel the generated power to the substations at load centers. Many electrical equipment are used for proper transmission and distribution of the generated power. The grid lines are such that:

GRID: The transmission system of a given area.

- Regional GRID: Different grids are interconnected through transmission lines.
- National GRID: Interconnection of several regional grids through tie lines.

Each grid operates independently, although power can be exchanged between various grids.
Economic loading of generators and interconnected stations:

Optimum economic efficiency is achieved when all the generators which are running in parallel are loaded in such a way that the fuel cost of their power generation is the minimum. The units then share the load to minimize the overall cost of generation. This economical approach of catering to the load requirement is called as ‘economic dispatch’. The main factor in economic operation of power systems is the cost of generating the real power. In any EPS, the cost has two components as under:

- **The Fixed Costs**: Capital investment, interest charged on the money borrowed, tax paid, labour, salary, etc. which are independent of the load variations.

- **The Variable Costs**: which are dependant on the load on the generating units, the losses, daily load requirements, purchase or sale of power, etc.

The current discussion on economic operation of power systems is concerned about minimizing the variable costs only.

Further, the factors affecting the operating cost of the generating units are: generator efficiency, transmission losses, fuel cost, etc. Of these, the fuel cost is the most important factor.

Since a given power system is a mix of various types of generating units, such as hydel, thermal, nuclear, hydro-thermal, wind, etc., each type of unit contributes its share for the total operating cost. Since fuel cost is a predominating factor in thermal (coal fired) plants, economic load dispatch (ELD) is considered usually for a given set of thermal plants in the foregoing discussion.

**PROBLEM OF ECONOMIC LOAD SCHEDULING:**

There are two problem areas of operation strategy to obtain the economic operation of power systems. They are: problem of economic scheduling and the problem of optimal
power flow.

* The problem of economic scheduling: This is again divided into two categories:

  - The unit commitment problem (UCP): Here, the objective is to determine the various generators to be in operation among the available ones in the system, satisfying the constraints, so that the total operating cost is the minimum. This problem is solved for specified time duration, usually a day in advance, based on the forecasted load for that time duration.

  - The economic load dispatch (ELD): Here, the objective is to determine the generation (MW power output) of each presently operating (committed or put on) units to meet the specified load demand (including the losses), such that the total fuel cost is minimized.

* The problem of optimal power flow: Here, it deals with delivering the real power to the load points with minimum loss. For this, the power flow in each line is to be optimized to minimize the system losses.

{Qn.: compare ELD and UCP and hence bring out their importance and objectives.}

PERFORMANCE CURVES:

The Performance Curves useful for economic load dispatch studies include many different types of input-output curves as under:

1. Input Output Curve: A plot of fuel input in Btu/Hr. as a function of the MW output of the unit.

2. Heat Rate Curve: A plot of heat rate in Btu/kWH, as a function of the MW output of the unit. Thus, it is the slope of the I-O curve at any point. The reciprocal of heat rate is termed as the ‘Fuel Efficiency’.
3. **Incremental Fuel Rate Curve**: A plot of incremental fuel rate (IFC) in Btu/kWh as a function of the MW output of the unit, where,

\[ \text{IFC} = \frac{\text{input}}{\text{output}} = \frac{\text{Incremental change in fuel input}}{\text{Incremental change in power output}} \]  

(1)

4. **Incremental Fuel Cost Curve**: A plot of incremental fuel cost (IFC) in Rs./kWh as a function of the MW output of the unit, where,

\[ \text{IFC in Rs./kWh} = \text{IFC in Btu/kWh} \times \text{Fuel cost in Rs./Btu} \]  

(2)

The IFC is a measure of how costlier it will be to produce an increment of power output by that unit.

The **Cost Curve** can be approximated by:

* Quadratic Curve by the function: 
  \[ \text{C}(P_i) = a_i + b_i P_i + c_i P_i^2 \]  
  Rs./Hr. \hspace{2cm} (3)

* Linear curve by the function: 
  \[ \frac{dC_i}{dP_i} = b_i + 2c_i P_i \]  
  Rs./MWHr. \hspace{2cm} (4)

Generally, the quadratic curve is used widely to represent the cost curve, with the IC curve given by the linear curve as above.

**CONSTRAINTS IN ECONOMIC OPERATION OF POWER SYSTEMS:**

Various constraints are imposed on the problem of economic operation of power systems as listed below:

1. **Primary constraints (equality constraints):**

   Power balance equations:
   \[ P_i - P_{Di} - P_l = 0; \quad Q_i - Q_{Di} - Q_l = 0; \quad i=\text{buses of the system} \]  
   \hspace{2cm} (5)

   where, 
   \[ P_l = \sum ||V_iV_jY_{ij}|| \cos (\delta_{ij} - \theta_{ij}); \]
   \[ Q_l = \sum ||V_iV_jY_{ij}|| \sin (\delta_{ij} - \theta_{ij}); \]
\( j = 1,2,\ldots,n, \) are the power flow to the neighboring system. \hspace{1cm} (6)

The above constraints arise due to the need for the system to balance the generation and load demand of the system.
2. Secondary constraints (inequality constraints):

These arise due to physical and operational limitations of the units and components.

\[ P_{i\text{min}} \leq P_i \leq P_{i\text{max}} \]
\[ Q_{i\text{min}} \leq Q_i \leq Q_{i\text{max}} \]

\( i = 1,2,\ldots,n \), the number of generating units in the system. (7)

3. Spare Capacity Constraints:

These are used to account for the errors in load prediction, any sudden or fast change in load demand, inadvertent loss of scheduled generation, etc. Here, the total generation available at any time should be in excess of the total anticipated load demand and any system loss by an amount not less than a specified minimum spare capacity, \( P_{SP} \) (called the Spinning Reserve) given by:

\[ P_{IG} \text{ (Generation)} \geq \sum_{i=1}^{n} P_i \text{ (Losses)} + P_{SP} + P_{Dj} \text{ (Load)} \] (8)

4. Thermal Constraints:

For transmission lines of the given system:

\[ S_{i\text{min}} \leq S_{bi} \leq S_{i\text{max}} \]

\( i = 1,2,\ldots,n_b \), the number of branches,

where, \( S_{bi} \) is the branch transfer MVA. (9)

5. Bus voltage and Bus angle Constraints:

Bus voltage and Bus angle Constraints are needed to maintain a flat bus voltage
profile and to limit the overloading respectively.

\[- V_{i \min} \leq V_i \leq V_{i \max} \quad i = 1,2,\ldots,n \]
\[- \delta_{ij \min} \leq \delta_{ij} \leq \delta_{ij \max} \quad i = 1,2,\ldots,n; j = 1,2,\ldots.m \] (10)

where, \( n \) is the number of nodes and \( m \) is the number of nodes neighboring each node with interconnecting branches.

6. Other Constraints:

In case of transformer taps, during optimization, it is required to satisfy the constraint:

\[ T_{i \min} \leq T_i \leq T_{i \max} \] (11)

where \( T_i \) is the percentage tap setting of the tap changing transformer used.

In case of phase shifting transformers, it is required to satisfy the constraint:

\[ PS_{i \min} \leq PS_i \leq PS_{i \max} \] (12)

where \( PS_i \) is the phase shift obtained from the phase shifting transformer used.

**SPINNING RESERVE**

Spinning reserve (SR) is the term used to describe the total amount of generation available from all the synchronized (spinning) units of the system minus the present load plus the losses being supplied. i.e.,

\[ Sp.Res., P_{SP} = \{ \text{Total generation, } \Sigma P_{IG}\} - \{ \Sigma P_{DJ}(\text{load}) + \Sigma P_l (\text{losses}) \} \] (13)
The SR must be made available in the system so that the loss of one or more units does not cause a large drop in system frequency. SR must be allocated to different units based on typical Council rules. One such rule is as follows:

‘SR must be capable of making up for the loss of the most heavily loaded unit in the system’

Reserves must be spread around the system to avoid the problem of ‘bottling of reserves’ and to allow for the various parts of the system to run as ‘islands’ whenever they become electrically disconnected.

{Qn.: Write a brief note on the following: Spinning Reserve, constraints in economic operation, performance curves}

SOLUTION TO ECONOMIC LOAD DISPATCH

{Qn.: Derive the EIC criterion for economic operation of power systems with transmission losses neglected, MW limits considered/ not considered}

The solution to economic load dispatch problem is obtained as per the equal incremental cost criterion (EIC), which states that:

‘All the units must operate at the same incremental fuel cost for economic operation’

This EIC criterion can be derived as per LaGrangian multiplier method for different cases as under.

CASE (i) Solution to ELD without inequality constraints:

Consider a system with N generating units supplying a load $P_D$ MW. Let the unit
MW limits and the transmission losses are negligible. Suppose the fuel cost of unit ‘i’ is given by:

\[ C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \] Rs./Hr. so that

\[ IC_i = dC_i/dP_i = b_i + 2c_i P_i \] Rs./MWHr. (14)

Hence, the total cost, 

\[ C_T = \sum C_i(P_i) \quad i = 1, 2, \ldots N \]

The ELD problem can thus be stated mathematically as follows:

| Minimize \[ C_T = \sum C_i(P_i) \quad i = 1, 2, \ldots N \] |
|---|---|
| Such that \[ \sum P_i = P_D \] (15) |

where, \( C_T \) is the total fuel cost of the system in Rs./Hr., \( P_D \) is the total demand in MW and \( P_i \) is the MW power output of unit i. The above optimization problem can be solved by LaGrange’s method as follows.

The LaGrange function \( L \) is given by:

\[ L = C_T + \lambda (P_D - \Sigma P_i) \] (16)

The minimum cost value is obtained when:

\[ \partial L/\partial P_i = 0; \quad \text{and} \quad \partial L/\partial \lambda = 0 \] (17)

\[ \partial C_T/\partial P_i - \lambda = 0 \quad \text{and} \quad P_D - \Sigma P_i = 0 \] (which is same as the constraint given)
Further, since the cost of a given unit depends only on its own power output, we have,
\[ \frac{\partial C_i}{\partial P_i} = \frac{\partial C_i}{\partial P_i} = \frac{\partial C_i}{\partial P_i} \quad i=1,2,\ldots, N \quad (18) \]

Thus,
\[ \frac{dC_i}{dP_i} - \lambda = 0 \quad i = 1,2,\ldots, N \quad \text{or} \]
\[ \frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \ldots = \frac{dC_i}{dP_i} = \ldots = \frac{dC_N}{dP_N} \quad (19) \]

\[ = \lambda, \quad \text{a common value of the incremental fuel cost of generator I, in Rs./MWHr.} \]

The equation above is stated in words as under:

“For the optimum generation (power output) of the generating units, all the units must operate at equal incremental cost (EIC)”

Expression for System Lambda, \( \lambda \):

Consider equation (14);
\[ IC_i = d(C_i)/dP_i = b_i + 2c_i P_i \]

Simplifying, we get,
\[ P_i = \frac{[\lambda - b_i]}{2c_i} \quad \text{MW} \quad \text{Rs./MWH} \]

\[ \sum P_i = P_D = \sum \frac{[\lambda - b_i]}{2c_i} \quad \text{r. so that} \]

Solving, we get,
\[ \lambda = \frac{P_D + \sum [b_i/2c_i]}{(\sum [1/2c_i])} \quad (20) \]

**CASE (ii) Solution to ELD with capacity constraints:**

Consider a system with \( N \) generating units supplying a load \( P_D \) MW. Let the unit MW limits be considerable and the transmission losses be negligible. Suppose the fuel cost of unit ‘i’ is given by:
\[ C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad \text{Rs./Hr.} \quad \text{so that the total cost,} \]
\[ C_T = \sum C_i(P_i) \quad i=1,2,\ldots, N \]
The ELD problem can now be stated mathematically as follows:

\[
\begin{align*}
\text{Minimize} \quad & C_T = \sum C_i(P_i) \quad i = 1, 2, \ldots N \\
\text{Such that} \quad & \sum P_i = P_D \text{ and} \\
& P_i^{\min} \leq P_i \leq P_i^{\max} \quad (21)
\end{align*}
\]

where, \( C_T \) is the total fuel cost of the system in Rs./Hr., \( P_D \) is the total demand in MW, \( P_i \) is the MW power output of unit \( i \), \( P_i^{\min} \) is the minimum MW power output and \( P_i^{\max} \) is the maximum power output by the unit \( i \).

The necessary conditions for the solution of the above optimization problem can be obtained as follows:

\[
\lambda_i = \frac{dC_i}{dP_i}
\]

\[
= \lambda_i \quad \text{Rs.} / \text{MWHr. for} \quad P_i^{\min} \leq P_i \leq P_i^{\max}
\]

\[
\frac{dC_i}{dP_i} \leq \lambda_i \quad \text{Rs.} / \text{MWHr. for} \quad P_i = P_i^{\max}
\]

\[
= \frac{dC_i}{dP_i} \geq \lambda \quad \text{Rs.} / \text{MWHr. for} \quad P_i = P_i^{\min}
\]  

(22)
From the above equations, if the outputs of the unit, according to optimality rule, is:

* Less than its minimum value, then it is set to \( P_{i\text{min}} \), the corresponding IC will be greater than the system \( \lambda \),
* More than its maximum value, then it is set to \( P_{i\text{max}} \), the corresponding IC will be less than the system \( \lambda \),
* With in its maximum and minimum values, then the corresponding IC will be equal to the system \( \lambda \).

In other words, the sequential procedural steps are as follows:

1. First, find the power output according to the optimality rule (EIC Criterion)

2. If the power output of any unit is less than its minimum value, then set the value to be equal to its \( P_{i\text{min}} \),

3. Similarly, if the power output of any unit is more than its maximum value, then set the value to be equal to its \( P_{i\text{max}} \),

4. Adjust the demand for the remaining units after accounting for the settings made for the above units (those units which have violated the limits)

5. Finally, apply the EIC criterion, for the remaining units. Here, the system lambda is determined by only those units whose power output values are with in the specified MW limits.
CASE (iii) Solution to ELD with Transmission losses considered-

**PENALTY_FACTOR METHOD**

{Qn.: Derive the EIC criterion for economic operation of power systems with transmission losses considered. Use penalty factor method}

Consider a system with N generating units supplying a load $P_D$ MW. Let the transmission losses be considerable. Suppose the fuel cost of unit ‘i’ is given by:

$$C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \text{ Rs./Hr.}$$

so that the total cost,

$$C_T = \sum C_i(P_i) \quad i = 1, 2, \ldots, N$$

Let $P_L$ be the total transmission losses in the system. The ELD problem can now be stated mathematically as follows:

Minimize $C_T = \sum C_i(P_i) \quad i = 1, 2, \ldots, N$

Such that $\sum P_i = P_D + P_L \quad (23)$

where, $C_T$ is the total fuel cost of the system in Rs./Hr., $P_D$ is the total demand in MW, $P_i$ is the MW power output of unit i, $P_L$ is the transmission losses in the system. This above optimization problem can be solved by LaGranje’s method as follows.

The LaGranje function L is given by:

$$L = C_T - \lambda (\sum P_i - P_D - P_L) \quad (24)$$

The minimum cost value is obtained when:
\[ \frac{\partial L}{\partial P_i} = 0; \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = 0 \quad (25) \]

\[ \frac{\partial C_T}{\partial P_i} - \lambda (1 - \frac{\partial P_L}{\partial P_i}) = 0 \quad \text{and} \quad \sum P_i - P_D - P_L = 0 \] (which is same as the constraint given)

Further, since the cost of a given unit depends only on its own power output, we have,

\[ \frac{\partial C_i}{\partial P_i} = \frac{\partial C_i}{\partial P_i} = \frac{dC_i}{dP_i} \quad i = 1, 2, \ldots, N \]

Thus,

\[ \frac{dC_i}{dP_i} - \lambda (1 - \frac{dP_L}{dP_i}) = 0 \quad i = 1, 2, \ldots, N \quad \text{or} \]

\[ \frac{dC_i}{dP_i} = \lambda (1 - \frac{dP_L}{dP_i}) \]

So that we have for optimal operation,

\[ \lambda = \frac{IC_i}{(1 - \frac{dP_L}{dP_i})} = \frac{IC_i (1 - \frac{dP_L}{dP_i})^{-1}}{P_{ni}} \quad IC_i \]

Where, Pni is the penalty factor of unit i = (1 - \frac{dP_L}{dP_i})^{-1} = (1 - \text{ITL}_i)^{-1}; \text{ITL}_i = \frac{dP_L}{dP_i} \text{is the Incremental Transmission Loss of unit i, and } \lambda \text{ is in Rs. MWHr.} \quad (26) \]

The equation above is stated in words as under:

‘For the optimum generation (power output) of the generating units, when the transmission losses are considered, all the units must operate such that the product of the incremental fuel cost and their penalty factor must be the same for all units’

Note: in equation (26), if losses are negligible as in case (i) above, then, \( \text{ITL}_i = \frac{dP_L}{dP_i} = 0 \), \( P_{ni} = 1.0 \) so that \( \lambda = IC_i \) \( i = 1, 2, \ldots, N \), as before.

**CASE (iv) Solution to ELD with Transmission losses considered- LOSS COEFFICIENTS METHOD**

\{Qn.: Derive the EIC criterion for economic operation of power systems with transmission losses considered. Use B- Coefficients method OR **
Derive an expression for the transmission loss as a function of the plant generations.

Consider a system with two generating units supplying currents $I_1$ and $I_2$ respectively to the load current $I_L$. Let $I_{k1}$ and $I_{k2}$ be the respective currents flowing through a general transmission branch element $k$ of resistance $R_k$, with current $I_k$ as shown in figure 1 below.

![Figure 1. Branch currents in a 2 unit system](image)

\[ I_k = I_{k1} + I_{k2} = N_{k1}I_1 + N_{k2}I_2 \]  \hspace{1cm} (27)

Where, $N_{k1}$ and $N_{k2}$ (assumed to be real values) are the current distribution factors of units 1 and 2 respectively.

It is assumed that the currents $I_{k1}$ and $I_L$ as well as $I_{k2}$ and $I_L$ have the same phase angle or they have a zero phase shift. Thus, they can be added as real numbers as under.
Let \[ I_1 = 3I_3 \cos \sigma_1 + j3I_3 \sin \sigma_1 \]
\[ I_2 = 3I_3 \cos \sigma_2 + j3I_3 \sin \sigma_2 \] (28)

Where \( \sigma_1 \) and \( \sigma_2 \) are the phase angles of currents. Consider now, the magnitude of current \( I_k \), in branch \( k \), given by

\[
I_k = \{N_{k1}I_1 \cos \sigma_1 + jN_{k1}I_3 \sin \sigma_1\} + \{N_{k2}I_2 \cos \sigma_2 + jN_{k2}I_3 \sin \sigma_2\}
\]

Thus,

\[
3I_k^2 = N_{k1}^2I_1^2 + N_{k2}^2I_2^2 + 2N_{k1}N_{k2}I_1I_3 \cos(\sigma_1-\sigma_2) \] (29)

However, we have,

\[
P_1 = \sqrt{3}V_1I_1 \cos \theta_1; \quad P_2 = \sqrt{3}V_2I_2 \cos \theta_2; \quad P_L = \Sigma 3I_k^2R_k \] (30)

where, \( P_1 \) and \( P_2 \) are the MW power output values by the units 1 and 2 respectively, \( V_1 \) and \( V_2 \) are the respective line voltages and \( \theta_1, \theta_2 \) are the respective power factor angles and \( P_L \) is the transmission loss in the system.

From equations (29) and (30),
after simplification, an expression for the transmission loss as a function of plant generation can be obtained as:

\[
P_L = \sum N_{k1}^2 R_k P_1^2/(V_1^2 \cos^2 \theta_1) + \sum N_{k2}^2 R_k P_2^2/(V_2^2 \cos^2 \theta_2) + 2 \sum N_{k1} N_{k2} R_k P_1 P_2 \cos(\sigma_1 - \sigma_2)/(V_1 V_2 \cos \theta_1 \cos \theta_2) \\
= B_{11} P_1^2 + B_{22} P_2^2 + 2 B_{12} P_1 P_2
\]

(31)

Where, the B coefficients are called as the loss coefficients. Thus, in general, for a system of n units we have,

\[
P_L = \sum \sum P_i B_{ij} P_j \quad \text{Where,}
\]

\[
B_{ij} = \sum \{\cos(\sigma_i - \sigma_j)/(V_i V_j \cos \theta_i \cos \theta_j) N_{ki} N_{kj} R_k \}
\]

(32)

Note:

1. The B coefficients are represented in units of reciprocal MW, (MW^{-1})
2. For a three unit system, equation (32) takes the form:
\[
P_L = B_{11} P_1^2 + B_{22} P_2^2 + B_{33} P_3^2 + 2 B_{12} P_1 P_2 + 2 B_{13} P_1 P_3 + 2 B_{23} P_2 P_3 \\
= P^T B P
\]

(33)

Where, \(P = [P_1 \ P_2 \ P_3]\), the vector of unit power output values and \(B = [B_{11} \ B_{12} \ B_{13}; B_{21} \ B_{22} \ B_{23}; B_{31} \ B_{32} \ B_{33}]\), the loss coefficient matrix for the 3 unit system.

3. The B coefficient matrix is a square, symmetric matrix of order n, n being the number of generating units present in the system.
4. The following are the assumptions made during the above analysis:
   - All load currents maintain a constant ratio to load current \((N_{ki} = \text{constant})\).
   - The voltage at any bus remains constant.
   - The power factor of each bus source is constant \((\theta_i = \text{constant})\).
• The voltage phase angle at load buses is constant ($\sigma_i=$constant).

5. The Incremental Transmission loss, ITL$_i$ of a given unit can be expressed in terms of its MW power output values as under:

Consider, $P_L = \sum \sum P_{jk}P_k$

$$= \sum \sum P_{jk}P_k + \sum P_{ji}P_i$$

$$= \sum \sum P_{jk}P_k + \sum P_{ji}P_i + B_{ii}P_i^2$$

Thus,

$$ITL_i = dP_L/dP_i = 0 + \sum B_{ik}P_k + \sum P_{ji} + 2 B_{ii}P_i$$

$$= 2 \sum P_kB_{ik}$$

(34)
Examples on Economic operation of power systems

Part A: Transmission losses negligible

Example-1:
The I-O characteristics of two steam plants can be expressed analytically as under (with $P_1$ and $P_2$ in MW):

\[
F_1 = (2.3P_1 + 0.0062P_1^2 + 25) \times 10^6 \text{kCals/Hr.}
\]
\[
F_2 = (1.5P_1 + 0.01P_2^2 + 35) \times 10^6 \text{kCals/Hr.}
\]

The calorific value of coal at plant#1 and plant#2 are respectively equal to 4000 kCals/kg. and 5000 kCals/kg. The corresponding cost of coal is Rs.55/- and Rs.65/-per Ton. Find the following: (i) Incremental Fuel Rate in kCals/MWHr (ii) Incremental Fuel Cost in Rs./MWHr and (iii) Incremental Production Cost in Rs./MWHr if the cost of other items can be taken as 10% of the incremental fuel cost/plant.

Solution:

(i) Incremental Fuel Rate in kCals/MWHr

\[
IFR_1 = \frac{dF_1}{dP_1} = (2.3 + 0.0124P_1) \times 10^6 \text{kCals/MWHr}
\]

\[
IFR_2 = \frac{dF_2}{dP_2} = (1.5 + 0.02P_2) \times 10^6 \text{kCals/MWHr}
\]

(ii) Incremental Fuel Cost in Rs./MWHr

\[
IC_1 = \left[ \frac{dF_1}{dP_1} \text{in kCals/MWHr} \right] \left[ \text{cost of coal in Rs./ton} \right] \left[ \text{calorific value}^{-1} = (2.3 + 0.0124P_1) \times 10^6 \times (55/4000) \times 10^{-3} \right]
\]

\[
= 31.625 + 0.1705P_1 \text{ Rs./MWHr.}
\]

\[
IC_2 = \left[ \frac{dF_2}{dP_2} \text{in kCals/MWHr} \right] \left[ \text{cost of coal in Rs./ton} \right] \left[ \text{calorific value}^{-1} = (1.5 + 0.02P_2) \times 10^6 \times (65/5000) \times 10^{-3} \right]
\]

\[
= 19.9 + 0.26P_2 \text{ Rs./MWHr.}
\]
(iii) *Incremental Production Cost* in Rs./MWHr if the cost of other items can be taken as 10% of the incremental fuel cost/plant.

Effective value of IC are given by:

\[ IC_{1\text{eff}} = 1.1 \left( IC_1 \right) \]

\[ = 1.1 \left( 31.625 + 0.1705 P_1 \right) \text{ Rs./MWHr.} \]

- \[ 34.7878 + 0.1875 P_2 \text{ Rs./MWHr.} \]

\[ IC_{2\text{eff}} = 1.1 \left( IC_2 \right) \]

\[ = 1.1 \left( 19.9 + 0.26 P_2 \right) \text{ Rs./MWHr.} \]

\[ = 21.45 + 0.286 P_2 \text{ Rs./MWHr.} \]

Example-2:
The incremental costs of a two unit system are given by: \[ IC_1 = (0.008 \, P_{G1} + 8.0) \]
\[ IC_2 = (0.0096 \, P_{G2} + 6.4) \]
Find the incremental cost and the distribution of loads between the two units for optimal operation for a total load of 1000 MW. What is this value if the same total load is equally shared among the two units?

Solution:
For the total load values of \( P_T = 1000 \text{ MW} \), if the load is shared equally among the two units then:
\[ P_{G1} = 500 \text{ MW}; \, P_{G2} = 500 \text{ MW} \quad \text{with} \]
\[ \lambda_1 = 12 \text{ Rs./MWHr} \quad \text{and} \quad \lambda_2 = 11.2 \text{ Rs./MWHr} \quad \text{(unequal lambda values)} \]
Now, for optimal operation, we have as per EIC principle, the IC’s to be equal. i.e.,
\[ IC_1 = IC_2; \quad P_T = P_1 + P_2 = 1000 \]
are the equations to be solved for the output power values. Thus,
\[ IC_1 = 0.008 \, P_{G1} + 8.0 = IC_2 = 0.0096 \, P_{G2} + 6.4 \]
\[ = 0.0096 \left( 1000 - P_{G1} \right) + 6.4 \]
Solving, we get, \( P_{G1} = 454.54 \text{ MW} \); \( P_{G2} = 545.45 \text{ MW} \)

Further, \( \lambda_{\text{system}} \) is calculated using any one of the IC equations as: \( \lambda_{\text{system}} = \lambda_1 = \lambda_2 = 11.64 \text{ Rs./MWHr.} \)

Thus, with \( \lambda_{\text{system}} = \lambda_1 = \lambda_2 = 11.64 \text{ Rs./MWHr.} \), the total load is optimally shared between the two units and the operating cost would be at its minimum.

Example-3:
The fuel costs in Rs./Hr. for a plant of three units are given by:
\( C_1 = (0.1P_1^2+40P_1+100); \) \( C_2 = (0.125P_2^2+30P_2+80); \) \( C_3 = (0.15P_3^2+20P_3+150); \)
Find the incremental cost and the distribution of loads between the three units for optimal operation for a total load of 400 MW, given that the max. and min. capacity limits for each of the units as 150 MW and 20 MW respectively.

Solution:
Consider the incremental cost curves given by:
\( \text{IC}_i = \frac{dC_i}{dP_i} = (2c_iP_i+b_i) \text{ Rs./MWHr} \)

\( \text{IC}_1 = \frac{dC_1}{dP_1} = (0.20P_1+40) \text{ Rs./MWHr} \)

\( \text{IC}_2 = \frac{dC_2}{dP_2} = (0.25P_2+30) \text{ Rs./MWHr} \) and

\( \text{IC}_3 = \frac{dC_3}{dP_3} = (0.30P_3+20) \text{ Rs./MWHr} \)

For the total load values of \( P_T = 400 \text{ MW} \), for optimal operation, as per EIC principle, the IC’s are equal. i.e., \( \text{IC}_1 = \text{IC}_2 = \text{IC}_3 \); and \( P_T = P_1 + P_2 + P_3 = 400 \text{ MW} \).

Also, the system lambda is given by:
\[ \lambda = \frac{P_D + \Sigma(b_i/2c_i)}{\Sigma(1/2c_i)} \quad i=1,2,3 \]

Substituting the values, we get after simplification, \( \lambda = 63.78 \text{ Rs./MWHr.} \).
Using this value of common system lambda, the MW output values of all the 3 units are obtained from their IC curves as:

\[ P_1 = 118.9 \text{ MW}, \ P_2 = 135.12 \text{ MW} \quad \text{and} \quad P_3 = 91.90 \text{ MW}. \]

(All the MW output values are found to be within their capacity limits specified)

Thus, with \( \lambda_{\text{system}} = \lambda_1 = \lambda_2 = \lambda_3 = 63.78 \text{ Rs./MWHr} \), the total load is optimally shared between the three units and the operating cost would be at its minimum.

Example-4:
The incremental costs of a two unit system are given by:

\[ IC_1 = 0.008 P_G1 + 8.0 \quad \text{; IC}_2 = 0.0096 P_{G2} + 6.4 \]

Find the incremental cost and the distribution of loads between the two units for optimal operation for a total load of 900 MW. Also determine the annual saving in cost in optimal operation as compared to equal sharing of the same total load.

Solution:
For a total load of \( P_T = 900 \text{ MW} \), if the load is shared equally among the two units then: \( P_{G1} = P_{G2} = 450 \text{ MW} \).

Now, for optimal operation, we have as per EIC principle, the IC’s to be equal. i.e., \( IC_1 = IC_2 \); \( P_T = P_1 + P_2 = 900 \) are the equations to be solved for the output power values.

Thus, \( IC_1 = 0.008 P_{G1} + 8.0 = IC_2 = 0.0096 (900 - P_{G1}) + 6.4 \)

Solving, we get,

\[ P_{G1} = 400 \text{ MW}; \ P_{G2} = 500 \text{ MW} \quad (\lambda = 11.2 \text{ Rs./MWHr.}) \]

The increase in cost of operation by Unit 1 if it supplies 450 MW (equal sharing) instead of 400 MW (optimal sharing) is given by:

\[ C_1 = \int IC_1 \ dP_{G1} = \int (0.008P_{G1} + 8) \ dP_{G1} = |(0.004P_{G1}^2 + 8P_{G1})|_{450}^{400} = Rs.570/\text{hr.} \]

Similarly, the decrease in cost of operation by Unit 2 if it supplies 450 MW (equal sharing) instead of 500 MW (optimal sharing) is given by:
\[ C_2 = \int IC_{G2} \, dP_{G2} = \int (0.0096P_{G2}^2 + 6.4) \, dP_{G2} = |(0.0096P_{G2}^2 + 6.4P_{G2})|_{500}^{450} = Rs. -548/hr. \]

Thus, the net saving in cost in optimal operation is given by:

Rs. 570 – 548 = Rs. 22/- per hour or it is equivalent to an annual saving in cost of (assuming continuous operation): Rs.(22)(24)(365) = Rs. 1,92,720/- PA.

Example-5:

The fuel costs of a two generator system are given by:

\[ C_1 = \alpha_1 + \beta_1P_1 + \gamma_1P_1^2; \quad C_2 = \alpha_2 + \beta_2P_2 + \gamma_2P_2^2 \]

Where, \[ \beta_1 = 40, \quad \beta_2 = 30, \quad \gamma_1 = 0.1, \quad \gamma_2 = 0.125, \quad \text{and} \quad \alpha_1, \alpha_2 \text{ are constants}. \]

How will the load of 150 MW be shared optimally between the two units? Also determine the saving in cost in Rs./Hr. in optimal operation as compared to equal sharing of the same total load.

Solution:

Consider the incremental cost curves given by:

\[ IC_1 = dC_1/dP_1 = \beta_1 + \gamma_1P_1 = 40 + 0.2P_1 \]

Rs./MWHr \quad \[ IC_2 = dC_2/dP_2 = \beta_2 + \gamma_2P_2 = 30 + 0.25P_2 \text{ Rs./MWHr} \]

For a total load of \( P_T = 150 \) MW, if the load is shared equally among the two units then: \( P_{G1} = P_{G2} = 75 \) MW.

Now, for optimal operation, we have as per EIC principle, the IC’s to be equal. i.e., \( IC_1 = IC_2; \quad P_T = P_1 + P_2 = 150 \) are the equations to be solved for the output power values.

Thus, \( IC_1 = 40 + 0.2P_1 = IC_2 = 30 + 0.25P_2 = 30 + 0.25(150 - P_1) \)

Solving, we get,

\[ P_1 = 61.11 \text{ MW}; \quad P_2 = 88.89 \text{ MW} \quad (\lambda = 52.222 \text{ Rs./MWHr.}) \]

The increase in cost of operation by Unit 1 if it supplies 75 MW (equal sharing) instead of 61.11 MW (optimal sharing) is given by:
\[ C_1 = \int IC_1 \ dP_{G1} = \int (40 + 0.2P_1) \ dP_1 = \left[ 40P_1 + 0.1P_1^2 \right]_{61.11}^{75} = \text{Rs.} 737.344/\text{hr.} \]

Similarly, the decrease in cost of operation by Unit 2 if it supplies 75 MW (equal sharing) instead of 88.89 MW (optimal sharing) is given by:

\[ C_2 = \int IC_2 \ dP_{G2} = \int (30 + 0.25P_2) \ dP_2 = \left| 30P_2 + 0.125P_2^2 \right|_{88.89}^{75} = \text{Rs.} -707/\text{hr.} \]

Thus, the net saving in cost in optimal operation is given by:

\[ \text{Rs.} 737.344 - 707 = \text{Rs.} 30.344/- \text{ per hour} \]

(or it is equivalent to an annual saving in cost of (assuming continuous operation): Rs. \((30.3)(24)(365) = \text{Rs.} 2,62,800/- \text{ PA}\)

**Example-6:**
The fuel cost function in Rs./Hr. for three thermal plants is given by the following
(with \(P\)'s in MW):

\[
\begin{align*}
F_1 &= 350 + 7.20P_1 + 0.0040P_1^2 \\
F_2 &= 500 + 7.30P_2 + 0.0025P_2^2 \\
F_3 &= 600 + 6.74P_3 + 0.0030P_3^2
\end{align*}
\]

Find the optimal schedule for a total load of 450 MW. Also compute the costs of operation for this schedule. Compare the same when the three generators share the same total load equally among them.

**Solution:**
Consider the IC curves in Rs./MWhr for the 3 units as under:

\[
\begin{align*}
IC_1 &= dF_1/dP_1 = 7.2 + 0.008P_1 \ \text{Rs.}/\text{MWhr} \\
IC_2 &= dF_2/dP_2 = 7.3 + 0.005P_2 \ \text{Rs.}/\text{MWhr} \ \text{and} \\
&= 6.74 + \\
IC_3 &= dF_3/dP_3 = 0.006P_3 \ \text{Rs.}/\text{MWhr}
\end{align*}
\]
For optimal operation, we have as per EIC, the common lambda of the system given

\[
\lambda = \frac{P_D + \Sigma (b_i/2c_i)}{\Sigma (1/2c_i)} \quad i = 1, 2, 3
\]

Substituting the values, we get after simplification, \( \lambda = \text{Rs. 8/ MWhr.} \)

Using this value of common system lambda, the MW output values of all the 3 units are obtained from their IC curves as:

\[ P_1 = 100 \text{ MW, } P_2 = 140 \text{ MW and } P_3 = 210 \text{ MW.} \]

The operating costs for this schedule are found by using the cost curves as:

\[ F_{T\text{ (Optimal operation)}} = F_1 + F_2 + F_3 = 1110 + 1571 + 2147.7 = 4828 \text{ Rs./Hr.} \]

Similarly, the operating costs for the equal sharing of total load are also found by using the cost curves as: (with \( P_1 = P_2 = P_3 = 150 \text{ MW.} \))

\[ F_{T\text{ (Equal sharing)}} = F_1 + F_2 + F_3 = 1520 + 1621.25 + 1078.5 = 4849.5 \text{ Rs./Hr.} \]

Thus, saving in cost in optimal operation is: 4849.5 – 4828 = Rs.21.75/- per hour.

Example-7:
Given that \( IC_1 = (40 + 0.2P_1) \) ; \( IC_2 = (30 + 0.25P_2) \) Calculate and tabulate the load shared by each unit for optimal operation if the total load varies from 50 to 250MW, in steps of 50MW, given that max.MW is 125 and min. MW is 20 for both the units.

Solution:

**Stage 1:** Consider the IC’s at \( P_{min} \):

\[ IC_1|P_1=P_{1\text{min}}=20\text{MW} = 40 + 0.2(20) = 44 \text{ Rs./MWHr} \]

\[ IC_2|P_2=P_{2\text{min}}=20\text{MW} = 30 + 0.25(20) = 35 \text{ Rs./MWHr} \]

Thus, \( IC_2 < IC_1 \); i.e., the EIC holds good only from the stage where, the system lambda is equal to 44 Rs./MWHr. Now find \( P_2 \) corresponding to this Lambda:
\[ P_2 | \lambda_2 = 44 = (44 - 30)/0.25 = 56 \text{ MW} \] so that then \[ P_{\text{Total}} = 20 + 56 = 76 \text{ MW}. \]

Thus, until \( P_T = 76 \text{ MW} \), EIC will not be feasible, Unit 1 will work at its minimum load 20 MW and all the additional load is shared by unit 2 alone till \( \lambda = 44 \text{ Rs./MWHr} \).

**Stage 2:** Consider the IC’s at \( P_{\text{max}} \): since the Unit 2 is expected to reach its max. limit earlier, find:

- \[ \text{IC}_2 | p_2 = p_{2\text{max}} = 125 \text{ MW} = 30 + 0.25(125) = 61.25 \text{ Rs./MWHr} \]
- \[ p_1 | \lambda_1 = 61.25 = (61.25 - 40)/0.2 = 106.25 \text{ MW} \]

so that then

\[ P_{\text{Total}} = 106.25 + 125 = 231.25 \text{ MW} \]

Thus, after \( P_T = 231.25 \text{ MW} \), EIC ceases to hold good; Unit 2 will work at its maximum load sharing of 125 MW only and all the additional load variations are shared by unit 1 alone until \( P_1 \) also reaches 125 MW.

**Stage 3:** In summary, EIC holds good only for \( P_T \) and \( \lambda_{\text{system}} \) values which satisfy the

\[ 76 \leq P_T \leq 231.25 \text{ MW} \quad \text{and} \quad 44 \leq \lambda_{\text{system}} \leq 61.25 \text{ Rs./MWHr} \]

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>( P_T )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( \lambda_{\text{system}} )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>50</td>
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<td>30</td>
<td>44</td>
<td>Unit 2 only shares the additional load</td>
</tr>
<tr>
<td>2.</td>
<td>76</td>
<td>20</td>
<td>56</td>
<td>44</td>
<td>Unit 1 and Unit 2 share the total load as per EIC Criterion. The system works with a common system lambda</td>
</tr>
<tr>
<td>3.</td>
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<td>33.33</td>
<td>66.67</td>
<td>46.67</td>
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<td>4.</td>
<td>150</td>
<td>61.11</td>
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<td>52.22</td>
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<td>5.</td>
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<td>111.11</td>
<td>57.78</td>
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<td>6.</td>
<td>231.25</td>
<td>106.25</td>
<td>125</td>
<td>61.25</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>250</td>
<td>125</td>
<td>125</td>
<td>61.25</td>
<td>Unit 1 only shares the additional load</td>
</tr>
</tbody>
</table>
Example-8:

Given that \( F_1 = 110 + 30P_1 + 0.09P_1^2 \) \( 12 \leq P_1 \leq 125 \) MW
\( F_2 = 135 + 12P_2 + 0.1P_2^2 \) \( 2 \leq P_2 \leq 125 \) MW

For the total load values of \( P_T = 100, 150 \) and \( 200 \) MW, the equations to be solved for the output power values are: \( IC_1 = IC_2; P_T = P_1 + P_2 \) and \( \lambda_{system} \) is calculated using any one of the IC equations. The values so obtained for the said range of load values are tabulated as under.

Calculate and tabulate the load shared by each unit for optimal sharing of the total load in the range 50-250 MW in steps of 100 MW. Also find \( \lambda_{system} \) in each case.

Solution:

Consider the incremental cost curves given by:

\[ IC_1 = \frac{dC_1}{dP_1} = 30 + 0.18P_1 \text{ Rs./MWHr} \]
\[ IC_2 = \frac{dC_2}{dP_2} = 12 + 0.2P_2 \text{ Rs./MWHr} \]

**Stage 1:** Consider the IC’s at \( P_{min} \):

\[ IC_1|_{P1=P_{1min}} = 30 + 0.18P_1 \text{ Rs./MWHr} \]
\[ IC_2|_{P2=P_{2min}} = 12 + 0.2P_2 \text{ Rs./MWHr} \]

Thus, \( IC_2 < IC_1 \); i.e., the EIC holds good only from the stage where, the system lambda is equal to 32.16 Rs./MWHr. Now find \( P_2 \) corresponding to this Lambda:

\[ P_2|_{\lambda=32.16} = 100.8 \text{ MW} \] so that then \( P_{Total} = 112.8 \text{ MW} \)

Thus, until \( P_T = 112.8 \) MW, EIC will not be feasible, Unit 1 will work at its...
minimum load, 12 MW and the additional load is shared by unit 2 alone till 
\( \lambda = 32.16 \) Rs./MWHr.

**Stage 2:** Consider the IC’s at \( P_{\text{max}} \): since the Unit 2 is expected to reach its max. limit earlier, find:

\[
\text{IC2|P2=P2max} = 125\text{MW} = 37 \text{ Rs./MWHr}
\]

\[
P_{1|\lambda=37} = 38.88 \text{ MW} \quad \text{so that then}
\]

\[
P_{\text{Total}} = 163.88 \text{ MW}
\]

Thus, after \( P_T = 163.88 \) MW, EIC ceases to hold good; Unit 2 will work at its maximum load sharing of 125 MW only and all the additional load variations are shared by unit 1 alone until \( P_1 \) also reaches 125 MW.

**Stage 3:** In summary, EIC holds good only for \( P_T \) and \( \lambda_{\text{system}} \) values which satisfy the limits: \( 112.8 \leq P_T \leq 163.88 \) MW and \( 32.16 \leq \lambda_{\text{system}} \leq 37 \) Rs./MWHr.

For the total load value of \( P_T = 150 \) MW, where the EIC holds good, the equations to be solved for the output power values are: \( IC_1=IC_2; \ P_T = P_1+P_2 \) and \( \lambda_{\text{system}} \) is calculated using any one of the IC equations. The values so obtained for the said range of load values are tabulated as under.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>( P_T )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( \lambda_{\text{system}} )</th>
<th>Remarks</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
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<td>32.16</td>
<td>Unit 2 only shares the additional load</td>
</tr>
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<td>112.8</td>
<td>12</td>
<td>100.8</td>
<td>32.16</td>
<td>Unit 1 and Unit 2 share the total load as per EIC Criterion. The system works with a common system lambda</td>
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<tr>
<td>4.</td>
<td>163.88</td>
<td>38.88</td>
<td>125</td>
<td>37</td>
<td>Unit 1 only shares the additional load</td>
</tr>
<tr>
<td>5.</td>
<td>250</td>
<td>125</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Example-9:
A system is fed by two steam plants with IC functions as under:

\[ IC_1 = 28 + 0.16P_1 \text{ Rs./MWHr} \]
\[ IC_2 = 20 + 0.25P_2 \text{ Rs./MWHr} \]

The maximum and minimum loads on the units are 100 MW and 10 MW respectively. Determine the minimum cost of generation for supplying a load as follows based on the EIC criterion:

<table>
<thead>
<tr>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>120</td>
</tr>
<tr>
<td>175</td>
</tr>
<tr>
<td>50</td>
</tr>
</tbody>
</table>

Duration
- 6 am
- 12 noon
- 2 pm
- 6-9 pm
- 9 pm – 12 Midnight

Solution:

**Stage 1:** Consider the IC’s at \( P_{\text{min}} \):

\[ IC_1|P_1=P_{1\text{min}} = 10\text{ MW} = 29.6 \text{ Rs./MWHr} \]
\[ IC_2|P_2=P_{2\text{min}} = 10 \text{ MW} = 22.5 \text{ Rs./MWHr} \]

Thus, \( IC_2 < IC_1 \); i.e., the EIC holds good only from the stage where, the system lambda is equal to 29.6 Rs./MWHr. Now find \( P_2 \) corresponding to this Lambda:

\[ P_2|\lambda=29.6 = 38.4 \text{ MW} \] so that then \( P_{\text{Total}} = 48.4 \text{ MW} \)

However all the required loads to be supplied are above this total load of 48.4 MW!

**Stage 2:** Consider the IC’s at \( P_{\text{max}} \):

\[ IC_1|P_1=P_{1\text{max}} = 100\text{ MW} = 44 \text{ Rs./MWHr} \]
\[ IC_2|P_2=P_{2\text{max}} = 100\text{ MW} = 45 \text{ Rs./MWHr} \]

\[ P_2|\lambda=44 = 96 \text{ MW} \] so that then \( P_{\text{Total}} = 196 \text{ MW} \)
Again, it is observed that all the required loads to be supplied are below this total load of 196 MW!

In summary, EIC holds good for all the load values specified. Now for the total load value of $P_T = P_1 + P_2$, where EIC is shown to holds good, the equations to be solved for the output power values are: $IC_1 = IC_2$; $P_T = P_1 + P_2$ and $\lambda_{\text{system}}$ is calculated using any one of the IC equations. The values so obtained for the said range of load values are tabulated as under.

Example-10:
Assume that the fuel input in Btu/Hr. for unit 1 and unit 2 of a plant are given by:

\[ F_1 = (P_1 + 0.024P_1^2 + 80) \times 10^6 \]
\[ F_2 = (6P_2 + 0.04P_2^2 + 120) \times 10^6 \]

The maximum and minimum loads on the units are 100 MW and 10 MW respectively. Determine the minimum cost of generation for supplying a load as follows with the fuel cost at Rs.2 per MBtu.

Solution:

Consider the incremental cost curves given by:

\[ IC_1 = \frac{dC_1}{dP_1} = 2 + 0.096 P_1 \text{ Rs./MWHr} \]
\[ IC_2 = \frac{dC_2}{dP_2} = 12 + 0.16 P_2 \text{ Rs./MWHr} \]

Consider the IC’s at $P_{\text{min}}$:

\[ IC_1|_{P_1=P_1\text{min}=10\text{ MW}} = 2.96 \text{ Rs./MWHr} \]
\[ IC_2|_{P_2=P_2\text{min}=10\text{ MW}} = 13.6 \text{ Rs./MWHr} \]

Thus, $IC_2 > IC_1$; i.e., the EIC holds good only from the stage where, the system
lambda is equal to 13.6 Rs./MWHr. Now find $P_1$ corresponding to this Lambda:

$$P_1|_{\lambda=13.6} = 120.833 \text{ MW} (> P_1^{\text{max}})$$

Rs./MWHr. However, this is not further feasible since the unit 1 reaches its max. value of 100 MW within this range!!! Hence, EIC ceases to exist for any given

Solution:

Consider the IC curves in Rs./MWHr for the 3 units as under:

- $IC_1 = dH_1/dP_1 = (1.1) \left(7.2 + 0.000284P_1\right)$ Rs./MWHr
- $IC_2 = dH_2/dP_2 = (1.0) \left(7.85 + 0.00388P_2\right)$ Rs./MWHr
- $IC_3 = dH_3/dP_3 = (1.0) \left(7.97 + 0.00964P_3\right)$ Rs./MWHr

For optimal operation, we have as per EIC, the common lambda of the system given by:

$$\lambda = \frac{P_D + \sum (b_i/2c_i)}{\sum (1/2c_i)}$$

Substituting the values, we get after simplification, $\lambda = 9.148$ Rs./MWHr.

Using this value of common system lambda, the MW output values of all the 3 units are obtained from their IC curves as:

- $P_1 = 393.2$ MW, $P_2 = 334.6$ MW and $P_3 = 122.2$ MW.

(All the MW output values are within their capacity limits specified)

Example-12:

Three plants of total capacity 500 MW are scheduled for operation to supply a total load of 310 MW. Find the optimal load schedule if the IC curves and limitations are:

Solution:

For optimal operation, we have as per EIC, the $\lambda_{\text{system}}$ given by:

$$\lambda_{\text{system}} = \frac{P_D + \sum (b_i/2c_i)}{\sum (1/2c_i)}$$

Substituting the values, we get after simplification,
\[ \lambda_{\text{system}} = 42 \text{ Rs./MWHr.} \]

Using this value of \( \lambda_{\text{system}} \), the MW output values of all the 3 units are obtained from their IC curves as:

\[ P_1 = 100 \text{ MW}, \quad P_2 = 10 \text{ MW} \quad \text{and} \quad P_3 = 200 \text{ MW}. \]

Thus, \( P_2 < P_2^{\text{min}} \) (of 20 MW) and \( P_1, P_3 \) are within the limits.

In such cases, for optimal operation, we set \( P_2 = P_2^{\text{min}} = 20 \)
And hence the total load to be shared only between the unit 1 and unit 3 is: 310-10=290 MW;

Solution:

Consider the IC curves in Rs./MWHr for the 3 units as under:

\[
\begin{align*}
IC_1 &= \frac{dH_1}{dP_1} = (0.9) (7.2 + 0.000284P_1) \text{ Rs./MWHr} \\
IC_2 &= \frac{dH_2}{dP_2} = (1.0) (7.85 + 0.00388P_2) \text{ Rs./MWHr} \\
IC_3 &= \frac{dH_3}{dP_3} = (1.0) (7.97 + 0.00964P_3) \text{ Rs./MWHr}
\end{align*}
\]

For optimal operation, we have as per EIC, the common lambda of the system given by:

\[
\lambda = \frac{P_D + \Sigma (b_i/2c_i)}{\Sigma (1/2c_i)} 
\]

Substituting the values, we get after simplification, \( \lambda = 8.284 \text{ Rs./MWHr} \).

Using this value of common system lambda, the MW output values of all the 3 units are obtained from their IC curves as:

\( P_1 = 704.6 \text{ MW} \), \( P_2 = 111.8 \text{ MW} \) and \( P_3 = 32.6 \text{ MW} \).

Thus, \( P_1 > P_1^{\text{max}} \) (of 600 MW); \( P_3 < P_3^{\text{min}} \) (of 50 MW) and \( P_2 \) is within the limits.

In such cases, for optimal operation, we have:

\( P_1^{\text{max}} = 600 \) so that \( \lambda_1 = 8.016 \text{ Rs./MWHr} \);
\( P_3^{\text{min}} = 50 \) so that \( \lambda_3 = 8.458 \text{ Rs./MWHr} \) and thus
\( P_2 = P_D - 600 - 50 = 850 - 650 = 200 \), so that \( \lambda_2 = 8.626 \text{ Rs./MWHr} \).
Thus between units 2 and 3, the optimal operation may be feasible since $IC_3 < IC_2$.

For this, we solve the equations:

$$7.85 + 0.00388P_2 = 7.97 + 0.00964P_3 \quad \text{and} \quad P_T = P_2 + P_3 = 250.$$ 

The solution thus yields:

$$\lambda_2 = \lambda_3 = 8.576 \text{ Rs./MWHr.;}$$

$$P_2 = 187.13 \text{ MW}; \quad P_3 = 62.82 \text{ MW}$$

with $P_1 = P_1^{\text{max}} = 600 \text{ MW}$ (fixed).

Example-14:

If the total load at a certain hour of the day is 400 MW for a 3 unit system, obtain the optimum generation schedule, if the IC curves of the three units are as under (with IC’s in Rs./MWHr. and $P_G$’s in MW):

$$P_{G1} = -100 + 50 (IC_1) - 2 (IC_1)^2$$
$$P_{G2} = -150 + 60 (IC_2) - 2.5 (IC_2)^2$$
$$P_{G3} = -80 + 40 (IC_3) - 1.8 (IC_3)^2$$

Solution:

Consider the EIC condition:

$$IC_1 = IC_2 = IC_3 = \lambda_{\text{system}} \quad \text{and} \quad P_T = P_{G1} + P_{G2} + P_{G3} = 400$$

Thus,
\[
400 = [-100+50(IC)-2(IC)^2] + [-150+60(IC)-2.5(IC)^2] + [-80+40(IC)-1.8(IC)^2] \, \text{i.e., } 6.3(IC)^2-150(IC)+730=0;
\]

Solving we get two solutions: IC=6.82 and IC=16.989 Rs./MWhr., of which, the lower and economical value is considered for further analysis: With IC= 6.82 Rs./MWhr, we have, \( P_{G1} = 148 \text{ MW} \), \( P_{G2} = 142.9 \text{ MW} \) and \( P_{G3} = 109.1 \text{ MW} \).

In EPS, the first step is to properly assess the load requirement of a given area where electrical power is to be supplied. This power is to be supplied using the available units such as thermal, hydroelectric, nuclear, etc. Many factors are required to be considered while choosing a type of generation such as: kind of fuel available, fuel cost, availability of suitable sites for major station, nature of load to be supplied, etc.

**Variable load:** The load is not constant due to the varying demands at the different times of the day. The EPS is expected to supply reliable and quality power. It should ensure the continuity of power supply at all times. *Qn.: write a note on the choice of the number and size of the generating units at a power station from economic operation point of view*

**Single unit Vs. multiple units:** the use of a single unit to supply the complete load demand is not practical since, it would not be a reliable one. Alternately, a large number of smaller units can be used to fit the load curve as closely as possible. Again, with a large number of units, the operation and maintenance costs will increase. Further, the capital cost of large number of units of smaller size is more as compared to a small number of units of larger size. Thus, *there has to be compromise in the selection of size and number of generating units within a power plant or a station.*

*Electric Power Systems (EPS): Operational Aspects:*

Dept. of EEE, Vtusolution.in
Electric energy is generated at large power stations that are far away from the load centers. Large and long transmission lines (grid lines) wheel the generated power to the substations at load centers. Many electrical equipment are used for proper transmission and distribution of the generated power. The grid lines are such that:

- **GRID**: The transmission system of a given area.
- **Regional GRID**: Different grids are interconnected through transmission lines.
- **National GRID**: Interconnection of several regional grids through tie lines.

Each grid operates independently, although power can be exchanged between various grids.

**Economic loading of generators and interconnected stations:**

Optimum economic efficiency is achieved when all the generators which are running in parallel are loaded in such a way that the fuel cost of their power generation is the minimum. The units then share the load to minimize the overall cost of generation. This economical approach of catering to the load requirement is called as ‘economic dispatch’. The main factor in economic operation of power systems is the cost of generating the real power. In any EPS, the cost has two components as under:

- **The Fixed Costs**: Capital investment, interest charged on the money borrowed, tax paid, labour, salary, etc. which are independent of the load variations.

- **The Variable Costs**: which are dependant on the load on the generating units, the losses, daily load requirements, purchase or sale of power, etc.

The current discussion on economic operation of power systems is concerned about minimizing the variable costs only. Further, the factors affecting the operating cost of the generating units are: generator efficiency, transmission losses, fuel cost, etc. Of these, the fuel cost is the most important factor. Since a given power system is a mix of various types of generating units, such as hydel, thermal, nuclear, hydro-thermal, wind, etc., each type of unit contributes its share for the total operating cost. Since fuel cost is a predominating factor in thermal
(coal fired) plants, economic load dispatch (ELD) is considered usually for a
given set of thermal plants in the foregoing discussion.

PROBLEM OF ECONOMIC LOAD SCHEDULING:
There are two problem areas of operation strategy to obtain the economic
operation of power systems. They are: problem of economic scheduling and the
problem of optimal power flow.

* The problem of economic scheduling: This is again divided into two categories:
  • The unit commitment problem (UCP): Here, the objective is to determine the
    various generators to be in operation among the available ones in the system,
    satisfying the constraints, so that the total operating cost is the minimum. This
    problem is solved for specified time duration, usually a day in advance, based on
    the forecasted load for that time duration.
  • The economic load dispatch (ELD): Here, the objective is to determine the
    generation (MW power output) of each presently operating (committed or put
    on) units to meet the specified load demand (including the losses), such hat the
    total fuel cost s minimized

* The problem of optimal power flow: Here, it deals with delivering the real
  power to the load points with minimum loss. For this, the power flow in each line is to
  be optimized to minimize the system losses.

{Qn.: compare ELD and UCP and hence bring out their importance and
objectives.}

PERFORMANCE CURVES:

The Performance Curves useful for economic load dispatch studies include many
different types of input-output curves as under:
1. Input Output Curve: A plot of fuel input in Btu/Hr. as a function of the MW output of the unit.

2. Heat Rate Curve: A plot of heat rate in Btu/kWH, as a function of the MW output of the unit. Thus, it is the slope of the I-O curve at any point. The reciprocal of heat rate is termed as the ‘Fuel Efficiency’.

3. Incremental Fuel Rate Curve: A plot of incremental fuel rate (IFC) in Btu/kWH as a function of the MW output of the unit, where, 
\[ \text{IFC} = \frac{\Delta \text{input}}{\Delta \text{output}} = \text{Incremental change in fuel input/ Incremental change in power output} \]

4. Incremental Fuel Cost Curve: A plot of incremental fuel cost (IFC) in Rs./kWH as a function of the MW output of the unit, where,
\[ \text{IFC in Rs./kWH} = \text{(Incremental fuel rate in Btu/kWH) (Fuel cost in Rs./Btu)} \]
The IFC is a measure of how costlier it will be to produce an increment of power output by that unit.

The Cost Curve can be approximated by:
- Quadratic Curve by the function: 
  \[ C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \text{ Rs./Hr.} \]
- Linear curve by the function: 
  \[ \frac{d(C_i)}{dP_i} = b_i + 2c_i P_i \text{ Rs./MWHr.} \]

Generally, the quadratic curve is used widely to represent the cost curve, with the IC curve given by the linear curve as above.

**CONSTRAINTS IN ECONOMIC OPERATION OF POWER SYSTEMS:**
Various constraints are imposed on the problem of economic operation of power systems as listed below:

1. **Primary constraints (equality constraints):**
   - Power balance equations:
     \[ P_i - P_{Di} - P_l = 0; \ Q_i - Q_{Di} - Q_l = 0; \ i=\text{buses of the system} \]
     where, 
     \[ P_l = \sum \sum V_i V_j Y_{ij} \cos(\delta_{ij} - \theta_{ij}) \]
     \[ Q_l = \sum \sum V_i V_j Y_{ij} \sin(\delta_{ij} - \theta_{ij}); \ j = 1,2,\ldots,n, \text{are the power flow to the neighboring system.} \]
The above constraints arise due to the need for the system to balance the generation and load demand of the system.

2. Secondary constraints (inequality constraints):
These arise due to physical and operational limitations of the units and components.
\[\text{Pi min} \leq \text{Pi} \leq \text{Pimax}\]
\[\text{Qi min} \leq \text{Qi} \leq \text{Qimax}\]
i = 1, 2, …, n, the number of generating units in the system.

3. Spare Capacity Constraints:
These are used to account for the errors in load prediction, any sudden or fast change in load demand, inadvertent loss of scheduled generation, etc. Here, the total generation available at any time should be in excess of the total anticipated load demand and any system loss by an amount not less than a specified minimum spare capacity, PSP (called the Spinning Reserve) given by:
\[\text{PlG (Generation)} - \text{Pl (Losses)} + \text{PSP} + \text{PDj (Load)}\]

SPINNING RESERVE
Spinning reserve (SR) is the term used to describe the total amount of generation available from all the synchronized (spinning) units of the system minus the present load plus the losses being supplied. i.e.,
\[\text{Sp. Res., PSP} = \{\text{Total generation, } \sum \text{PlG}\} - \{\sum \text{PDj (load)} + \sum \text{Pl (losses)}\}\]

The SR must be made available in the system so that the loss of one or more units does not cause a large drop in system frequency. SR must be allocated to different units based on typical Council rules. One such rule is as follows:
‘SR must be capable of making up for the loss of the most heavily loaded unit in the system’ Reserves must be spread around the system to avoid the problem of
‘bottling of reserves’ and to allow for the various parts of the system to run as ‘islands’, whenever they become electrically disconnected.

{Qn.: Write a brief note on the following: Spinning Reserve, constraints in economic operation, performance curves}

SOLUTION TO ECONOMIC LOAD DISPATCH

{Qn.: Derive the EIC criterion for economic operation of power systems with transmission losses neglected, MW limits considered/ not considered} The solution to economic load dispatch problem is obtained as per the equal incremental cost criterion (EIC), which states that: ‘All the units must operate at the same incremental fuel cost for economic operation.” This EIC criterion can be derived as per LaGrangian multiplier method for different cases as under.

CASE (i) Solution to ELD without inequality constraints:
Consider a system with N generating units supplying a load PD MW. Let the unit MW limits and the transmission losses are negligible. Suppose the fuel cost of unit ‘i’ is given by:

\[ C_i(P_i) = a_i + b_iP_i + c_iP_i^2 \text{ Rs./Hr.} \]

so that

\[ \frac{d(C_i)}{dP_i} = b_i + 2c_iP_i \text{ Rs./MW.Hr.} \]

Hence, the total cost, \( C_T = \sum C_i(P_i) \) for \( i = 1,2,\ldots N \)

The ELD problem can thus be stated mathematically as follows:

<table>
<thead>
<tr>
<th>Minimize</th>
<th>( C_T = \sum C_i(P_i) ) for ( i = 1,2,\ldots N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Such that</td>
<td>( \sum P_i = P_D )</td>
</tr>
</tbody>
</table>

where, \( C_T \) is the total fuel cost of the system in Rs./Hr., \( P_D \) is the total demand in MW and \( P_i \) is the MW power output of unit i. The above optimization problem can be solved by LaGranje’s method as follows.

The LaGranje function \( L \) is given by:
\[ L = C_T + \lambda \left( P_D - \Sigma P_i \right) \]

The minimum cost value is obtained when:
\[ \frac{\partial L}{\partial P_i} = 0; \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = 0 \]
\[ \frac{\partial C_T}{\partial P_i} - \lambda = 0 \quad \text{and} \quad P_D - \Sigma P_i = 0 \quad (\text{w}) \]

Further, since the cost of a given unit depends only on its own power output, we have,
\[ \frac{\partial C_i}{\partial P_i} = \frac{\partial C_i}{\partial P_i} = \frac{dC_i}{dP_i} \quad i = 1, 2, \ldots, N \quad (18) \]

Thus,
\[ \frac{dC_i}{dP_i} - \lambda = 0 \quad i = 1, 2, \ldots, N \quad \text{or} \]
\[ \frac{dC_i}{dP_1} = \frac{dC_2}{dP_2} = \ldots = \frac{dC_i}{dP_i} = \ldots = \frac{dC_N}{dP_N} = \lambda, \text{ a common value of the incremental fuel cost of generator } I, \text{ in Rs./MWHr.} \]

The equation above is stated in words as under:

“For the optimum generation (power output) of the generating units, all the units must operate at equal incremental cost (EIC)”

**Expression for System Lambda, \( \lambda \):**

Consider equation (14):

\[ IC_i = \frac{d(C_i)}{dP_i} = \frac{b_i + 2c_i P_i}{\Sigma P_i} \quad \text{Rs./MWHr.} \]

Simplifying, we get,
\[ P_i = \frac{[\lambda - b_i]/2c_i}{\Sigma P_i} \quad \text{MW} \]

so that
\[ \Sigma P_i = P_D = \Sigma \frac{[\lambda - b_i]/2c_i}{\Sigma P_i} \quad (19) \]

Solving, we get,
\[ \lambda = \frac{P_D + \Sigma \left[ b_i/2c_i \right]}{\Sigma \left[ 1/2c_i \right]} \quad (20) \]

CASE (iv) Solution to ELD with Transmission losses considered- LOSS

COEFFICIENTS METHOD
Fig. 1. Branch currents in a 2 unit system

\[ I_k = I_{k1} + I_{k2} = N_{k1}I_1 + N_{k2}I_2 \]

Where, \( N_{k1} \) and \( N_{k2} \) (assumed to be real values) are the current distribution factors of units 1 and 2 respectively.

It is assumed that the currents \( I_{k1} \) and \( IL \) as well as \( I_{k2} \) and \( IL \) have the same phase angle or they have a zero phase shift. Thus, they can be added as real numbers as under.

Let

\[ I_1 = I_1^\sigma_1 = I_1\cos\sigma_1 + jI_1\sin\sigma_1 \]
\[ I_2 = I_2^\sigma_2 = I_2\cos\sigma_2 + jI_2\sin\sigma_2 \]

Where \( \sigma_1 \) and \( \sigma_2 \) are the phase angles of currents.

Consider now, the magnitude of current \( I_k \), in branch \( k \), given by

\[ I_k = (N_{k1}I_1\cos\sigma_1 + jN_{k1}I_1\sin\sigma_1) + (N_{k2}I_2\cos\sigma_2 + jN_{k2}I_2\sin\sigma_2) \]
\[ = (N_{k1}I_1\cos\sigma_1 + N_{k2}I_2\cos\sigma_2) + j(N_{k1}I_1\sin\sigma_1 + N_{k2}I_2\sin\sigma_2) \]

Thus,

\[ I_k^2 = N_{k1}I_1^2 + N_{k2}I_2^2 + 2N_{k1}N_{k2}I_1I_2\cos(\sigma_1-\sigma_2) \]

However, we have,

\[ P_1 = \bar{V}_1I_1\cos\sigma_1; \]
\[ P_2 = \bar{V}_2I_2\cos\sigma_2; \]
\[ P_L = S_3I_kR_k \]
where,
p1 and p2 are the mw power output values by the units 1 and 2 respectively,
v1 and v2 are the respective line voltages and θ1, θ2 are the respective power factor angles and PL is the transmission loss in the system. after simplification, an expression for the transmission loss as a function of plant generation can be obtained as:

\[
\text{PL} = \frac{SNk1 \cdot 2RkP1 \cdot 2(V1 \cdot 2\cos q1) + SNk2 \cdot 2RkP2 \cdot 2(V2 \cdot 2\cos q2) + 2\cdot SNk1Nk2RkP1P2 \cdot \cos (\theta_1 - \theta_2)}{(V1V2\cos q1\cos q2)}
\]

= B11P1 2 + B22P2 2 + 2 B12P1P2

Where, the B coefficients are called as the loss coefficients. Thus, in general, for a system of n units we have,

\[
\text{PL} = \sum \text{Pi} \cdot B_{ij} \cdot \text{Pj}
\]

Where,

\[
B_{ij} = \sum \frac{\cos(\delta_i - \delta_j)}{(V_iV_j\cos q_i\cos q_j)} \cdot N_{ki}N_{kj}R_k
\]

Note:

1. The B coefficients are represented in units of reciprocal MW, (MW-1)
2. For a three unit system, equation (32) takes the form:

\[
\text{PL} = B11P1^2 + B22P2^2 + B33P3^2 + 2 B12P1P2 + 2 B13P1P3 + 2 B23P2P3 = \text{PTBP}
\]

Where, \(P = [P1 \ P2 \ P3]\), the vector of unit power output values and
\[B = [B11 \ B12 \ B13; B21 \ B22 \ B23; B31 \ B32 \ B33]\]

the loss coefficient matrix for the 3 unit system.

3. The B coefficient matrix is a square, symmetric matrix of order n, n being the number of generating units present in the system.

4. The following are the assumptions made during the above analysis:

- All load currents maintain a constant ratio to load current (Nki=constant).
- The voltage at any bus remains constant.
- The power factor of each bus source is constant (qi=constant).
- The voltage phase angle at load buses is constant (θi=constant).

5. The Incremental Transmission loss, ITLi of a given unit can be expressed in
terms of its MW power output values as under:

Examples on Economic operation of power systems

**Part B: Transmission losses considered**

Example-1:

The costs of two units at the busses connected through a transmission line are (with \( P_1 \) and \( P_2 \) in MW): 
\[
IC_1 = 15 + 0.125 P_1; \quad IC_2 = 20 + 0.05 P_2
\]

If 125 MW is transmitted from unit-1 to the load at bus-2, at which the unit-2 is present, a line loss of 15.625 MW is incurred. Find the required generation for each of the units and the power received by the load when the system lambda is Rs.24.0 per MWHr. Use Penalty Factor method.

**Solution:**

With unit-2 not contributing to the line loss, it is due to the unit-1 alone, and hence,

\[
dP_1/dP_2 = ITL_2 = 0; \quad \text{where,} \quad P_l = B_{11} P_1^2; \quad \text{i.e.,} \quad B_{11} = P_l / P_1^2 = 15.625/125^2 = 10^{-3} \text{ MW}^{-1}
\]

Thus,

\[
P_L = 10^{-3} P_1^2 \quad \text{so that} \quad \frac{dP_L}{dP_1} = ITL_1 = 2(10^{-3})P_1 \quad \text{MW}
\]

Hence we have,

\[
IC_1 = 15 + 0.125 P_1 = \lambda (1 - ITL_1) = 24 \quad \{1 - 2(10^{-3})P_1 \}
\]

\[
IC_2 = 20 + 0.05 P_2 = \lambda (1 - ITL_2) = \lambda = \frac{24}{2}
\]

Solving, we get, \( P_1 = 52 \text{ MW} \) and \( P_2 = 80 \text{ MW} \).

Total loss = Total Generation – Total Load = \((P_1 + P_2) - P_l \text{load}\)

\[
= (52 + 80) - 15.625 \quad = 116.4 \text{ MW}.
\]

Example-3:

In a power system, the ICs of the two units are (with \( P_1 \) and \( P_2 \) in MW):

\[
IC_1 = 0.01 P_1 + 20; \quad IC_2 = 0.015 P_2 + 22.5 \text{ Rs./ MWHr.}
\]
The system is running under optimal scheduling with \( P_1 = P_2 = 100 \) MW. If the incremental transmission loss of unit 2 is 0.2, find the penalty factors and the incremental transmission loss of unit 1.

Solution:
For optimal power dispatch, we have, \( P_{\lambda A} = P_{\lambda B} = \lambda_{\text{System}} \)
i.e., \((0.01 P_1 + 20) P_{n1} = (0.015 P_2 + 22.5) P_{n2} \) where,
\[
P_{n2} = \{1 - \text{ITL}_2\}^{-1} = \{1 - 0.2\}^{-1} = 1.25 \text{ so that,}
\]
\[
[0.01 (100) + 20] P_{n1} = [0.015 (100) + 22.5] 1.25
\]
Solving, we get,
\[
P_{n1} = 1.4286 \text{ and}
\]
Since, \( P_{n1} = \{1 - \text{ITL}_1\}^{-1} \) we get, \( \text{ITL}_1 = 0.3. \)

Example-4:
A hydro plant and a steel plant are to supply a common load of 90 MW for a week (168 Hrs.), the unit characteristics are as under:
Hydro Plant: \( C_1 = 300 + 15 P_H \) acre feet/ Hr. \( 0 \leq P_H \leq 100 \) MW
Steam Plant: \( H_S = 53.25 + 11.27 P_S + 0.0213 P_S^2 \) \( 125 \geq P_S \geq 50 \) MW
If the Hydro plant is limited to 10000 MWH of energy usage, find the sharing of load during the period as a means of economic operation schedule.

Solution:
Since load=90 MW, total energy=90(168) = 15128 MWH
Energy to be supplied by steam plant, \( E_S = 15128 - 10000 = 5128 \) MWH. Now, the steam plant has the max. efficiency when \( (H_S = a + b P_S + c P_S^2) \); Power, \( P = \sqrt{a/c} = \sqrt{53.25/0.0213} = 50 \) MW.

Number of hours of operation of the steam plant at its maximum efficiency to supply

5128 MWH of energy is given by: \( \{5128/50\} = 102.4 \) Hrs.
Thus, the final operating schedule is as under:

Total Hours of operation = 168 Hours per week

Steam plant at 50 MW and

Hydro plant at 40 MW : both operate for 102.4 Hrs./week

Only hydro plant

at 90 MW: operates for remaining period of (168-102.4) = 65.6 Hrs./week.

Example-5:

Three plants A, B, C supply \( P_1 = 100 \) MW, \( P_2 = 200 \) MW and \( P_3 = 300 \) MW. Calculate the transmission loss in the network in p.u. and the incremental transmission losses in all the three units A, B and C, if the loss coefficient matrix of the system on 100 MVA

\[
B = \begin{bmatrix}
0.01 & -0.001 & 0.02 & -0.003 & -0.002 \\
0.001 & -0.002 & 0.003 & 0.03 \\
\end{bmatrix}
\]

Solution:

\[
P_L = P^T BP
\]

\[
= B_{11}P_1^2 + B_{22}P_2^2 + B_{33}P_3^2 + 2B_{12}P_1P_2 + 2B_{13}P_1P_3 + 2B_{23}P_2P_3
\]

\[
= 0.308 \text{ pu} = 30.8 \text{ MW}
\]

\[
\text{ITL}_1 = 2 \sum P_kB_{ik}
\]

i.e., \( \text{ITL}_1 = 2[P_1B_{11} + P_2B_{12} + P_3B_{13}] = 0.004 \text{ pu} = 0.4 \text{ MW} \)

i.e., \( \text{ITL}_2 = 2[P_1B_{21} + P_3B_{22} + P_3B_{23}] = 0.06 \text{ pu} = 6 \text{ MW} \)

i.e., \( \text{ITL}_3 = 2[P_1B_{31} + P_2B_{32} + P_3B_{33}] = 0.082 \text{ pu} = \)}
8.2 MW.

Example-6:

Two units are at two busses connected through a transmission line. If 100 MW is transmitted from plant 1 at bus-1 to the load at bus-2, a line loss of 10 MW is incurred. The IC curves of the units are, $IC_1=0.02P_1+16; IC_2=0.04P_2 +20$ Rs./MWHr. If $\lambda_{system} = 26$ Rs./MWHr., the no-load fuel costs are Rs.250/- and Rs.350/- per hour for unit-1 and unit-2 respectively, determine the following:

(xvii) The values of $P_1, P_2$ and the received load for optimal operation?

(xviii) The optimum values of $P_1, P_2$ for the above received load, if the system losses are accounted for but not coordinated.

(xix) Total fuel costs in Rs./Hr. for both the parts above.
Solution:

Consider: \( I_L = I_a + I_b; \) \( I_{L1} = I_a = 1 - j0.15 \) and \( I_{L2} = I_b = 0.5 - j0.1 \)

\[
\begin{align*}
I_{L1}/I_L &= 0.6649 \\
I_{L2}/I_L &= 1 - 0.6649 = 0.3351 \\
\end{align*}
\]

The current distribution factors are as follows:

\[
\begin{align*}
Na1 &= Na2 = 0.6649 \\
Nc1 &= -0.3351 \\
Nb1 &= Nb2 = 0.3351 \\
Nc2 &= 0.6649 \\
\end{align*}
\]

\[
\begin{align*}
V_1 &= 1.0 \quad 0^\circ; \quad V_2 = 1 + Ic \cdot Zc = 1.0176 \quad 2.76^\circ \\
IG_1 &= Ia - Ic = 0.8 - j0.1 = 0.8062 \quad -7.12^\circ \\
IG_2 &= Ib + Ic = 0.7 - j0.15 = 0.7159 \quad -12.09^\circ \\
\end{align*}
\]

Thus, \( \sigma_1 = -7.12^\circ; \ \sigma_2 = -12.09^\circ; \ \theta_1 = -7.12^\circ \) and \( \theta_2 = 14.85^\circ \)

So that the PF values are: \( \cos \theta_1 = 0.9923 \) and \( \cos \theta_2 = 0.9966 \)

Now, the loss coefficients are given by the expression:

\[
B_{ij} = \sum \cos(\sigma_i - \sigma_j) \cdot (V_iV_j\cos\theta_i\cos\theta_j) \cdot N_{ki}N_{kj}R_k
\]

So that

\[
B_{11} = \frac{\sum Na_1^2R_k/(V_1^2\cos^2\theta_1)}{1/(V_1^2\cos^2\theta_1)} \cdot \{N_a^2R_a^2+N_b^2R_b^2+N_c^2R_c^2\} = 0.01462 \quad \text{pu}
\]

\[
B_{22} = \frac{\sum Nk_2^2R_k/(V_2^2\cos^2\theta_2)}{1/(V_2^2\cos^2\theta_2)} \cdot \{N_a^2R_a^2+N_b^2R_b^2+N_c^2R_c^2\} = 0.2175 \quad \text{pu}
\]

and

\[
B_{12} = \frac{\sum Nk_1Nk_2R_k\cos(\sigma_1 - \sigma_2)}{(V_1V_2\cos\theta_1\cos\theta_2)} \cdot \{N_aN_bR_a^2+N_bN_a^2R_b^2+N_cN_eR_c^2\} = 0.0079 \quad \text{pu}
\]
Example-8:

Figure below shows a system with plant-1 and plant-2 connected to bus-1 and bus-2 respectively. There are two loads and 3 branches. The bus-1 is the reference bus with 1.0 \( \text{pu} \) voltage. Base MVA is 100. Calculate the loss formula coefficients in \( \text{pu} \) and in MW\(^{-1} \) units, if the branch currents and branch impedance values are as under:

\[
\begin{align*}
I_a &= 2.0 - j \ 0.50 \text{ pu} & Z_a &= 0.06 + j \ 0.24 \text{ pu} \\
I_b &= 1.6 - j \ 0.40 \text{ pu} & Z_b &= 0.03 + j \ 0.12 \text{ pu} \\
I_c &= 1.8 - j \ 0.45 \text{ pu} & Z_c &= 0.03 + j \ 0.12 \text{ pu}
\end{align*}
\]

Solution:

Consider:

\[
\frac{I_c}{(I_b + I_c)} = 0.5294 \quad \frac{I_b}{(I_b + I_c)} = (1-0.5294) = 0.4706
\]

The current distribution factors are as follows:

\[
\begin{align*}
N_a1 &= -0.5294; & N_b1 &= N_b2 = 0.4706 \\
N_a2 &= 0.4706; & N_c1 &= N_c2 = 0.5294
\end{align*}
\]

\[
\begin{align*}
V_1 &= 1.0 \ \text{pu}; & V_2 &= 1 + I_a Z_a = 1.319 \ 20^\circ \\
I_1 &= I_b - I_a = -0.4+j0.1 = I_1 = 166^\circ; & I_2 &= I_a + I_c = I_2 = -14^\circ
\end{align*}
\]

Thus, \( \sigma_1 = 166^\circ; \ \sigma_2 = -14^\circ; \ \theta_1 = 166^\circ \) and \( \theta_2 = 20+14 = 34^\circ \)

So that the PF values are: \( \cos \theta_1 = 0.97 \) and \( \cos \theta_2 = 0.829 \)

Now, the loss coefficients are given by the expression:

\[
B_{ij} = \Sigma \Sigma \frac{(\cos(\sigma_i - \sigma_j))/(V_i V_j \cos \theta_i \cos \theta_j)}{N_{ki} N_{kj} R_k}
\]

So that

\[
\begin{align*}
B_{11} &= \Sigma \Sigma N_{k1}^2 R_k/(V_1^2 \cos^2 \theta_1) = 0.0338 \text{ pu} = 0.3387(10^{-3}) \text{ MW}^{-1} \\
B_{22} &= \Sigma \Sigma N_{k2}^2 R_k/(V_2^2 \cos^2 \theta_2) = 0.0237 \text{ pu} = 0.237(10^{-3}) \text{ MW}^{-1} \quad \text{and} \\
B_{12} &= \Sigma \Sigma N_{k1} N_{k2} R_k \ \cos(\sigma_1 - \sigma_2)/(V_1 V_2 \cos \theta_1 \cos \theta_2) = 9.6073(10^{-5}) \text{ pu}
\end{align*}
\]
Example-9:

The following table gives the unit charges of a 3 unit system. Initially the unit 2 is ON for 4 hours. Min. up time is 3 hours for all the units. The priority schedule is 2-1-3. Find the unit commitment schedule.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Max. MW</th>
<th>Incr. Cost Rs./MW</th>
<th>No load cost Rs./Hr.</th>
<th>Start up cost Rs.</th>
<th>Load pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>75</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>75</td>
<td>100</td>
<td>100</td>
<td>1-2 200 3-4 500</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>5-6 200 7-8 200</td>
</tr>
</tbody>
</table>

Solution:

Based on the priority schedule we have the UC combination and the load supplied worked out as per the table below:

<table>
<thead>
<tr>
<th>Unit combination</th>
<th>A: 010</th>
<th>B: 110</th>
<th>C: 111</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. capacity in MW</td>
<td>200</td>
<td>400</td>
<td>500</td>
</tr>
</tbody>
</table>

Consider the distribution of load in the given pattern:

Stage a:
1-2 for 2 hours: 200 MW

It is combination-a with unit 2 only ON. Thus

Operating cost is \( 75(200)^2 + \text{start up cost of unit } 2 = 30000 + 100 = \text{Rs. 30100.00} \)

Stage b:
3-4 for 2 hours: 500 MW

It is combination-c with unit all the units ON. Thus

Operating cost is \( 75(200)^2 + 75(200)^2 + \text{start up cost of unit } 1 + 100(100)^2 + \text{start up cost of unit } 3 = 30000 + 30000 + 100 + 20000 + 200 = \text{Rs. 80300.00} \)

Stage c: 5-6 for 2 hours: 200 MW

It is combination-d with the unit-2 ON at full load and units 1 & 3 ON at no load.
for one hour (due to min. up time of 3 hours for unit-1 and unit-3) and with only
the unit-2 ON at full load for the next one hour. (Now, the unit-2 does not have
any start up cost component). Thus,
Operating cost is = \{75(200) + \text{no load cost of unit 1} + \text{no load cost of unit 3}\} +
75(200) = 15000 + 100 + 100 + 15000 = Rs. 30200.00

Stage d:
7-8 for 2 hours: 200 MW
It is combination-a again with unit 2 only ON. Thus
Operating cost is = 75(200)2 = 30000 = Rs. 30000.00 (there is no start up cost
now)

Thus, the total operating cost
\[= \text{Rs. 30100.00} + \text{Rs. 80300.00} + \text{Rs. 30200.00} + \text{Rs. 30000.00}\]
\[= \text{Rs. 1,70,600/-}\]

Example-10:
The unit charges of a 3 unit system are as under. Initially the unit 1 is ON for 4
hours. Min. up time is 2 hours for all the units. The priority schedule is 1-2-3.
Find the unit commitment schedule.

<table>
<thead>
<tr>
<th>Unit #</th>
<th>Max. MW</th>
<th>Incr. Cost Rs./MW</th>
<th>No load cost Rs./Hr.</th>
<th>Start up cost Rs.</th>
<th>Load pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>900</td>
<td>200</td>
<td>400</td>
<td>Hrs. MW</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1-2 200</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>950</td>
<td>150</td>
<td>100</td>
<td>3-4 300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5-6 400</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>1000</td>
<td>150</td>
<td>100</td>
<td>7-8 200</td>
</tr>
</tbody>
</table>

Solution:
Based on the priority schedule we have the UC combination and the load
supplied worked out as per the table below:

<table>
<thead>
<tr>
<th>Unit combination</th>
<th>A: 100</th>
<th>B: 110</th>
<th>C: 111</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. capacity in MW</td>
<td>200</td>
<td>300</td>
<td>400</td>
</tr>
</tbody>
</table>
Thus, the total operating cost

\[ \text{Total Operating Cost} = (\text{unit 1 for 2 hrs. (200 MW)} + \text{start up cost of unit-1}) + (\text{units 1 & 2 for 2 hours (200+100 MW)} + \text{start up cost of unit-2}) + (\text{units 1,2,3 for 2 hrs. (200+100+100 MW)} + \text{start up cost of unit-3}) + \text{(only unit 1 for the next 2 hrs. (200 MW))} \]

\[ = \text{Rs. 360400.00} + \text{Rs. 550100.00} + \text{Rs. 750100.00} + \text{Rs. 360000.00} \]

\[ = \text{Rs. 20,20,600/-} \]
UNIT -6


6 Hours

Introduction

The Power System needs to be operationally secure, i.e. with minimal probability of blackout and equipment damage. An important component of power system security is the system’s ability to withstand the effects of contingencies. A contingency is basically an outage of a generator, transformer and or line, and its effects are monitored with specified security limits. The power system operation is said to be normal when the power flows and the bus voltages are within acceptable limits despite changes in load or available generation. From this perspective, security is the probability of a power system’s operating point remaining in a viable state of operation. System security can be broken down into TWO major functions that are carried out in an operations control centre:

(i) Security assessment and (ii) security control.

The former gives the security level of the system operating state. The latter determines the appropriate security constrained scheduling required to optimally attaining the target security level. Before going into the static security level of a power system, let us analyze the different operating states of a power system. The states of power system are classified into FIVE states:

Normal
Alert
Emergency
Extreme Emergency and
Restorative.

Fig below depicts these states and the ways in which transition can occur from one state to another.
The operation of a power system is usually in a normal state. Voltages and the frequency of the system are within the normal range and no equipment is overloaded in this state. The system can also maintain stability during disturbances considered in the power system planning. The security of the power system is described by Thermal, voltage and stability limits. The system can also withstand any single contingency without violating any of the limits. The system transits into the emergency state if a disturbance occurs when the system is in the alert state. Many system variables are out of normal range or equipment loading exceeds short-term ratings in this state. The system is still complete. Emergency control actions, more powerful than the control actions related to alert state, can restore the system to alert state. The emergency control actions include fault clearing, excitation control, fast valving, generation tripping, generation run back-up, HVDC modulation, load curtailment, blocking of on-load tap changer of distribution system transformers and rescheduling of line flows at critical lines. The extreme emergency state is a result of the occurrence of an extreme disturbance or action of incorrect of ineffective
emergency control actions. The power system is in a state where cascading outages and shutdown of a major part of power system might happen. The system is in unstable state. The control actions needed in this state must be really powerful. Usually load shedding of the most unimportant loads and separation of the system into small independent parts are required.

- Factors Affecting Power System Security
- Contingency Analysis

- Illustrative Example
- Procedure
- Methods of Security Analysis

- Study with fast but approximate algorithms
- Detailed analysis with AC power flow (Ex: NR Method)
- Contingency Selection and Detailed analysis Introduction
- System Security involves Procedures involved which ensure that the Power System can operate when components fail.

For Example:
- Maintaining Adequate Spinning Reserve ensures that when a generator fails, the remaining units can take up the deficit load without too much of a drop in frequency
- Taking care of Transmission Line Flows When generation is dispatched ensures that, when a Line is taken out, the remaining lines do not get overloaded

- The time at which failures occur are unpredictable
- Any initiating event should not result in Cascading Failures leading to Blackout
- Major Functions of System Security
  - System Monitoring
  - Contingency Analysis
  - Security Constrained Optimal Power Flow (SCOPF) System Monitoring
• Provides up-to-date information
• Telemetry Systems: Critical Quantities are measured and transmitted to a central location
• Measured Quantities
  – V, I, Line Flows, f, Unit Outputs, C.B Status, Transformer Taps etc

• Measured data is very large
• Difficult for human operators to process
• Digital Computers at control centres
  – Gathers Telemetered Data
  – Process this Data
  – Stores the Data in a Data Base
  – Generate Alarms
  – Operator can display this data
  – Combines data with system models for on-line and offline analysis
  – Telemetering is combined with Supervisory Control to provide operators with a capability to control components such as switches etc (SCADA) Contingency Analysis

• Power System Problems due to events such as Transmission Line and Generator Outages etc, can cause serious damage within very short duration
• Contingency Analysis Programs
  – Model the events before they occur
  – Combined with standard analysis procedures such as Power Flow etc to study the system
  – Generate Operator Alarms

• Other Features
  – Fast Solution Methods
  – Automatic Contingency Selection
Automatic Initialization of Contingency Power Flows using Actual System Data and State Estimation Procedures

Security Constrained Optimal Power Flow (SCOPF)
- Third Major Security Function
- Contingency Analysis Combined with OPF
- Four States of Power System

Optimal Dispatch (Figure A)
- State Prior to a Contingency
- Optimal w.r.t Economic Operation
- May not be secure
- Post Contingency (Figure B)

State after contingency has occurred
A Security Violation may have occurred
Ex: Tn Line Flow or Bus Voltage outside Limit

Secure Dispatch (Figure C)

State with no contingency outages
Corrections to dispatch parameters taking security violations into account

Secure Post Contingency (Figure D)

State when contingency occurs at the base operating condition- with corrections

Summary of Contingency Analysis
- Line and Generator outages may result in line flow and Bus Voltage limit violations
- The way in which lost generation is distributed among remaining units is important
• Contingency Analysis procedures are used to predict effects of outages
• Lost Generation is picked up by units in its own area as well as neighboring areas
• Contingency analysis procedures Model
• Single Event outages (Ex: 1 Line, 1 Generator out)
• Multiple Event Outages (2 Line, 2 Generators)
• All Credible Outages are Studied one after another
• The C.A procedure tests lines and voltages against their respective limits
• Difficulties
  • Speed of Solution
  • Selection of All Credible outages
  • Changed system conditions over time
Overview of Security Analysis

- Thousands of outages may have to be studied before they occur
• A Security Analysis procedure run at an Energy Control Centre must be very fast
• Three Techniques commonly used
  – Study the Power System with Approximate but fast Algorithms (DC Power Flow Methods, Linear Sensitivity factors)
  – Select only important cases for detailed analysis (Contingency Selection)
  – Use Multiple Processors or Vector Processors: running cases in parallel (Still in research stage)
UNIT 7
SYSTEM MONITORING AND CONTROL: Introduction, Energy management system, the basis of power system state estimation (PSSE), mathematical description of PSSE process, minimization technique for PSSE, Least Square estimation, Error and detection in PSSE, System security and emergency control.

6 Hours

Power system state estimation (PSSE) has been traditionally performed at regional control centers with limited interaction. However, due to the deregulation of energy markets, large amounts of power are transferred over high-rate, long-distance lines spanning several control areas [11]. These so-called tie lines, originally constructed for emergency situations, are now fully operational and must be accurately monitored. Since any control area can be strongly affected by events and decisions elsewhere, independent system operators (ISOs) can no longer operate in a truly independent fashion. The ongoing penetration of renewable sources further intensifies inter-area power transfers, while their intermittent nature necessitates more frequent state acquisition. At the same time, the advances in metering infrastructure are unprecedented: phasor measurement units (PMUs) provide finely-sampled voltage and current phasors, synchronized across the grid; smart meters reach the distribution level; and networked processors are being installed throughout the grid. The abundance and diversity of measurements offer advanced monitoring capabilities, but processing them constitutes a major challenge, which is exacerbated in the presence of malicious data attacks and bad data. There are two key issues in modernizing the power grid monitoring infrastructure: Firstly, PSSE should be performed at the interconnection level. Yet an interconnection may include thousands of buses, while 2-3 measurements per state are typically needed. Requiring also real-time processing along with resilience to corrupted data render centralized state estimation computationally formidable. Further, a centralized approach is vulnerable and is not flexible when it comes to policy and privacy issues. Secondly, decentralizing information processing for the power grid can be performed at several hierarchies. PMU measurements can be processed by phasor data concentrators (PDCs) [26]; conventional supervisory control and data acquisition (SCADA) measurements together with PDC fused data can be aggregated by the ISO; and finally, estimates from ISOs can be merged at the interconnection level. These considerations corroborate that distributed PSSE and bad data analysis are essential for realizing the smart grid vision.
Existing distributed methods for PSSE and bad data analysis are reviewed in Section II. The
PSSE problem, its unique requirements and challenges are highlighted in Section III. In
Section IV, a new distributed PSSE methodology is developed. Based on the alternating
direction method of multipliers a systematic cooperation between local control centers is
enabled with unique features: it facilitates several practical PSSE formulations; it lowers the
overhead for inter-area information exchanges; its convergence is guaranteed regardless of
local observability or parameter tuning; and the resultant algorithm can be executed by solvers
already in use at local control centers. Building on this framework, a robust decentralized
estimator is derived in Section V. Different from the conventional two-step bad data analysis,
the novel approach implements Huber’s M-estimator in a decentralized manner, while PSSE is
accomplished jointly with bad data removal. Leveraging sparsity of the introduced bad data
vectors, the new algorithm augments standard PSSE solvers by a few iterations. The novel
robust decentralized algorithms are numerically evaluated in Section VI, and the paper is
wrapped up in Section VII. Regarding notation, lower- (upper-) case boldface letters denote
column vectors (matrices), and calligraphic letters stand for sets.

State estimation

The tool used for power system monitoring and which is central for all subsequent grid
analysis is the state estimator. The objective of state estimation is to estimate the voltage
phasor at each bus of the monitored electrical grid so that it is as consistent as possible with the
measurements made on the grid. An important aspect of the problem is the redundancy,
meaning that there are more measured variables than state variables to estimate. The network
equations are given in 1 and give the relations between the measurement vector \( z \) and the state
vector \( x \)

\[
z = h(x) + e; \quad (1)
\]

where \( e \) is the measurement noise vector [2, 5]. Because of the redundancy, there are more
measurements than state variables to estimate so the system is over determined.

This is why a statistical method must be used to get an estimation of the state variables. The
**Weighted Least Squares (WLS)** method is used for power system state estimation. The
method ensures that the performance index or sum of squared residuals $J$ is minimized. $J(x) = (z - h(x))^T R^{-1} (z - h(x)); (2)$

where $R$ is the covariance matrix which gives the variance of each measurement. The algorithm used to solve this equation depends on the type of measurement. Minimization of $J$ gives the equation below:

$$l(x) = @J(x)/@x = H^T (x) R^{-1} (z - h(x)) = 0; (3)$$

where $H = @h(x)/@x$ is the Jacobian matrix of $h$ with respect to the state variables. Resolution of equation 3 depends on the linearity of $h(x)$, which depends on the measurements used.

The measurement residual $r$ can be computed as

$$r = z - h(bx); (4)$$

where $bx$ is the estimated state vector, and thus represents the difference between the actual measurement and the estimated measurement. The measurement residual $r$ can also be expressed as $r = S e; (5)$

where $S$ is the residual sensitivity matrix and represents the sensitivity of the measurement residuals to the measurement errors and $e$ is the measurement noise vector. The sensitivity matrix $S$ can be calculated as $S = I K; (6)$

where $I$ is the identity matrix. The residual sensitivity matrix $S$ has certain properties $S$ is not symmetric unless the variances associated with all measurement types are equal. $S: S = S: R; S: R; S: T = S: R$

Using equation (5), the covariance of the measurement errors called the residual covariance matrix can be determined $= SR; (7)$

Measurements provided by the SCADA system are the voltage, active or reactive line power and active or reactive injected power, while PMUs will provide voltage phasors at the bus and current phasors owing in the transmission line. The $h(x)$ functions between power measurements and state variables are non-linear so the relations must be linearized during the resolution process and converge to the solution after some iterations. In the case of phasor measurements, all $h(x)$ functions are linear, so the computation time needed by the state estimator is shorter when only phasor measurements are used. It can be shown that when the input of a state estimator includes measurements to which a normalized noise has been added then indices $J$ and $J_t$ can be evaluated statistically [5].
E(J) = m n; (8)  
E(Jt) = n (9)  

where m is the number of measurements available, n is the number of state variables to estimate, J and Jt are the performance indices. The sum of the squared residuals is shown to have distribution. This can be deduced from the property of a sum of squared random variables, which have a normal distribution. The redundancy is particularly important for bad data detection and elimination.

UNIT-8
POWER SYSTEM RELIABILITY: Introduction, Modes of failures of a system, Generating system and its performance, derivation of reliability index, reliability measure for N-unit system, cumulative probability outages-Recursive Relation, Loss of load probability, Frequency and duration of a state.

8 Hours

Reliability Indices

This analysis relies on two general classes of information to estimate the reliability: component reliability parameters and system structure. Using system structure and component performance data, we can evaluate the reliability of specific load points or the whole distribution system. The structure information is achieved by the circuit traces presented previously.

In the following paragraphs the performance data is discussed. Predictive reliability techniques suffer from data collection difficulties. Simplifying assumptions (default values) are required for practical analysis of distribution systems.

8.1. Functional characterization

The availability of component functionally is characterized by the following indices:

• Annual Failure Rate = the annual average frequency of failure
• Annual Down Time = the annual outage duration experienced at a load point.

These indices are computed for each segment in the feeder. All load points within a segment experience the same failure rate and down time.
In the reliability analysis program, failure rates and repair times from field data are preferred. When this data is not available, default values are fetched from a table in the relational database which has generic average failure rates and repair times for each type of device.

8.2. Reliability Indices Calculation
After finding the reliability analysis sets for the segment of interest $S$, we can calculate the reliability indices. First assume there is a single failure incident.

**Probability of failure**
- Chance that a component will fail
- Probabilistic value with no unit
- May be difficult to interpret

**Frequency of failure (failure rate)**
- In terms of number of failure within specified time
- Easier to predict from history
- Express in per hour, per day, per year

**Reliability** is a measure of the ability of the power system to deliver electricity to all points of utilization within accepted standards and in the amount desired, for the period of time intended, under the operating conditions intended.

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RELIABILITY

Adequacy  SECURITY

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**Adequacy** : relates to the existence of sufficient facilities within the system to satisfy the consumer load demand at all times; taking into account scheduled/ unscheduled outages
- assessed using the power flow (AC/DC) solutions.

**Security** : ability of the electric systems to respond to sudden disturbances arising within that system, such as electric short circuits
- assessed using dynamic calculation.
Power Systems Reliability Indexes

**Deterministic indexes**
- Do not take into account the uncertainties that affect reliability
- Simple calculation and require less data
- Percentage reserve
- Reserve margin as the largest unit online

**Probabilistic indexes**
- Reflect uncertainties in the system
- Loss of load probability (LOLP)
- Probability that generation will not meet demand in a year
- Loss of load frequency (LOLF)
- How often does the system fail in a year
- Expected energy not supplied (EENS)

**Deterministic criteria**
N-m contingency analysis System with ‘N’ components should be able to serve peak load when loss ‘m’ components Sometimes called security analysis

**Probabilistic criteria**
Loss of load expectation, for example, 1 day in 10 years